DIRECT CURRENTS

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C. E. MAGNUSSON

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DIRECT CURRENTS

BY

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First Edition

McGRAW-HILL BOOK COMPANY, Inc.
NEW YORK: 370 SEVENTH AVENUE
LONDON: 6 & 8 BOUVERIE ST., E. C. 4
1929

PRINTED IN THE UNITED STATES OF AMERICA

THE MAPLE PRESS COMPANY, YORK, PA.

PREFACE

This text book presents in logical order the basic principles of the electric circuit and the characteristics of direct-current electric machines, apparatus, and distribution systems. Combined with the companion volumes on "Alternating Currents" and "Electric Transients," it forms a series or set of electrical engineering texts, covering the fundamental laws of electrical phenomena as applied to practical electrical engineering problems.

Material for the book has been gathered from all available sources, as is necessarily the case in the preparation of a text in so well-established field. No attempt is made to give references or cite sources of material used except for Rules and Standards and the photographs and data so kindly furnished by manufacturing companies for illustrations.

Grateful acknowledgment is extended to Prof. G. R. Shuck, Prof. G. L. Hoard, Mr. M. T. Crawford, Prof. A. V. Eastman, and Mr. L. B. Robinson for helpful suggestions. The author desires particularly to express appreciation and extend sincere thanks to Prof. R. E. Lindblom for his constructive criticisms, for checking equations and quantitative data, and for many valuable suggestions.

C. Edward Magnusson

University of Washington, Seattle,
August, 1929

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DIRECT CURRENTS

CHAPTER I

INTRODUCTION

Electrical engineering deals with the generation, transmission, and distribution of electric energy and its application to a great variety of practical purposes in several extensive and widely separated fields. It deals with the design, manufacture, sale, installation, operation, and maintenance of electric machines, equipment, and appliances and with innumerable related problems. In all branches and phases of electrical engineering, however, the central and coordinating theme is *electric energy* or, more specifically the *laws of the electric circuit.*

To gain insight into any division or phase of electrical engineering it is of prime importance to first acquire clear concepts of the basic laws of the electric circuit. The term, "electric circuit," is here used in its broad or general meaning and, therefore, includes magnetic and dielectric circuits and fields as coexistent and essential phases of the currents flowing in the electric circuit. In a limited sense electric currents merely transfer electric energy from one place to another in the electric circuit, but it should be kept in mind that in the surrounding space and coexistent with the electric current are magnetic and dielectric fields; in which, energy is stored as the current increases and returned to the electric circuit proper when the current decreases. This interaction of the magnetic and dielectric fields with electric currents in storing and automatically returning energy is a factor of great importance both in the theory and application of electric energy. Thus, it is mainly on the basis of energy storage and the interaction of the magnetic and dielectric fields with electric circuits that the basic principles, as well as the practical applications, of electric energy are generally grouped into three divisions:

1. Direct currents.
2. Alternating currents.
3. Electric transients.

These divisions are not "water-tight compartments" but overlap and interact so that the purpose of the grouping is largely to emphasize certain specific characteristics of the electric circuit in each case. For example, alternating currents flow in the armatures of direct-current generators and direct currents are used in the field circuits of alternators; but the output, the purpose for which the machine is designed, is *direct currents* in the one and *alternating currents* in the other. Experience has shown that in the study of electrical phenomena progress can be made to best advantage by first mastering the laws of direct currents and their application in the design and operation of electrical machinery and then undertaking a like study of alternating currents, followed by electric transients. A brief statement of the main theme in each division follows.

Electric Transients.—Electric transients relate to phenomena occurring when the voltages and currents in electric circuits change rapidly in magnitude. Transient electric phenomena, as the term implies, are usually of short duration and relate to what occurs in an electric circuit between periods of stable conditions. Any disturbance, even a small change in load or voltage, causes a readjustment of the energy content of the electric circuit or system of circuits and hence produces transient conditions. Starting or stopping of generators and motors, sudden changes in load, switching surges, arc-overs, arcing grounds, lightning disturbances and similar sudden changes or impulses produce electric transient phenomena in electric circuits.

Alternating Currents.—In alternating-current circuits the voltage and current change in direction many times each second. Fifty, one hundred or one hundred and twenty reversals per second, or as usually expressed, 25, 50, or 60 cycles per second, are considered standard frequencies for alternating-current power systems. Much higher frequencies or number of reversals per second are used in telephony and radio.

The increase, decrease and reversal in direction of the current causes corresponding changes in the magnetic field produced by the current. The rapid changes in the energy stored in the magnetic field and the induction produced by the changes in the magnetic and dielectric fields greatly complicate conditions in the electric circuit, as well as the equations required for making quantitative computations.

Direct Currents.—In direct-current circuits the impressed voltages and the flow of the currents are continuously in one direction. The laws for direct-current circuits as usually stated also require that the voltage and current for any particular case shall be constant in magnitude. The equations representing the laws of direct-current circuits apply, quantitatively, only under constant or steady conditions. In obtaining test data on direct-current machinery and power circuits care must be exercised in having conditions steady; that is, constant voltage, current, speed, etc., while taking each set of observations. It is evident that under the specified unidirectional, constant-current conditions there will be no change in the magnetic and dielectric fields, and therefore no change in the magnetically and dielectrically stored energy. Moreover, with constant magnetic and dielectric fields, no inductive effects will be produced. The laws for direct-current circuits, therefore, are simpler than the corresponding equations for alternating currents. For this reason it is advisable to first gain a firm grasp of the laws of direct currents and their application to meet industrial requirements, and let this be followed by a study of the more complex laws of alternating currents and electric transients.

In this book the essential characteristics of direct currents are discussed in a systematic manner as outlined in the table of contents. As an introduction to the more detailed analysis of the several factors or phases that enter into direct-current systems, preliminary general statements of five basic principles of the electric circuit are made in this chapter. Students for whom this book is intended will have had courses in Physics dealing with the fundamentals of the electric circuit so that these statements are not new but more in the nature of a brief review. Therefore attention is directed to the following five important basic concepts relating to electric circuits:

(*a*) Principle of the conservation of energy.
(*b*) The electric field.
(*c*) Electro-magnetic induction.
(*d*) Ohm's law.
(*e*) Kirchhoff's laws.

(*a*) *The Principle of the Conservation of Energy.*—The principle of the conservation of energy is the foundation or basic assumption on which the superstructure of physics and engineering rests. Energy is considered to be a physical entity that can be trans-

formed into several forms, as heat, light, electricity, chemical reactions, mechanical work, etc. During all changes and transformations the total amount of energy remains unchanged; that is, energy can neither be created nor destroyed; all its manifestations merely indicate a change in form or location. Energy is measured in well-defined units in its several forms. Thus for heat: gram-calories, British thermal units, etc.; for mechanical energy: foot-pounds, dyne-centimeters, kilogram-meters, etc.; for electrical energy: joules, watt-hours, kilowatt-hours, etc. The numerical ratio between the several units, as foot-pounds and kilowatt-hours, have been determined to a high degree of accuracy.

It is of prime importance that the study of the electric circuit and its application to electrical machinery and appliances should be made *from the energy point of view.* On the basis of the principle of the conservation of energy, consider the energy relations in a belt-driven electric generator. The machine receives from the belt a certain amount of mechanical energy and at the same time delivers a somewhat smaller amount of electric energy to the load in the external electric circuit. The difference in the mechanical energy received and the electrical energy delivered is dissipated by friction, windage and electrical losses inside of the machine, which appear as heat. That is, all of the energy involved in the transformation produced by the generator is accounted for; no part is destroyed.

For the generator:

Mechanical energy received = electrical energy delivered
+ energy losses in the generator (friction, windage and heat due to electrical losses).

Similarly, in the reverse process of changing electric energy to mechanical work in the motor, the amount of mechanical energy delivered by the motor is less than the electric energy received from the power line, by the amount dissipated in the machine due to friction, windage, and heat developed.

For the motor:

Electric energy received = mechanical energy delivered
+ energy losses in the motor (friction, windage, and heat due to electrical losses).

The applications of the principle of the conservation of energy are innumerable in all engineering fields; in fact, the physical universe may be considered as consisting of energy changes in time and space.

(*b*) *The Electric Field. Lines of Force.*—Electric currents flow in conductors forming closed circuits. Around the currents magnetic fields are formed which in magnitude and direction depend in each case on the magnitude and direction of the current in the circuit. Likewise, a dielectric field exists between any parts of the circuit for which there is a difference of potential. The combined effect of the magnetic and dielectric properties of the space surrounding the electric circuit is known as the *electric field.* The existence and properties of the electric field are visualized by means of *magnetic and dielectric lines of force.* These lines of force are endowed with certain definite properties and, hence, by means of a set or system of appropriate lines of force a graphical representation may be made of the physical properties of any specific electric field. Thus the direction of the magnetic lines of force is definitely related to the direction of current flow in the circuit; the density of the number of lines is proportional to the field intensity; lines in the same direction repel; lines of opposite direction attract; moreover, the magnetic lines themselves form closed circuits. In general, for currents in straight wire conductors the intensity of the magnetic field is greatest at the surface of the conductor and varies inversely as the distance from the center.

(*c*) *Electromagnetic Induction.*—Electromagnetic induction, or the generation of voltage magnetically, was discovered by Michael Faraday in 1831 and independently by Joseph Henry. Faraday found that if a conductor moved so as to cut the magnetic lines of force in an electric field, *voltage would be generated in the conductor, in magnitude directly proportional to the time rate of cutting lines of force.* If the conductor is in the form of a closed circuit, the voltage generated by electromagnetic induction will cause a current to flow in the circuit.

The importance of Henry's or Faraday's discovery is beyond measure; it may well be compared to Columbus' discovery of America. All generators, motors, and transformers, as well as most other electric apparatus and appliances, are primarily based on the simple relation that voltage generated or induced by

electromagnetic induction is directly proportional to the time rate of cutting magnetic lines of force.

(*d*) *Ohm's Law.*—In the electric circuit the electromotive force or voltage is the factor or propelling force that causes the electric current to flow in the circuit. Just as friction opposes or retards the flow of water in a pipe, so the flow of the electric current is limited by the resistance of the conductor in the electric circuit. The quantitative relation between the electromotive force, resistance and current in electric circuits was discovered by Ohm in 1827. For direct currents and in a simple circuit Ohm's law states: *The current is directly proportional to the impressed voltage and inversely proportional to the resistance,* as expressed by equation (1).

$$\mathrm{I(current)} \propto \frac{E\ (voltage)}{R\ (resistance)} \tag{1}$$

Ohm's law may also be written as in equation (2); that is, the

$$E \propto RI \tag{2}$$

That is, the voltage is directly proportional to the product of the resistance and the current.

(*e*) *Kirchhoff's Laws.*—Kirchhoff established two laws relating to the currents and voltages, respectively, in electric circuits.

First Law:

The sum of all the currents flowing at any point in an electric circuit is zero.

$$\Sigma I = 0.$$

Second Law:

The sum of all the electromotive forces in an electric circuit equals zero.

$$\Sigma E = 0.$$

Like Ohm's law, Kirchhoff's laws are of fundamental importance and may be applied to all types and forms of electric circuits. A full discussion of Kirchhoff's laws and their application to typical electric circuits is found in Chap. II. The laws are stated here to direct attention to their great importance in the solution of electric-circuit problems.

Other laws of the electric circuit, as Joule's law, Lenz's law, reaction of electric current and magnetic field, the power equation, energy storage in magnetic and dielectric fields, etc., are likewise basic but not so elemental in establishing the nature of the electric circuit as the above stated concepts or principles.

CHAPTER II

THE ELECTRIC CIRCUIT

Electrons. Electronic Charge.—In 1897 J. J. Thomson published the results of his epoch-making investigations on the nature of electric conduction through gases. His experiments proved that very small particles of matter carrying electric charges formed the basic mechanism in electric conduction. Each of these minute particles, at first called "corpuscles" but now known as *electrons*, has a definite mass, equivalent to 1/1,848 that of the hydrogen atom, the smallest division of matter known before these investigations were made. The extreme minuteness of these particles arouses interest, but what is of much greater importance, the electrons possess, or are carried by, definite charges of electricity. These fundamental discoveries led to very many extensive investigations on the properties of electrons and their relation to electrical phenomena and the constitution of matter. In all the widely varied conditions under which electrons have been studied, the electric charges have been found to be of unvarying magnitude; that is, a definite unit or constant quantum of negative electricity is always associated with each electron. This elemental quantity of electricity, called the *electronic charge*, is a natural unit of fundamental importance in the study of electrical phenomena.

The concept of electricity as minute, discreet, elemental units or electronic charges forms the basis of the *electron theory* which is generally accepted as the most serviceable system yet developed for coordinating present knowledge of electrical phenomena. To the electrons, or more properly to the negative electronic charges, with the corresponding positive charges on the protons, or positive nuclei, are ascribed the properties of electric fields, (the space surrounding magnets, electric charges and electric currents) considered as innate characteristics of each elemental unit. Electrons at rest produce electrostatic phenomena while electrodynamic effects result from electrons or ions in motion.

Electric Current.—On the basis of the electron theory, electric currents consist of, or are formed by, the drift or flow of electrons and protons or ions through a conductor. The flow of an electric current in a copper wire may be compared to the flow of water in a pipe. In the comparison the electrons would correspond to the water molecules. Metals and many other materials that offer little resistance to electric currents flowing through them are termed *conductors*. Other substances, like glass, rubber or air, are called *insulators* and offer such high resistance that very few electrons will pass through even thin layers. The ratio of the conductivity (ability to conduct electric currents) of a good conductor like copper to that of a good insulator like rubber is exceedingly large—in the order of 10^{22}—and for this reason it is possible to confine electric currents to definite paths as in wires, comparable to the flow of water in pipes.

The practical unit of electric current is the *ampere*.

The quantitative value of the ampere may be defined in several ways. On the basis of the electron theory 1 amp. of electric current flows in a conductor when $6.281 \cdot 10^{18}$ electronic charges per second pass at a uniform rate the given point in the circuit. The legal definition of 1 amp. in the United States is based on the chemical effects of the current (Chap. XIX); that is, 1 amp. is that unvarying current which, when passed through a solution of silver nitrate in water, in accord with certain specifications, deposits silver at the rate of 0.0011180 g. per second. For definitions based on the electrostatic and electromagnetic systems, see Chap. VII. The ampere may also be determined by its relation to the volt and ohm as shown under Ohm's law in this chapter. Instruments for measuring electric currents are called *ammeters*. (Chap. VIII.) The commonly used symbols for electric current are i, I, etc. (Chap. VII).

Illustrations.—(*a*) An ordinary 60-watt incandescent lamp connected to a 120-volt circuit takes 0.5 amp.

(*b*) A 5-hp. direct-current motor operating on 220-volt mains and carrying full load requires approximately 20 amp.

Electromotive Force.—It should be noted that *electromotive force, electric pressure, electric potential,* and *voltage* are essentially equivalent terms and that it is merely a question of custom or choice which one would be used in the varied conditions under which electromotive forces occur.

In order to make water flow through a pipe a difference in the pressure at the two ends of the pipe is necessary. Likewise to make an electric current flow through a conductor, as a copper wire, a difference in electric pressure or voltage must exist between the two ends of the conductor. Electromotive force or voltage may be generated in several ways (Chap. V) but the more important sources are electromagnetic induction and chemical reactions.

The practical unit of electromotive force is the *volt*.

In the United States the volt is legally defined (Act of Congress, July 12, 1894) in terms of the electromotive force produced by a Clark standard cell, which under the specified conditions gives 1.434 volts. A more fundamental definition is based on the applications of Ohm's law using the definition of the ampere and ohm as a basis. A useful subsidiary standard approved by the International Electrotechnical Commission (London, 1908) is the Weston-Cadmium cell, which at 20°C. gives 1.083 volts. For definitions based on the electrostatic and electromagnetic c.g.s. systems, see Chap. VII. Instruments for measuring voltage are called *voltmeters*. (Chap. VIII.) The generally used symbols for voltage are e, E, v, V, etc. (Chap. VII).

Illustrations.—The voltages on residence electric-light circuits are from 110 to 125 volts; small motors, 110, 220, or 440 volts; trolley voltage on street-car systems 650 to 750 volts; trolley voltage C.M. St. P. and P. R. R. 3,000 volts; transmission line voltage between Seattle and Spokane 110,000 volts.

Resistance, Conductance.—That property of materials which hinders or opposes the flow of electric currents is called *electrical resistance*. It may be compared to friction which resists the flow of water through pipes. For direct currents the reciprocal for resistance is called *electrical conductance*, which may be considered as a measure of the ease with which an electric current flows through the given conductor. Resistance is generally used in dealing with electric circuits but when solving problems in parallel circuits it is usually more convenient to use conductance.

The practical unit of resistance is the *ohm* and the practical unit of conductance is the *mho*.

The legal definition of the ohm is the resistance, at the temperature of melting ice, offered to an unvarying electric current by a column of mercury, of constant cross-sectional area, having a mass of 14.4521 g. and a length of 106.30 cm. A subsidiary standard, approved by the International Electrochemical Com-

mission in 1913 and of more convenient form, may be defined as follows: the resistance of a standard annealed copper wire, at a temperature of 20°C., 1 meter in length, and of uniform cross-section of 1 sq. mm. is $\frac{1}{58}$ ohm, or 0.017241 ohm. For the relation of the ohm to the electrostatic and electromagnetic c.g.s. systems of units (see Chap. VII). The symbols for resistance in general use are r, R, etc. (Chap. VII.)

Illustrations.—The resistance of No. 14 A.W.G. copper wire used in wiring lighting circuits in residences is approximately 2.58 ohms per 1,000 ft. When lighted the resistance of a 50-watt, 110-volt tungsten lamp is approximately 250 ohms. The field circuit of a typical 50 hp., 550 volt direct-current motor is approximately 200 ohms. The armature resistance of a large dynamo is a fraction of an ohm.

Ohm's Law.—The fundamental relation between voltage, resistance, and current in an electric circuit is expressed by Ohm's law: *When a steady direct current flows in a circuit the magnitude of the current is directly proportional to the impressed voltage and inversely proporional to the resistance.*

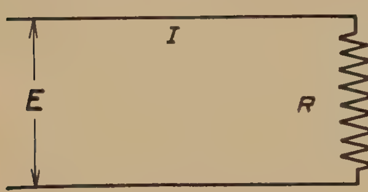


Fig. 1.—Simple electric circuit.

$$I \,\alpha\, \frac{E}{R} \qquad (1)$$

The simplest form of electric circuit is shown in Fig. 1.

The volt, ohm, and ampere as defined above are practical units in a consistent system and, hence, if the electromotive force E is expressed in volts, the resistance R in ohms and the current I in amperes, Ohm's law is expressed by equation (2).

$$I = \frac{E}{R} \qquad (2)$$

Obviously Ohm's law is also expressed by equations (3) and (4).

$$E = RI \qquad (3)$$

$$R = \frac{E}{I} \qquad (4)$$

The factor RI in equation (3) is generally referred to as the *resistance voltage drop* or the *resistance drop.* Equation (2) applies to direct currents under steady or constant conditions. For alternating currents or when direct currents are rapidly increasing or decreasing in magnitude, equation (2) must be

modified so as to include the effects produced by energy storage in the dielectric and magnetic fields of the circuit.

Circuit Connections.—In a *series circuit* the several parts or sections are so arranged that all of the current flows successively

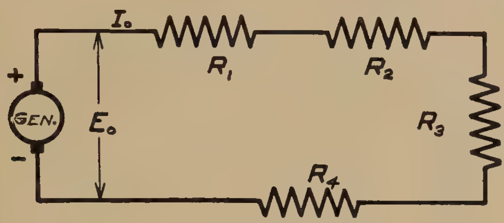


FIG. 2.—Series circuit.

through all sections; that is, the several resistances in the circuit are connected in series. A typical series circuit is shown in Fig. 2.

In *parallel or multiple circuits* the several sections are so arranged that the current divides into two or more parts which

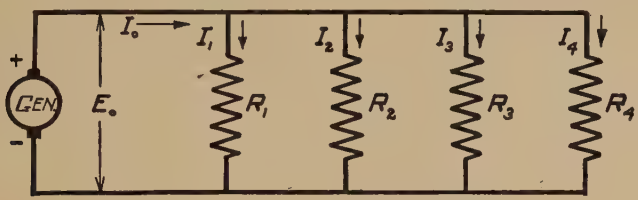


FIG. 3.—Parallel or multiple circuits.

return to the starting point or source by more than one path or route. Connections for a typical parallel circuit are illustrated by Fig. 3.

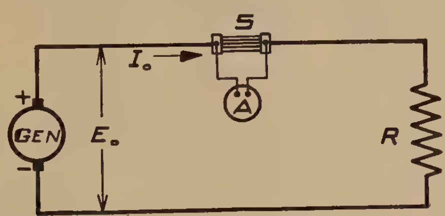


FIG. 4.—Shunt circuit with ammeter.

A *shunt* or a *shunt circuit* is a section of low resistance placed in parallel with a circuit having comparatively large resistance. The high resistance circuit is said to be *shunted* by the low-resistance shunt circuit. Shunts are frequently used in connection

with measuring instruments, especially ammeters and wattmeters. In Fig. 4 is shown a millivoltmeter A attached to a shunt S and calibrated so as to indicate the total amperes of current I_0 flowing in the main circuit.

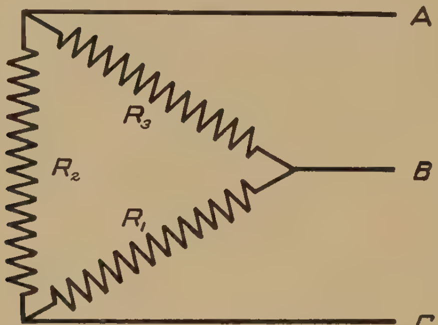


Fig. 5.—Delta connection.

Many combinations of series and parallel circuits occur in commercial electric systems. The *Delta* and *Y* or *Star* connections shown in Figs. 5 and 6 are of special interest as they are generally used in alternating-current three-phase transmission

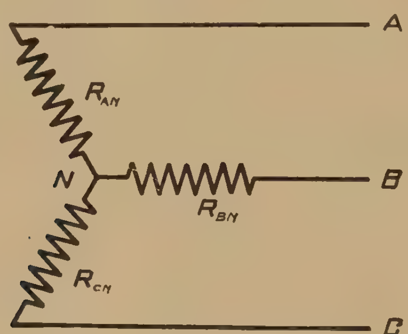


Fig. 6.—Y or star connection.

and distribution systems. Circuit arrangements of greater complexity are called *networks*, or compound circuits. A small section of a distribution network is shown in Fig. 7.

Distribution and transmission systems both in the power and communication fields form in most cases complicated networks by the combination of many simple series and parallel circuits.

Kirchhoff's Laws.—The *first law* states: *The sum of all the currents flowing through a point in any electric circuit equals zero.*

$$\Sigma I = 0 \tag{5}$$

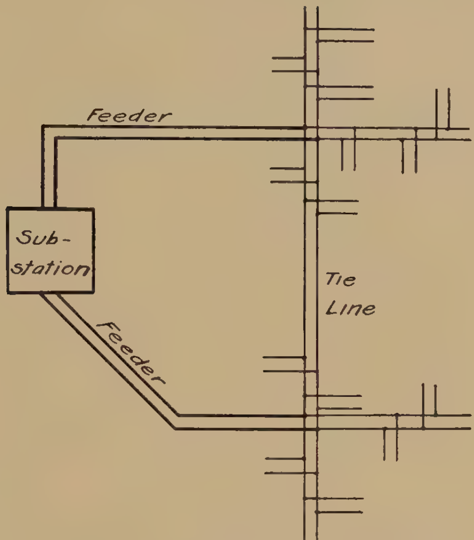


FIG. 7.—Section of distribution network.

In Fig. 8 five conductors connect at point A, four at B and two at C.

If the direction of the flow of the current from the generator is indicated by the arrow on the line PA, then the other arrows

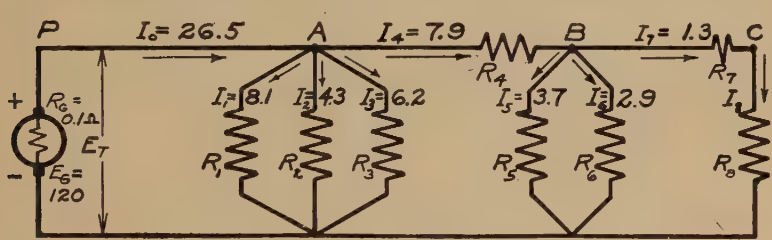


FIG. 8.—To illustrate Kirchhoff's first law.

indicate the directions of the currents in other parts of the circuit and the corresponding plus and minus signs must be assigned as to whether I_1, I_2, etc., flow toward or away from the point selected. Thus for point A on the basis of the first law, without stating the direction of the flow in each circuit:

$$I_0 + I_1 + I_2 + I_3 + I_4 = 0 \tag{6}$$

Substituting numerical values in equation (6) with plus and minus signs to indicate the direction of flow with respect to point A, gives equation (7).

$$26.5 - 8.1 - 4.3 - 6.2 - 7.9 = 0 \tag{7}$$

Similar equations may be written for points B and C.

In a series circuit the total resistance must necessarily be equal to the sum of the several resistances as all sections are connected in series. In the circuit shown in Fig. 9 the total resistance R equals the sum of R_G, the resistance inside the generator, and R_1,

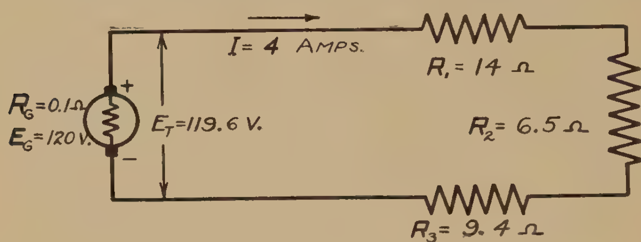


FIG. 9.—Series circuit.

R_2 and R_3, the external resistance in circuit, as expressed by equation (8).

$$R = R_G + R_1 + R_2 + R_3 \tag{8}$$

From equation (3) (Ohm's law) and equation (8), the total generated voltage E_G is given by equation (9).

$$E_G = (R_G + R_1 + R_2 + R_3)I$$
$$E_G = R_GI + R_1I + R_2I + R_3I \tag{9}$$

Letting the resistance voltage drops R_GI, R_1I, R_2I, R_3I, be represented by E_G', E_1, E_2, E_3, respectively, as the resistance drops represents voltage, and indicating the direction in each case gives equation (10) and equation (11).

$$E_G = E_G' + E_1 + E_2 + E_3 \tag{10}$$
$$E_G - E_G' - E_1 - E_2 - E_3 = 0$$

or

$$E_G + (-E_G') + (-E_1) + (-E_2) + (-E_3) = 0 \tag{11}$$

Kirchhoff's *second law* states the relation expressed by equation (11); that is, *the sum of all the voltages in a series electric circuit equals zero.*

$$\Sigma E = 0 \tag{12}$$

The minus signs in equation (11) indicate that the voltages represented by the resistance drops were in the opposite direction to

the impressed electromotive force. Voltages in direction opposite to the initial source used as reference base are usually called *counter-electromotive forces*, and in this respect, resistance voltage drops may be considered as counter-electromotive forces in the circuit.

The second law applies likewise to circuits in which more than one source of voltage is impressed as illustrated by Fig. 10. The arrows indicate the conventional directions of the impressed voltages and, also, the resistance voltage drops while the letters refer to resistance, current, and voltage, with the corresponding numbers expressing the respective quantities in ohms, amperes, and volts. Electric batteries are indicated by long and short parallel bars as illustrated in Fig. 10. The long bar indicates the positive terminal and the short bar the negative pole of each cell.

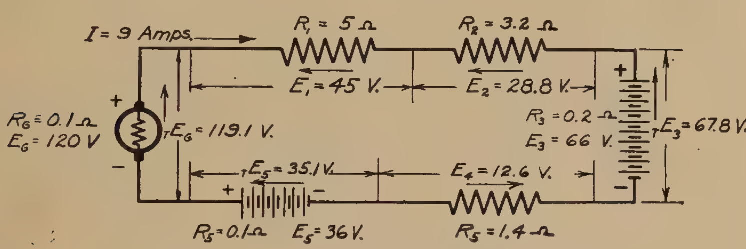


FIG. 10.—Illustrating Kirchhoff's second law.

Since Kirchhoff's law includes the complete circuit the resistance voltage drops inside the generator and the storage batteries must be included in the equation. The notation must be made so as to show both the generated and terminal voltages as well as the internal resistance voltage drops for the generator and the batteries.

Let E_G = voltage generated in the machine.
$_TE_G$ = generator terminal voltage.
$E_G' = R_GI$ = voltage drop inside generator.
E_3 = voltage generated in battery (3).
$_TE_3$ = battery (3) terminal voltage.
$E_3' = R_3I$ = voltage drop inside battery (3).
E_5 = voltage generated in battery (5).
$_TE_5$ = battery (5) terminal voltage.
$E_5' = R_5I$ = voltage drop inside battery (5).

Kirchhoff's second law applied to the circuit in Fig. 10 gives equation (13).

$$E_G + E_G' + E_1 + E_2 + E_3 + E_3' + E_4 + E_5 + E_5' = 0 \quad (13)$$

Substituting numerical values in equation (13) and noting the direction indicated by the arrows for E_3 and E_5 and considering resistance voltage drops as counter-electromotive forces with respect to the direction of current flow gives equation (14).

$$120 - 0.1I - 5I - 3.2I - 66 - 0.2I - 1.4I + 36 - 0.1I = 0 \quad (14)$$

Therefore the current $I = 9$ amp.

Substituting numerical values for I in equation (14),

$$120 - 0.9 - 45 - 28.8 - 66 - 1.8 - 12.6 + 36 - 0.9 = 0$$

It should be noted from equations (13) and (14) that the generated terminal voltage ${}_TE_G$ could be substituted for $E_G - E'$; that for

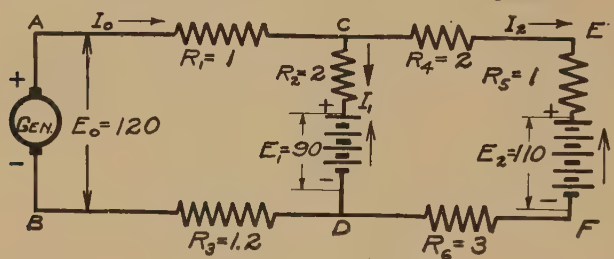


FIG. 11.—Complex circuit.

the batteries ${}_TE_3 = E_3 + E_3'$ and ${}_TE_5 = E_5 - E_5'$. Therefore, in the application of Kirchhoff's second law the terminal voltages of the given sources of voltage in the circuit may be used in place of the respective generated voltages and internal voltage drops, as illustrated for the circuit in Fig. 11.

The circuit shown in Fig. 11, and for the data given, will require the application of Kirchhoff's laws as well as Ohm's law to obtain the required solution. The circuit connections and the quantitative values and direction of the sources of electromotive force and the magnitude of the several resistances are given on the diagram in Fig. 11. Generator terminal voltage, $E_0 = 120$ volts; storage battery terminal voltage $E_1 = 90$ volts; storage battery terminal voltage $E_2 = 110$ volts; while $R_1 = 1$; $R_2 = 2$; $R_3 = 1.2$; $R_4 = 2$; $R_5 = 1$; $R_6 = 3$ ohms, respectively. The

directions of the flow of the currents is assumed to be as indicated by the arrows (solid heads). If the assumption in any case should be contrary to the actual condition this will become evident in the solution, since in those cases the currents will be of negative value, that is, will be of minus sign which indicates that the direction is opposite to that originally assumed. With the direction of the currents assumed the direction of the voltage consumed by the current in passing through the given resistance, that is, the + or − signs for the resistance drops in the voltage equation are determined. The direction of the voltages produced by the generator and the batteries is indicated by the + and − signs on the terminals. The voltage is positive in the direction from the − to the + terminal inside the battery and, therefore, likewise in the positive direction if passing from the + to the − terminals in the circuit outside the battery.

For the circuit in Fig. 11 and with the data as given in the diagrams, let it be required to find the values for I_0, I_1, and I_2 in amperes.

Since the problem has three unknowns, it will require three independent equations to obtain a solution. Two of the required equations may be obtained by applying Kirchhoff's second law to the circuit, considering the currents at the point A. For the path or circuit $BACDB$, keeping in mind the conventions as already explained, the voltages are expressed by equation (15).

$$E_0 + (-R_1 I_0) + (-R_2 I_1) + (-E_1) + (-R_3 I_0) = 0 \quad (15)$$

Substituting the given numerical values for R_1, R_2, R_3, E_1 and E_0,

$$120 - I_0 - 2I_1 - 90 - 1.2I_0 = 0 \quad (16)$$

or

$$2.2I_0 + 2I_1 = 30 \quad (17)$$

Similarly for the path or circuit $BACEFDB$ equation (18) is obtained by the application of Kirchhoff's second law.

$$E_0 + (-R_1I_0) + (-R_4I_2) + (-R_5I_2) + (-E_2) + (-R_6I_2) + (-R_3I_0) = 0 \quad (18)$$

Substituting the given numerical values for R_1, R_4, R_5, R_6, R_3, E_2 and E_0, gives equation (19)

$$120 - I_0 - 2I_2 - I_2 - 110 - 3I_2 - 1.2I_0 = 0 \quad (19)$$

or,

$$2.2I_0 + 6I_2 = 10 \quad (20)$$

The third required independent equation is obtained by the applications of Kirchhoff's first law to the circuit, considering the currents at the point C.

$$I_0 + (-I_1) + (-I_2) = 0 \tag{21}$$

or

$$I_0 = I_1 + I_2 \tag{22}$$

By combining the three simultaneous equations (17), (20) and (22) the numerical values of I_0, I_1 and I_2 are readily obtained.

$$I_0 = 6.75 \text{ amp.} \tag{23}$$
$$I_1 = 7.56 \text{ amp.} \tag{24}$$
$$I_2 = -0.81 \text{ amp.} \tag{25}$$

The negative sign for I_2 shows that this current flows in the opposite direction to that originally assumed and that hence to show the actual direction of current flow the arrow for I_2 in Fig. 11 should be reversed. From the direction of the currents as determined by equations (23), (24), and (25) it is evident that under the given conditions the 90-volt battery E_1 is being charged and receives energy both from the generator E_0 and the 110-volt battery E_2.

Resistance of Conductors.—The property of electric conductivity is to some extent possessed by all materials but the relative magnitude or range is very great. Specific conductance, or conductivity γ, may be defined as the conductance in mhos per $\overline{\text{cm.}}^3$, or circular mil-foot at 0°C. (or some specified temperature) of the given material. Resistivity ρ or specific resistance is the reciprocal of conductivity and is defined as the resistance in ohms per $\overline{\text{cm.}}^3$, or circular mil-foot, at 0°C. (or some specified temperature) of the given material. Materials like metals having relatively high conductivity are classed as "good conductors" while others like mica, rubber, glass, oils, and air, having very low conductivity, are termed "good insulators." In between these groups there are many substances difficult to classify and which may be grouped as "poor conductors" or "poor insulators." Impurities, changes in temperature, deterioration, and especially the presence of moisture greatly affect the resistivity of insulators so that the quality of the material must be carefully selected and adequate protection provided against moisture and high temperatures.

Primarily three independent factors, *resistivity, geometric form* and *temperature,* affect the resistance of any conductor. The *first factor* is the resistivity or specific resistance of the material of which the conductor is made. The resistivity is merely the resistance of a unit cube of the material at a specified temperature. The resistivity of annealed copper as ordinarily used in electric distribution systems is $1.724 \cdot 10^{-6}$ ohms per $\overline{\text{cm.}}^3$ (sometimes called the ohm-centimeter) at 20°C. The symbol for resistivity is the Greek letter ρ (rho). The resistivity may also be given in terms of other units as per cubic inch or per circular mil-foot and at any specified temperature.

The *second factor* is the geometrical form or shape of the conductor with respect to the direction or path of current flow. If the path traversed by the electric current is called the length of the conductor and the sectional area at right angles to the flow of the current the cross-section, then the resistance R of the conductor varies directly as its length l and inversely as its cross-section A as expressed by equation (35).

$$R \propto \frac{l}{A} \tag{35}$$

If there be no change in temperature the resistance of the conductor is given by equation (36), provided ρ, l and A are given in consistent quantitative units.

$$R = \rho\frac{l}{A} \tag{36}$$

If in equation (36), the resistivity ρ is given in ohms per $\overline{\text{cm.}}^3$ (ohm-centimeter), the length l in centimeters and the cross-section A in $\overline{\text{cm}}^2$., the resistance R will be in ohms. In American practice the cross-sections of electric conductors, as wires and cables, are generally given in *circular mils* and the length in feet. A *circular mil* is defined as the area of a circle whose diameter is 1/1,000 in. Since the areas of circles are directly proportional to the squares of their respective diameters it follows that cross-sectional areas expressed in circular mils are given by the square of the diameter measured in mils. Thus the cross-section of a cable of circular cross-section ½ in. (500 mils) in diameter must be $\overline{500}^2$ or 250,000 circular mils. Likewise, the cross-sectional area of a cable of circular cross-section, 1 in. (1,000 mils) in diameter would be expressed by, $\overline{1{,}000}^2$ or 1,000,000 circular mils.

TABLE I

A.W.G.	Diameter at 20°C. in mils	Cross-section at 20°C., circular mils	Pounds per 1,000 feet	Ohms per 1,000 feet	Ohms per mile
				20°C. 68°F.	20°C. 68°F.
0000	460.0	211,600	640.5	0.04901	0.259
000	409.6	167,800	507.9	0.06180	0.326
00	364.8	133,100	402.8	0.07793	0.411
0	324.9	105,500	319.5	0.09827	0.519
1	289.3	83,690	253.3	0.1239	0.654
2	257.6	66,370	200.9	0.1563	0.825
3	229.4	52,640	159.3	0.1970	1.040
4	204.3	41,740	126.4	0.2485	1.312
5	181.9	33,100	100.2	0.3133	1.654
6	162.0	26,250	79.46	0.3951	2.086
7	144.3	20,820	63.02	0.4982	2.630
8	128.5	16,510	49.98	0.6282	3.317
9	114.4	13,090	39.63	0.7921	4.182
10	101.9	10,380	31.43	0.9989	5.274
11	90.74	8,234	24.92	1.260	6.650
12	80.81	6,530	19.77	1.588	8.386
13	71.96	5,178	15.68	2.003	10.57
14	64.08	4,107	12.43	2.525	13.33
15	57.07	3,257	9.858	3.184	16.81
16	50.82	2,583	7.818	4.016	21.20
17	45.26	2,048	6.200	5.064	26.74
18	40.30	1,624	4.917	6.385	33.71
19	35.89	1,288	3.899	8.051	42.51
20	31.96	1,022	3.092	10.15	53.61
21	28.46	810.1	2.452	12.80	67.60
22	25.35	642.4	1.845	16.14	85.24
23	22.57	509.5	1.542	20.36	107.5
24	20.10	404.0	1.223	25.67	135.5
25	17.90	320.4	0.9699	32.37	170.9
26	15.94	254.1	0.7692	40.81	215.5
27	14.20	201.5	0.6100	51.47	271.7
28	12.64	159.8	0.4837	64.90	342.7
29	11.26	126.7	0.3836	81.83	432.1
30	10.03	100.5	0.3042	103.2	544.9
31	8.928	79.70	0.2413	130.1	687.0
32	7.950	63.21	0.1913	164.1	866.4
33	7.080	50.13	0.1517	206.8	1,092.4
34	6.305	39.75	0.1203	260.9	1,377.6
35	5.615	31.52	0.09542	329.0	1,737.1
36	5.000	25.00	0.07568	414.8	2,190.4
37	4.453	19.83	0.06001	523.1	2,762.0
38	3.965	15.72	0.04759	659.6	3,482.9
39	3.531	12.47	0.03774	831.8	4,391.8
40	3.145	9.888	0.02993	1,049.0	5,538.0

In Table I the cross-sectional area of standard size wires are expressed in circular mils.

In the c.g.s. system the $\overline{\text{cm}}.^3$ is the unit of conductor used for defining resistivity. If the circular mil is used as the cross-section

and the foot as the length, the circular mil-foot becomes the unit of conductor volume for defining the resistivity. A conductor having a cross-section of a circular mil and 1 ft. in length is called a *circular mil-foot*, frequently abbreviated to *mil-foot*.

If in equation (36) the resistivity ρ is expressed in ohms per circular mil-foot, the length l in feet and the cross-section A in circular mils the resistance R will be in ohms. A circular mil-foot of annealed copper has a resistance of 10.37 ohms at 20°C. To illustrate the application of equation (36) let it be required to find the resistance of a copper cable of circular cross-section, ½ in. in diameter and 4,000 ft. long. The resistance of this cable at 20°C. is obtained by equation (37).

$$R = \rho\frac{l}{A} = \frac{10.37 \times 4{,}000}{250{,}000}\text{ ohms} = 0.166\text{ ohms} \tag{37}$$

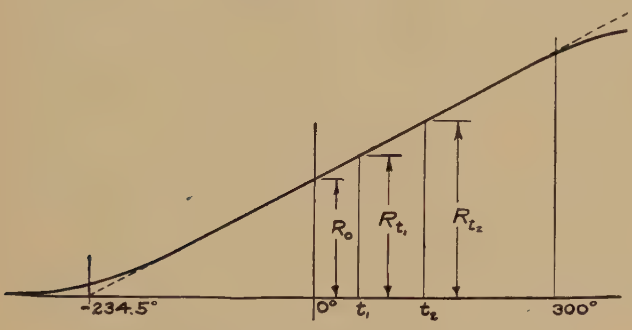


FIG. 12.—Resistance-temperature curve for copper.

The *third factor* affecting the resistance of electric conductors is the temperature, since the resistivity of most materials varies with changes in temperature. For metals the resistivity, within the temperature range of practical operation of electric machinery and appliances, is a linear function of the temperature. Thus in Fig. 12 is shown the resistance-temperature graph for copper. For metallic conductors the resistance-temperature characteristic is essentially a straight line from −75° to 300°C. but deviates for greater ranges as indicated by the full and broken curves in Fig. 12. If the straight-line part of the curve is extended as indicated by the broken line in Fig. 12 it will intercept the axis of abscissæ at the point representing −234.5°C. on the temperature scale. As stated above, the resistivity of copper at 20°C. is 1.724 microhms. If it be assumed that the resistivity

is a linear function of the temperature the change in the resistivity per degree change in temperature at 20°C. would be,

$$\rho_{20} = \frac{1.724}{234.5° + 20°} \text{ or } 0.0068 \text{ microhms per degree Centigrade} \quad (38)$$

TABLE II.—ELECTRICAL RESISTIVITY FACTORS

Material	Resistivity, ohms per cm.^{3} at 20°C.	Resistivity, ohms per circular mil-foot at 20°C.
Metals:		
Aluminum	2.828×10^{-6}	17.01
Copper	1.724×10^{-6}	10.37
Iron	10.0×10^{-6}	60.15
Lead	22.0×10^{-6}	132.3
Mercury	95.78×10^{-6}	576.1
Platinum	10.0×10^{-6}	60.15
Silver	1.59×10^{-6}	9.56
Tungsten	5.6×10^{-6}	33.69
Zinc	5.8×10^{-6}	34.89
Resistor Alloys:		
Constantan (Cu, 60; Ni, 40)	8.7×10^{-5}	523.3
Excello	9.2×10^{-5}	553.4
Manganin (Cu, 84; Mn, 12; Ni, 4)	4.4×10^{-5}	264.7
Nichrome	10.0×10^{-5}	601.5
Carbon:		
Amorphous lamp filament	3.5×10^{-2}	21.05×10^{4}
Metalized lamp filament	4.7×10^{-4}	28.27×10^{2}
Graphite	3.0×10^{-4}	18.05×10^{2}
Electrolytes:		
Ethel alcohol	3×10^{5}	18.05×10^{11}
Distilled water	5×10^{5}	30.08×10^{11}
River water	10×10^{4}	$6 \text{ to } 60 \times 10^{9}$
Sea water	30 approx.	180×10^{6}
NaCl, saturated solution	4.4	26.47×10^{6}
$CuSO_4$, saturated solution	29	174.4×10^{6}
Insulators:		
Glass	10^{14} to 10^{15}	$6 \text{ to } 60 \times 10^{20}$
Hard rubber	10^{15} to 10^{18}	$6 \text{ to } 6{,}000 \times 10^{21}$
Ice	2.8×10^{8} at 0°C.	16.84×10^{14}
Paraffin oil	10^{16}	6×10^{22}
Porcelain	10^{14}	6×10^{20}
Transformer oils	10^{13} to 10^{15}	$6 \text{ to } 600 \times 10^{19}$

Hence the resistivity at any temperature using centigrade scale and c.g.s. units is expressed by equation (39)

$$\rho_t = 1.724[1 + 0.0068(t - 20°\text{C.}) \quad \text{microhms} \quad (\text{resistance per } \overline{\text{cm.}}^3) \tag{39}$$

If the circular mil-foot is used as the unit for resistivity then the change in resistivity per degree centigrade at 20°C. would be,

$$\frac{10.37}{234.5° + 20°} \text{ or } 0.0407 \text{ microhms per degree Centigrade} \tag{40}$$

Therefore the resistivity at any temperature t, centigrade scale and the circular mil-foot unit is expressed by equation (41)

$$\rho_t = 10.37[1 + 0.0407(t - 20°\text{C.}) \text{ ohms (resistance per circular mil-foot)} \tag{41}$$

Hence the resistance of a metallic conductor of length l and constant cross-sectional area A is expressed by equation (42) in which the values ρ_t, l, and A must be given in consistent units as illustrated for copper by equations (39) and (41).

$$R_t = p_t\frac{l}{A} \tag{42}$$

For copper conductors with l in cm., A in cm.^{2}, t in centigrade degrees and ρ_t from equation (39),

$$R_t = 1.724[1 + 0.0068(t - 20°\text{C.})]\frac{l}{A} \text{ microhms} \tag{43}$$

$$= 1.724[1 + 0.0068(t - 20°\text{C.})]\frac{l}{A}\, 10^{-6} \text{ ohms} \tag{44}$$

For l in feet, A in circular mils, t in centigrade degrees, and ρ_t from equation (41),

$$R_t = 10.37[1 + 0.0407(t - 20°\text{C.})]\frac{l}{A} \text{ ohms} \tag{45}$$

The effect of variation in temperature on the resistance as shown by the straight line part of the graph in Fig. 12 may be expressed by equation (46)

$$R = R_1[1 + \alpha_1(t - t_1)] \tag{46}$$

In equation (46) R is the resistance of the given conductor at temperature t,

R_1 is the resistance at the reference temperature t_1

and α_1 is the *resistance temperature coefficient*, whose numerical value depends on the conductor material, the reference temperature t_1 and the units in which R, t and t_1 are expressed.

The resistance temperature coefficient α_1 may be defined as the increase in the resistance of the conductor material per degree rise in temperature, divided by the resistance R_1 at the reference temperature.

For the case in which the reference temperature is 0°C., letting R_0 and α_0 represent the resistance and the resistance-temperature coefficient at 0°C., respectively, equation (46) becomes

$$R_t = R_0(1 + \alpha_0 t) \tag{47}$$

This is the equation of a straight line which if extended would intercept the reference line at a temperature T' at which the conductor material would have zero resistance. For copper $T' = -234.5$°C. and hence α_0 for copper is expressed by equation (48)

$$\alpha_0 = \frac{1}{234.5} = 0.00427 \tag{48}$$

At 20°C. for copper the resistance-temperature coefficient,

$$\alpha_{20} = \frac{1}{234.5 + 20} = 0.00393 \tag{49}$$

From the above statements it is evident that the relation between α_0 and α_1, the resistance-temperature coefficients at 0°C. and t_1°C. respectively, is expressed by equation (50)

$$\alpha_1 = \frac{\alpha_0}{1 + \alpha_0 t_1} \tag{50}$$

The resistance-temperature coefficient at 0 and 20°C. of the more generally used conductor materials are given in Table III.

Table III.—Resistance-temperature Coefficients

Material	α_0 for 0°C.	α_{20} for 20°C.
For the common non-magnetic metals, average value	+0.0042	+0.0039
Annealed copper, international standard	+0.00427	+0.00393
Hard-drawn aluminum	+0.00423	+0.00390
Pure iron	+0.00625	+0.00555
Soft steel	+0.00424	+0.00390
Platinum	+0.00390	+0.00362
Tungsten	+0.00494	+0.00450
Constantan (Cu, 60; Ni, 40)	....	+0.00001
Manganin (Cu, 84; Mn, 12; Ni, 4)	....	+0.000006
Nichrome	....	+0.0004
Carbon, amorphous lamp filament	....	−0.001
Carbon, metalized lamp filament	....	+0.001

The rapid change in resistance due to temperature rise in the filament of a tungsten incandescent lamp is shown graphically by the oscillogram in Fig. 13. A constant direct-current voltage was impressed on the lamp in the same way as when the light in a room is turned on by throwing a switch. Time is represented on the axis of abscissæ. The alternating current wave near the top of the oscillogram, called the timing wave, measures the time, since each complete cycle represents $\frac{1}{100}$ sec. That is, the fre-

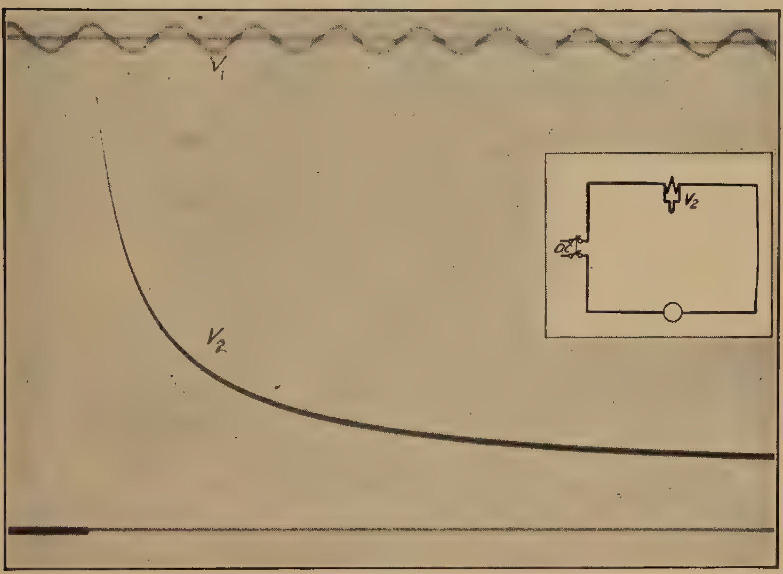


FIG. 13.—Starting transient of a 50-watt 120-volt tungsten lamp. $E = 118$ volts; $V_1 = 100 \sim$.

quency of the timing wave is 100 cycles per second. The ordinates of the curve represent the values of the current at successive instants. Notice that at start the current was more than seven times the final or permanent value. Since the impressed voltage was constant the resistance is at every instant the reciprocal of the current at that instant.

Example.—From the data given in Fig. 13 plot the corresponding resistance-time curve for the tungsten lamp.

Unlike the general characteristics of metals, carbon has a negative temperature coefficient. This is shown graphically by the oscillogram in Fig. 14. The circuit conditions, timing wave and

procedure in taking the oscillogram in Fig. 14 were the same as for the tungsten lamp in Fig. 13.

Note that for the carbon lamp the current increases for a short period of time after the switch is closed; that is, although the impressed voltage is constant the current increases with the rise in temperature of the carbon filament. Since the resistance under the given conditions is at each instant the reciprocal of the current

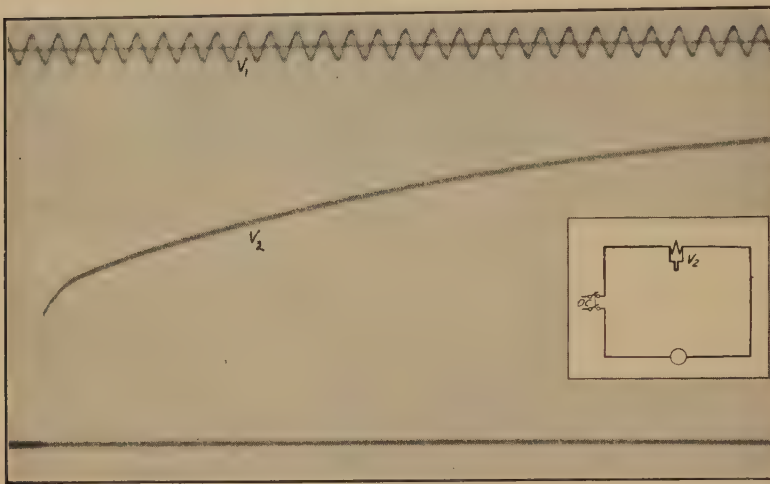


FIG. 14.—Starting transient of a 60-watt, 120-volt carbon lamp. $E = 118$ volts; $V_1 = 100 \sim$.

ordinate, it is evident that for the carbon filament the resistance decreases as the temperature increases.

Example.—From the data given in Fig. 14 plot the resistance-time curve corresponding to the current-time curve on the oscillogram.

Resistance of Circuits.—Ohm's and Kirchhoff's laws form the basis for the solution of networks and no special rules or methods are required. In a straight series circuit as Fig. 2 obviously the total resistance must be equal to the sum of all the component sections. In a simple parallel circuit, as Fig. 3, the total conductance of the circuit equals the sum of the conductances of the several sections.

$$G_0 = G_1 + G_2 + G_3 + G_4 \tag{51}$$

Since conductance is the reciprocal of resistance the relations shown in equation (51) may be represented by equation (52).

$$\frac{1}{R_0} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \tag{52}$$

If the values for R_1, R_2, R_3 and R_4 are known, it is simplest to make numerical substitutions in equation (52); or the algebraic value for R_0 can first be found, as in equation (53), and then substitute the numerical values.

$$R_0 = \frac{R_1R_2R_3R_4}{R_1R_2(R_3 + R_4) + R_3R_4(R_1 + R_2)} \tag{53}$$

If it is desired to find the resistance between two given points in a distribution network, as for example A and B in Fig. 15, successive application of the principles established by Ohm and Kirchhoff will give the desired solution. Starting at the section farthest away from the points A and B for which the resultant total resistance is desired the successive steps would be as follows:

Between the points G and H the numbers on the circuits indicating ohms,

$$\frac{1}{R_{GH}} = \frac{1}{6} + \frac{1}{2} + \frac{1}{3} \therefore R_{GH} = 1 \text{ ohm} \tag{54}$$

Hence, for the path EGH the resistance will be 5 + 1 or 6 ohms.

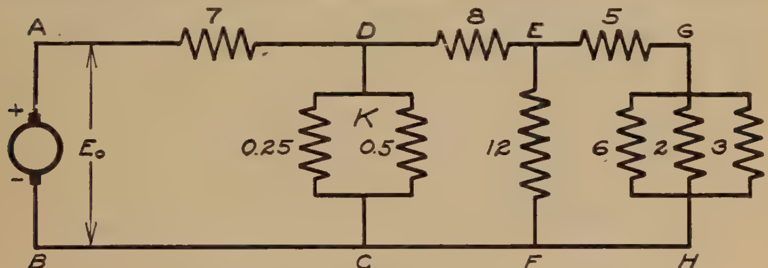


FIG. 15.

Between the points for the parallel paths $EGHF$ and EF the equivalent resistance is found by equation (55),

$$\frac{1}{R_{EF}} = \frac{1}{12} + \frac{1}{6} \therefore R_{EF} = 4 \text{ ohms} \tag{55}$$

For the path DEF the resistance is evidently 4 + 8 or 12 ohms.

For the path DKC

$$\frac{1}{R_{DKC}} = \frac{1}{0.25} + \frac{1}{0.5} \therefore R_{DKC} = \frac{1}{6} \text{ ohm} \tag{56}$$

Hence, for the parallel paths between DC the resistance is given by equation (57)

$$\frac{1}{R_{DC}} = \frac{1}{12} + \frac{6}{1} \therefore R_{DC} = \frac{12}{73} \text{ ohm} \tag{57}$$

In series with the resistance R_{DC} are 7 ohms and hence the equivalent resistance of the network from A to B is,

$$R_{AB} = 7\frac{12}{73} = 7.016 \text{ ohms} \tag{58}$$

Two simple circuit connections, the *delta* and *star*, are of special interest as they are used extensively in alternating-current three-phase systems.

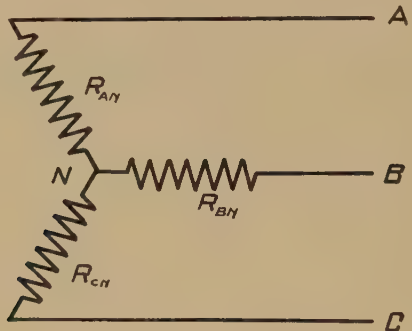


FIG. 16.—Star connection.

The Y or *star* connection is shown in Fig. 16. The transmission lines and distribution mains are connected to the points A, B, and C. The common point N is termed the *neutral* and is generally grounded.

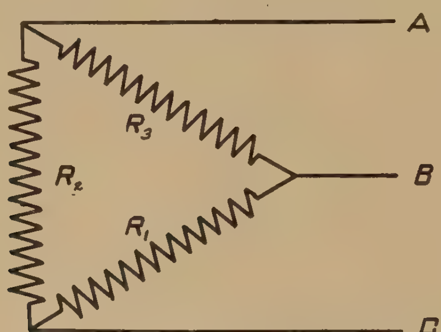


FIG. 17.—Delta connection.

The resistance between two-line wires as AB or BC or CA would in each case be the sum of the corresponding sections to the neutral point. This resistance from A to B would be $R_{AN} + R_{BN}$. In most commercial circuits the resistances R_{AN}, R_{BN} and R_{CN} are of equal value and hence the resistance between any pair of mains is simply twice the resistance from one main to neutral.

In the *delta* connection shown in Fig. 17 the total resistance between A and B is given by equation (59), which the student should verify.

$$R_{AB} = \frac{R_1R_3 + R_2R_3}{R_1 + R_2 + R_3} \tag{59}$$

In commercial systems are found electrical networks differing widely both in form and complexity. All may be analyzed by the application of Ohm's and Kirchhoff's laws, provided sufficient quantitative data on the component elemental sections are available.

Occasionally the desired solution for an electric network may be obtained more readily by substituting *equivalent circuits* for

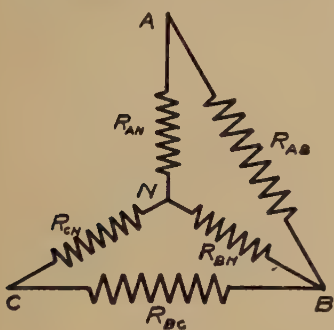


FIG. 18.

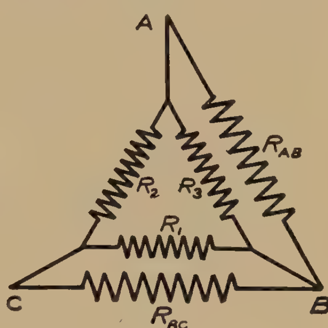


FIG. 19.

portions of the actual system in the given problem. An equivalent circuit carries the same current and consumes an equal amount of power as the actual circuit and the substitution of the equivalent circuit does not in any way alter the currents or voltages in other parts of the given network. Thus in Fig. 18 the three resistances R_{AN}, R_{BN}, R_{CN} connected in star can be replaced by an equivalent delta circuit as illustrated in Fig. 19 by the three resistances R_1, R_2, R_3 connected in delta. For given values of R_{AN}, R_{BN} and R_{CN} the corresponding values for R_1, R_2 and R_3 to form the equivalent delta circuit (Fig. 19), may be obtained by applying equations (60), (61), and (62).

$$R_1 = \frac{R_{AN}R_{BN} + R_{BN}R_{CN} + R_{CN}R_{AN}}{R_{AN}}, \tag{60}$$

$$R_2 = \frac{R_{AN}R_{BN} + R_{BN}R_{CN} + R_{CN}R_{AN}}{R_{BN}}, \tag{61}$$

$$R_3 = \frac{R_{AN}R_{BN} + R_{BN}R_{CN} + R_{CN}R_{AN}}{R_{CN}}. \tag{62}$$

Conversely if the values for R_1, R_2, and R_3 of the delta circuit (Fig. 19) are given, the values for the corresponding resistances R_{AN}, R_{BN} and R_{CN} that will produce the equivalent star circuit in Fig. 18 may be obtained from equations (63), (64), and (65).

$$R_{AN} = \frac{R_2 R_3}{R_1 + R_2 + R_3} \tag{63}$$

$$R_{BN} = \frac{R_1 R_3}{R_1 + R_2 + R_3} \tag{64}$$

$$R_{CN} = \frac{R_2 R_1}{R_1 + R_2 + R_3} \tag{65}$$

Leakage Current.—The specific resistances or resistivities of insulators, like rubber, glass, air and oil, are very large, in the order of 10^{22} times that of copper, an enormously large ratio. In general, however, experimental observations tend to prove that although the resistance may be large whenever a difference of potential exists a current flows through the dielectric, thus forming an electric circuit. This is the true *leakage current.*

In commercial electric distribution systems the leakage current may be of considerable magnitude even if the insulation is in good condition. The various compositions used for insulating electric conductors vary in resistivity and deteriorate more or less with the length of time in service. The length of the path for the leakage current through the insulation is short, a fraction of an inch, but the cross-section, for example over the distribution system in a city like Seattle, is very large. Hence while the resistivity of the insulating material may be large, still, over the total area of insulation involved a considerable leakage current may flow. The dimensions of length and cross-section of the dielectric are generally so irregular that it becomes impracticable to determine the resistance by means of space measurements and the resistivity of the material. Hence, while the law for the leakage current is simple, the practical determination of the average length and cross-section of the irregularly shaped dielectric cannot be made by direct measurements and, therefore, direct application is impossible. The total equivalent conductance of any machine or system is usually determined by measuring the leakage current for any impressed voltage and then applying Ohm's law. Let ${}_lI$ be leakage current, ${}_lg$ leakage conductance through the insulation, and E the voltage.

$${}_lI = {}_lgE \tag{66}$$

In equation (66) the measurements must be made with direct currents. In all cases the leakage current represents a transformation of electric energy into heat, light, or chemical reaction. The process is not reversible and the energy consumed by the leakage current represents a drain of energy from the electric circuit similar to the resistance losses in the metallic circuit. Leakage through the insulation is always undesirable. The energy loss may not be very large and in most cases the heat generated gives no troublesome rise in temperature, but usually the leakage current causes changes in the chemical composition of the insulating materials that rapidly destroy their insulating properties.

Three factors of special importance affecting both the leakage conductance and the rate of deterioration of the insulating properties of the dielectrics should be mentioned:

1. Moisture
2. Temperature
3. Voltage gradient

Even small amounts of *moisture* affect the leakage conductance to a marked degree. The leakage current is greatly increased and chemical reactions follow that cause deterioration at a more or less rapid rate. In many cases, electrical apparatus can not be protected from moisture, as in outdoor installations, and selection of the most serviceable insulation for the given conditions must be made. In other cases, as for the oil insulation in transformers, extreme care is exercised in removing all the water, and in keeping the oil protected against moisture.

The permissible *temperature* rise for various kinds of electrical apparatus is in almost all cases based directly on the specific characteristics of the insulating materials used. The temperature range for commercial operation is seldom above 100°C. and in most cases considerably less, because at higher temperatures the rate of chemical change becomes too great, causing a rapid deterioration of the insulating properties of the dielectric. Experimental observations show that the weakening effect on the insulation properties of organic compounds may increase in proportion to the fifth or even sixth power of the rise in temperature. It is therefore the rate of chemical change in the dielectric that limits the permissible rise in temperature and the rating of most electrical machinery as may be readily seen from the standardization rules of the A.I.E.E.

The *voltage gradient* at any point in the dielectric, as shown in Chapter IX, is in most cases of greater importance than the total voltage impressed on the circuit in determining the stresses and resultant strains and possible ruptures in the dielectric. Dielectrics are *electrically elastic* and have a more or less definite elastic limit above which the insulation breaks down or ruptures. In the irregularly shaped dielectric between two conductors the stresses are not uniform and the elastic limit may be reached in the layers near the conductors at a much lower total voltage than at points farther away. It is, therefore, necessary to so design electrical apparatus that at no point shall the voltage gradient exceed the elastic limit of the insulating materials, and to provide safety appliances that will protect the apparatus against excessive voltage gradients.

In applying Ohm's and Kirchhoff's laws to electric circuits the leakage currents must be taken into account and considered as flowing through circuits in parallel with the metallic conductors in the system.

PROBLEMS

1. Given a series circuit such as Fig. 2 in which R_1, R_2, R_3 and R_4 are respectively 2, 3.4, 7 and 6.6 ohms. If the generator maintains a constant voltage of 110 volts, find I_0 and the voltage across each resistance.

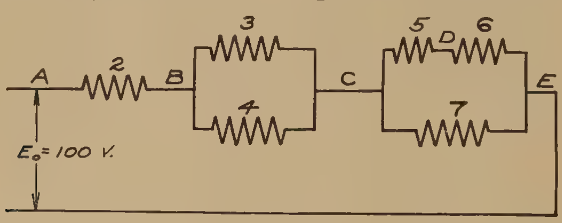


Fig. 20.

2. A generator supplies a constant voltage of 120 volts and is to be used for charging a battery whose voltage is 110 volts. Neglecting the internal resistance, how much resistance must be placed in series with the battery in order that the charging current shall not exceed 15 amp.?

3. A 125-volt motor takes a field current of 4.2 amp. when cold and 3.9 amp. when hot. What is the field resistance in each case?

4. Given a parallel circuit as in Fig. 3 in which R_1, R_2, R_3 and R_4 are 10.5, 6.8, 7.2 and 8.4 ohms respectively. Find the current through each resistance and the total current when E_0 equals 100 volts.

5. In Fig. 20 let the numbers represent the resistances in ohms. If E_0 = 100 volts, find:

(*a*) The total current.

(*b*) The voltage across each resistance.

(*c*) The current passing through each resistance.

6. The ammeter of Fig. 4 has a full-scale deflection when 50 millivolts are impressed across its terminals. What should be the resistance of the shunt if it is desired that the meter have full-scale deflection when 1,000 amp. flow in the load circuit?

7. In Fig. 21 find:

(a) The current in each section or part of the circuit diagram.

(b) The voltage from B to F and from C to E.

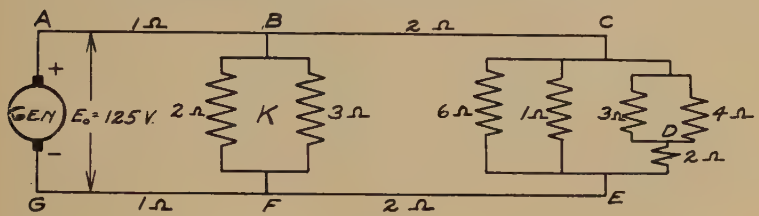


FIG. 21.

8. Figure 22 shows a circuit having a battery of 96 volts whose internal resistance is 1 ohm. Find the direction and value of the current in each part of the circuit and the terminal voltage of the battery.

9. In Fig. 11 find the currents in the circuits if the voltage of the 110-volt battery be increased to 120 volts.

10. What is the resistance of a copper busbar 80 ft. long made up of four 4 by ¼ in. copper bars in parallel?

11. What is the resistance of a tubular copper bus 45 ft. long; inside diameter 1¾ in.; outside diameter 2¼ in.?

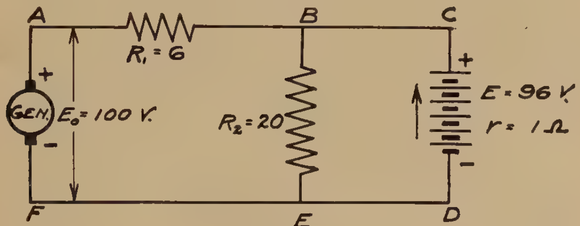


FIG. 22.

12. How many circular mils in a round rod ¾ in. in diameter? How many square mils? If the rod is made of copper and 42 ft. in length, what is its resistance?

13. A factory requires a current of 600 amp. at not less than 2,300 volts. The power station is 700 ft. away and maintains a constant voltage of 2,350 volts. Find the size of wire required. Give answer both in terms of the diameter and in circular mils.

14. Figure 23 shows a trolley system 5 miles long. The trolley wire is No. 0000 hard-drawn copper wire having a resistance of 0.265 ohms per mile. A 1,500,000-c.m. feeder runs parallel to the trolley wire for a distance of 4 miles and is tapped every 1,000 ft. to the trolley wire. Each of the rails has

a resistance of 0.06 ohms per mile. Both rails are bonded together 3 miles from the power station and connected to a 1,000,000-c.m. return feeder as shown. The power station has a constant voltage of 550 volts. A car at the end of the line takes 600 amp.

(a) Find the voltage from trolley to rails at the car.

(b) A battery of 530 volts having an internal resistance of 0.038 ohm is placed at the end of the line and connected to the trolley and rails in parallel

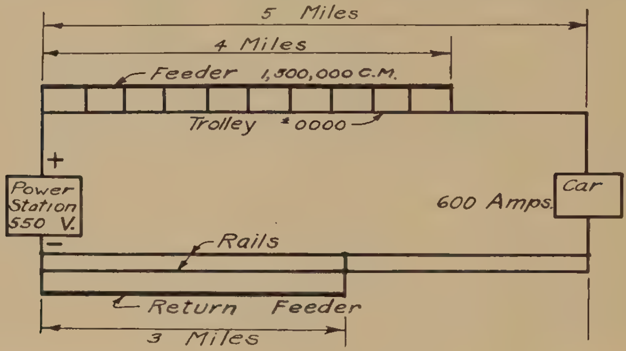


FIG. 23.

with the car. If the car takes 600 amp. what is the voltage from trolley to rails at the car?

(c) How much current is supplied by the battery?

(d) Disconnect the car from the trolley so that no current will flow through the car. Find the charging current in the battery under the given conditions.

15. Figure 24 shows two substations A and B feeding a street car trolley system. The voltage at A is 660 volts and at B is 640 volts. If the street

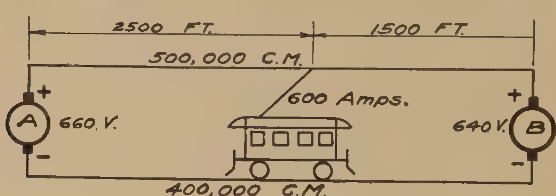


FIG. 24.

car takes 600 amp. what does each station supply and what is the voltage from trolley to rails at the car?

16. Two substations A and B are feeding a trolley system from which two cars are taking current as indicated in Fig. 25. Find the current supplied by each substation and the voltage from trolley to rails at each car.

17. Figure 26 shows a typical distribution system. A substation at AB is supplying current to loads at CD, EF, and GH. The voltage at the substation is maintained constant at 250 volts. The feeder lines running from

the substation to *CD*, *EF*, and *GH* are each 2,000,000 c.m. copper and are 2,600, 2,000, and 3,000 ft. long respectively. The tie lines from *CD* to *EF* and *EF* to *GH* are of 500,000 c.m. and each 1,200 ft. long. The load at *CD* is 1,200 amp.; at *EF*, 700 amp. and at *GH* is 1,000 amp.

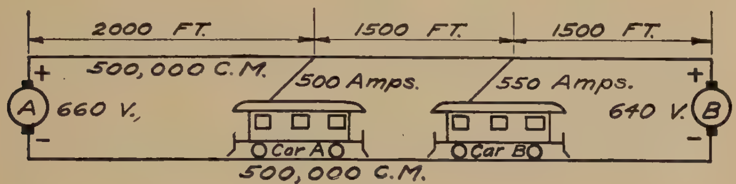


FIG. 25.

Find the voltage at each of the load points and the currents in all lines. Check results by noting that difference in voltage between load points equals the *RI* voltage drop in the tie line between load points.

18. Repeat Problem 17 using the same data except that all the negative return and negative tie wires are 1,000,000 c.m. in cross-section instead of the same size as the positive conductors.

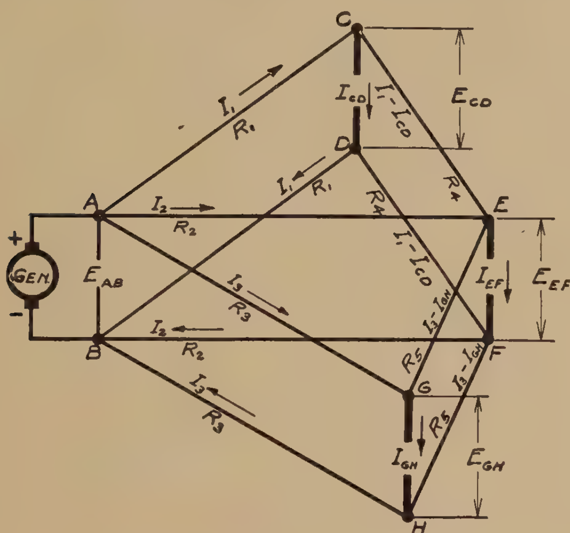


FIG. 26.

19. The resistance of a shunt-field winding of a motor is 120 ohms when measured at a room temperature at 22°C. After several hours operation at full load the resistance is again measured and found to be 132 ohms. Find the temperature in the field winding.

20. A transmission line has a resistance of 2.68 ohms at 20°C. If the minimum winter temperature is 10°F. and the summer maximum temperature is 100°F., what are the maximum and minimum resistances of the line?

21. A copper-clad steel cable has an outside diameter of 1 in. The steel core is ¾ in. in diameter. What is the resistance of 1 mile of this cable if the resistivity of copper is 1.732 microhms per $\overline{\text{cm.}}^3$ and steel is 21.6 microhms per $\overline{\text{cm.}}^3$? If the above values are at 20°C. what would be the resistance of the cable at 75°C.? The resistance temperature coefficient of the steel core may be taken as 0.0036 per degree Centigrade per ohm at 20°C.

22. In Fig. 17 let R_{AN}, R_{BN}, and R_{CN} equal 6.8, 7.6, and 5.2 ohms respectively. Find the resistances in the equivalent delta connected circuits similar in form to Fig. 18.

23. In Fig. 18 let R_1, R_2, and R_3 equal 6.1, 7.2, and 8.3 ohms respectively. Find the resistances in the equivalent star-connected circuits, similar in form to Fig. 17.

24. In Fig. 19 let $R_{AN} = 2$ ohms, $R_{BN} = 4$ ohms, $R_{CN} = 5$ ohms, $R_{AB} =$ 6 ohms and $R_{BC} = 8$ ohms. Find the resistance from A to C.

25. If 100 volts are impressed from A to C find the currents in the several resistances in Problem 24.

26. A 10-volt battery, a 10-ohm resistance, and a 6-ohm resistance are all connected in series. The middle points of the two resistances are connected together by a conductor having negligible resistance. The 10-ohm resistance is in addition shunted by a 7-ohm resistance. Find the current through the battery.

CHAPTER III

THE MAGNETIC CIRCUIT

Magnetic Flux. Lines of Force.—The space surrounding magnets and conductors carrying electric currents is termed the *magnetic field.* Physically the magnetic field is invisible; but, in order to more readily keep in mind the laws of electromagnetic phenomena, magnetic fields are visualized and pictured as consisting of continuous *tubes of magnetic force* or *lines of force* endowed with definite properties. This method has proved extremely useful for both practical and theoretical purposes. The designer of electrical machinery bases his computations on lines of force, or magnetic flux tubes, as the elements of which the *magnetic flux* is composed; theoretical investigations on electromagnetism are generally based on the distribution and interaction of lines of force; and even electricians use tubes or lines of force as a guide in the repair and installation of electrical appliances. The expressions, "magnetic lines of force," "lines of force in a magnetic field," "tubes of magnetic force," "magnetic flux lines," "magnetic flux," "lines of magnetic induction," "lines of induction" or "tubes of induction" are essentially equivalent terms. The "line" may be considered as the axis of the "tube" and to represent the magnetic flux contained in the volume of the "tube." The use of lines of flow and lines of force dates back to early observations on magnetism. Thus Euler in 1761 writes of "lines of flow of the magnetic fluid." But it was Faraday who made the concept of lines of force of basic importance in the study of magnetic and dielectric phenomena. He conceived a simple system of lines of force having definitely specified properties with which he pictured the physical characteristic of dielectric and magnetic fields and their interaction with electric currents, and thereby gave chart and compass on the electromagnetic-dielectric main. Faraday's concepts, which were developed and stated in the form of comprehensive mathematical equations by Maxwell, form the foundation and framework of the classical electromagnetic-dielectric theory.

In order to represent graphically the physical characteristics of magnetic fields, magnetic lines of force are conceived as endowed with the following specific properties:

1. All magnetic lines of force are continuous and closed upon themselves; that is, each magnetic flux line forms a closed circuit.

2. The direction of a line of force at any point follows the resultant of the magnetic forces acting on a north pole placed at the given point.

3. Magnetic lines of force in the same direction repel and in opposite directions attract one another.

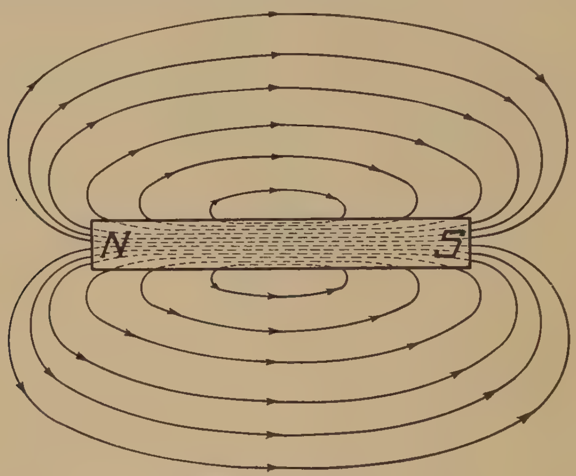


Fig. 1.—Magnetic field of bar magnet.

4. The strength of a magnetic field is measured by the density of the lines of force.

A longitudinal cross-section of the magnetic field of a bar magnet is shown in Fig. 1. As indicated by arrow heads in the figure the lines of force are always assumed to proceed from the north pole and to return to the steel bar at the south end. This is a conventional assumption and forms the basis for the customary system of directional notation of magnetic lines of force.

The practical units of magnetic flux are the *maxwell* and the *weber*.

$$1 \text{ maxwell} = \text{one magnetic flux line} \tag{1}$$

$$1 \text{ weber} = 10^8 \text{ maxwells} \tag{2}$$

The basic definition of the maxwell in the electromagnetic c.g.s. system may be found under Systems of Units, Chap. VII. The symbol for magnetic flux is the Greek letter ϕ or Φ.

Illustrations.—The field poles of a 5-hp., 120-volt, 1,800-r.p m., direct-current motor has approximately 2,000,000 maxwells of magnetic flux. If the cross-section of the field pole is 150 $\overline{\text{cm}}.^2$ and the distribution of the magnetic flux is uniform there would be 13,350 maxwells or lines of force per $\overline{\text{cm}}.^2$ of the field pole face.

Magnetomotive Force.—By means of magnetic lines of force considered as possessing the four specific properties described in the preceding paragraph, it is possible to produce graphical representations of the physical characteristics of the magnetic fields, which are of great assistance in acquiring knowledge of the basic laws of magnetism. It is likewise of fundamental importance to gain clear concepts of the interaction and interdependence of electric currents with magnetic fields. The electric current flowing in a conductor generates a *magnetomotive force* that produces magnetic lines of force around the current, or in the magnetic circuit, in much the same manner as a voltaic battery in an electric circuit provides electromotive force that causes an electric current to flow in the circuit. Referring to Fig. 2, the basic relations may be stated as follows:

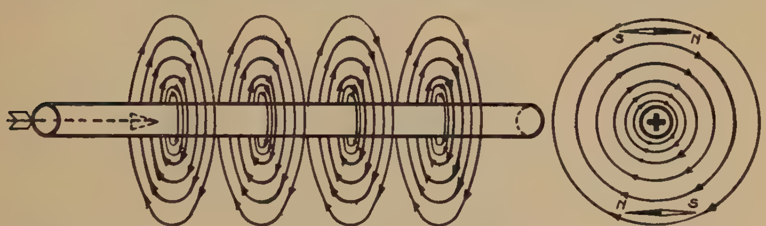


Fig. 2.—Magnetic field around a straight conductor carrying an electric current.

1. All electric currents are encircled by magnetic lines of force; that is, the electric circuit is interlinked with a magnetic circuit.

2. The magnetic lines of force are in space position at right angles to the flow of the electric current.

3. The direction of the lines of force is clockwise or like the turning of a right-handed screw with the forward motion of the screw following the flow of the electric current. Holding a wire in the right hand and letting the thumb indicate the direction of the flow of current, then the fingers are in the direction of the lines of force around the current.

4. The number of lines of force produced by the current is proportional to the current if the magnetic circuit does not contain magnetic materials.

5. If a conductor carrying a current is bent so as to form a loop as in Fig. 3, all the flux lines inside the loop will be in one direction and all the flux lines outside the loop will have the opposite direction. A longer wire may be looped several times and formed into a coil or solenoid as shown in Fig. 4. The magnetic flux lines inside the solenoid are in the same direction and hence

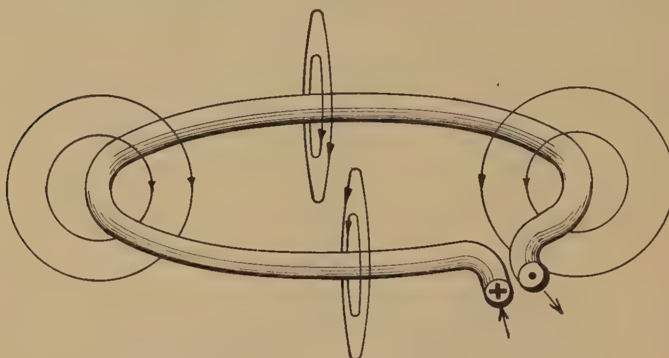


FIG. 3.—Magnetic field of electric current flowing in a loop.

add, producing a resultant flux as indicated by the flux lines in Fig. 4. Hence if the conductor (example, an insulated copper wire) carrying the electric current forms a coil, as the field windings of dynamos, motors, etc., the magnetomotive force is directly proportional to both the magnitude of the current and the number of turns in the coil, that is, the *magnetomotive force is directly*

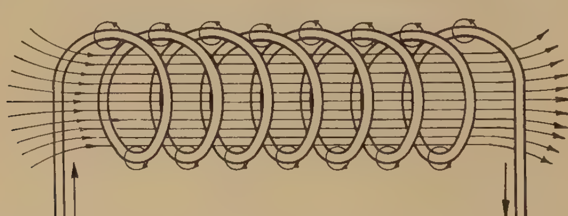


FIG. 4.—Magnetic field around a solenoid carrying an electric current.

proportional to the ampere-turns of the circuit. This basic relation of the magnetomotive force, $\mathfrak{F}$, the current I, and number of turns N is expressed by equation (3).

$$\mathfrak{F} \;\alpha\; NI \tag{3}$$

The practical units of magnetomotive force are *ampere-turns* and the *gilbert.* In a coil of N turns carrying a current of I

amp., the magnetomotive force expressed in gilberts is given by equation (6):

$$\mathfrak{F} = 4\pi NI10^{-1} = 0.4\pi NI \text{ gilberts} \tag{4}$$

$$1 \text{ gilbert} = \frac{1}{0.4\pi} \text{ ampere-turns} \tag{5}$$

$$\mathfrak{F} = NI \text{ ampere-turns} \tag{6}$$

The constants 4π and 10^{-1} are introduced by the definitions of the unit magnetic pole and the unit of current in the electromagnetic c.g.s., or absolute system of units (Chap. VII). A unit magnetic pole is assumed to have 4π lines of force; that is, one line of force for each square centimeter of the surface of a sphere of 1 cm. radius surrounding the pole. This definition unfortunately brings the 4π factor into many of the electromagnetic equations. The 10^{-1} factor comes from the definition of the unit of current in the absolute system. As explained in Chap. VII the ab-ampere equals 10 amp.

Reluctance. Permeance. Permeability.—The *reluctance* of a magnetic circuit corresponds to the resistance of the electric

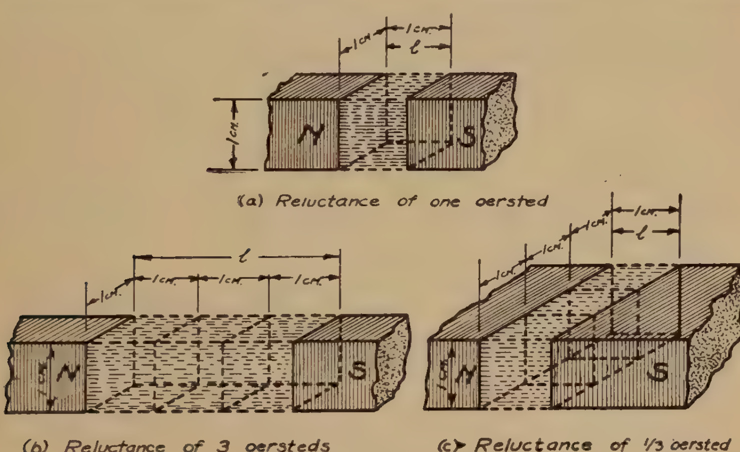


FIG. 5.—Reluctance.

circuit. *Reluctivity* corresponds to resistivity and is the specific reluctance of the given substance, that is, the reluctance per cm.^{3}. *Permeance* of a magnetic circuit corresponds to conductance of the electric circuit and is the reciprocal of the reluctance. *Permeability* is the reciprocal of reluctivity. The permeability of empty space is unity and for most materials very nearly unity and considered as unity in practical computations. Only a

comparatively small group of materials, principally those containing iron, nickel, cobalt, manganese, or chromium have permeabilities larger than unity.

The equations for the quantitative value of reluctance in a magnetic circuit are identical in form with the expressions for resistance in an electric circuit. The reluctance of a magnetic circuit is directly proportional to its length and inversely proportional to its cross-section, and to the permeability of the material carrying the magnetic flux. Let $\mathcal{R}$ represent reluctance; l, length of circuit; A, cross-sectional area of circuit, at right angles to the magnetic flux, and μ the permeability.

$$\mathcal{R} \ \alpha \ \frac{l}{\mu A} \tag{7}$$

The practical unit of reluctance is the *oersted*. If l is given in centimeters, A in $\overline{\text{cm}}.^2$ and μ, the permeability, then the reluctance $\mathcal{R}$ in equation (8) would be in oersteds.

$$\mathcal{R} = \frac{l}{\mu A} \text{ oersteds} \tag{8}$$

Ohm's Law Applied to Magnetic Circuit.—Ohm's law applies to magnetic circuits in precisely the same way as to electric

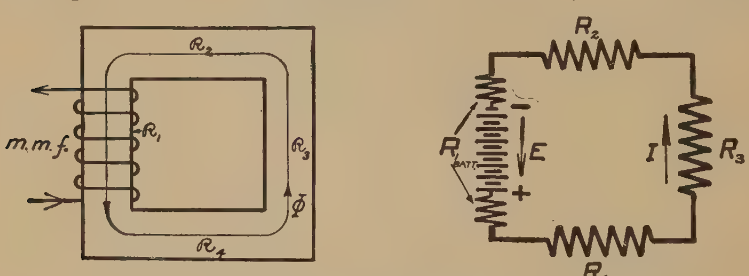


FIG. 6.—Illustrating equation (9).

circuits, by substituting magnetomotive force, magnetic flux, and reluctance, respectively for electromotive force, current and resistance in the electric circuit.

$$\text{magnetic flux} \ \alpha \ \frac{\text{magnetomotive force}}{\text{reluctance}}; \text{ or } \phi \ \alpha \ \frac{\mathcal{F}}{\mathcal{R}} \tag{9}$$

If the magnetic flux ϕ is given in maxwells, the magnetomotive force $\mathcal{F}$ in gilberts, and the reluctance $\mathcal{R}$ in oersteds, Ohm's law for the magnetic circuit is expressed by equation (10).

$$\phi = \frac{\mathcal{F}}{\mathcal{R}}, \text{ or } \mathcal{R} = \frac{\mathcal{F}}{\phi}, \text{ or } \mathcal{F} = \phi\mathcal{R} \tag{10}$$

Equation (11) is obtained from equations (4), (7), and (9).

$$\phi = \frac{\frac{4}{10}\pi NI}{l/\mu A} = \frac{4\pi\mu ANI}{10l} \text{ maxwells} \tag{11}$$

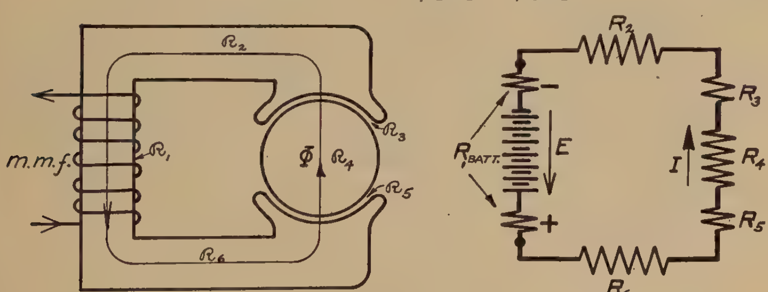
Kirchhoff's Laws Applied to Magnetic Circuit.—Kirchhoff's laws may, like Ohm's law, be applied to magnetic circuits in the same manner as to electric circuits by merely substituting magnetomotive force for electromotive force, magnetic flux for current, and reluctance for resistance.

The *reluctance drops* are considered equivalent to magnetomotive forces in the opposite direction to the impressed magnetomotive force producing the magnetic flux, in the same manner and for the same reason that resistance drops in electric circuits are equivalent to counter-electromotive forces in the electric circuit.

Since all magnetic lines form closed circuits, it is evident that Kirchhoff's current law will apply for any point in the magnetic circuit.

Reluctance of Magnetic Circuits.—In a series circuit the total reluctance equals the sum of the separate reluctances of the several sections. Thus for the magnetic circuit in Fig. 7

$$\mathcal{R}_1 = \mathcal{R}_1 + \mathcal{R}_2 + \text{etc.} = \frac{l_1}{\mu_1 A_1} + \frac{l_2}{\mu_2 A_2} + \text{etc.} \tag{12}$$



(a) (b)

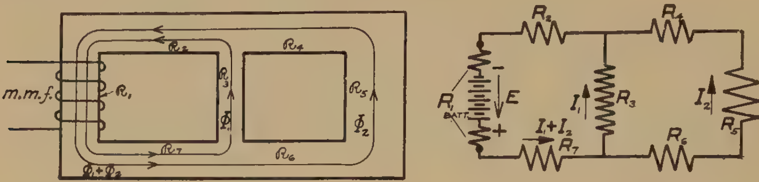
(a) Series magnetic circut of a two pole generator.
(b) Corresponding series electric circuit,

Fig. 7.—Series magnetic circuit.

In parallel circuits the total reluctance is the reciprocal of the sum of the reciprocals of the several reluctances as expressed by equation (13).

$$\mathcal{R} = \frac{1}{\frac{1}{\mathcal{R}_1} + \frac{1}{\mathcal{R}_2} + \text{etc.}} = \frac{1}{\frac{\mu_1 A_1}{l_1} + \frac{\mu_2 A_2}{l_2} + \text{etc.}} \tag{13}$$

Complex magnetic circuits are combinations of simple series and parallel magnetic circuits and may be analyzed in the same manner as complex electric circuits.



(a) (b)

(a) Parallel magnetic circuit.
(b) Corresponding parallel electric circuit.

FIG. 8.—Combination of series and parallel magnetic circuits.

Leakage Flux.—The ratio of the conductivity of good conductors of electric currents, like copper, to that of good insulators, like rubber, is very large, in the order of 10^{22}. As a consequence, under ordinary conditions, electric currents can be confined to definite paths. Thus, the current used for lighting in buildings is kept almost completely within the copper wires and the leakage

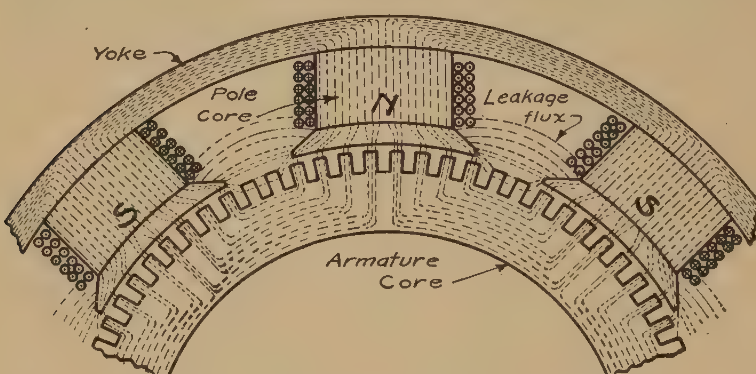


FIG. 9.—Showing leakage flux in dynamo field.

through the insulation is negligible. For some commercial currents the conditions are such that good insulation can not be provided, as for example, in the return circuit through the track rails of an electric street-railway system. It is desired that all the current passing through the street-car motor should return to the generator through the track rails; but the contact of the rails with the street paving does not provide good insulation and hence a considerable portion of the electric current leaks into the ground and returns to the generating station by devious

routes, frequently causing damage to gas and water mains by electrolysis.

Unfortunately, no good insulators exist for magnetic lines of force and the ratio between the poorest and the best magnetic flux conductors is comparatively small. Thus the permeability of all non-magnetic materials is very nearly unity and that of the best available magnetic materials, as good commercial iron and steel under normal operating conditions, less than one thousand.

In direct-current generators and motors the permeability of the iron core and field yokes varies from 50 to 600. It is evident that the reluctance of magnetic circuits or paths between pole tips, as illustrated in Fig. 9, is not very much greater than the circuit through which the useful magnetic flux passes; that is, from a north pole to the armature core to south pole and completing the circuit through the field yoke. The lack of good magnetic insulation, that is, materials having high reluctivity, and the relatively small range in the permeability of magnetic materials are factors that make it impossible to avoid having considerable *leakage flux* in the magnetic circuits of all electric machinery.

In the application of Ohm's and Kirchhoff's laws to magnetic circuits the leakage flux must be taken into account. This greatly complicates the circuits and frequently approximations and assumptions of equivalent circuits must be made in order to make solutions possible. Moreover, the geometric form of magnetic circuits, as for example the fields of a dynamo or motor, are not so readily expressed numerically in terms of length and cross-section as the dimensions of conductors in electric circuits, which generally consist of long wires of uniform cross-section. Hence, although the basic laws for the magnetic circuit are almost identical to those of the electric circuit, their application for finding quantitative solutions of practical magnetic circuits is much more difficult and the results obtained have a lower degree of accuracy than similar values for electric circuits.

PROBLEMS

1. (*a*) What is the reluctance of a round steel rod whose length is 18 in. and diameter 2 in., if the permeability is 1,200?

(*b*) Find the reluctance of a steel rod 8 cm. long, 3 cm. in diameter, if the permeability of the steel is 950.

2. (a) In Fig. 10 what is the total reluctance of the circuit if the permeabilities of the materials in A_1, A_2, A_3, and A_4 are 600, 900, 600, and 700 respectively?

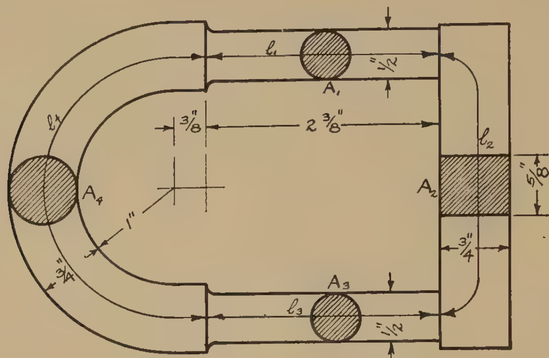


FIG. 10.

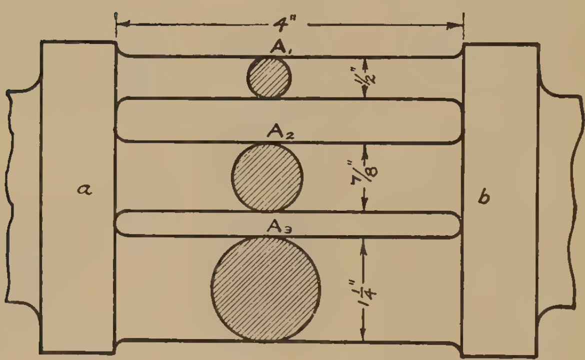


FIG. 11.

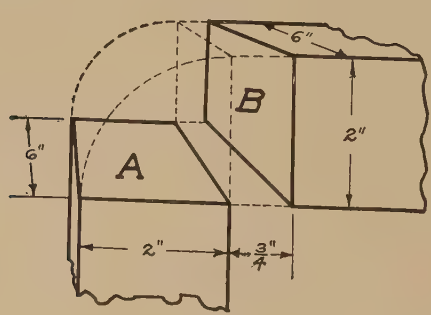


FIG. 12.

(b) How much current must flow through a coil of 120 turns in order to produce a flux of 20,000 lines in the circuit?

3. (*a*) Find the reluctance of the circuit in Fig. 11 from *a* to *b* if the permeabilities of the materials in A_1, A_2, and A_3 are 200, 250, and 160 respectively.

(*b*) How many ampere-turns would be necessary to produce a flux density of 55,000 lines per square inch in A_3?

(*c*) What would be the flux densities per square centimeter in A_1 and A_2 (no leakage assumed) if the flux density in A_3 were 55,000 lines per square inch?

4. In Fig. 12 are shown two steel surfaces *A* and *B* which have a magnetic potential of 410 ampere-turns between them. Assume that the lines of flux are composed of arcs of circles and straight lines as indicated. Find the total amount of flux from *A* to *B*.

CHAPTER IV

THE DIELECTRIC CIRCUIT

Dielectrics.—Any classification of materials necessarily implies that some characteristic property is used as the basis for the grouping. When considered in their relation to the electric circuit, materials may be grouped as conductors and non-conductors or insulators. In connection with the magnetic circuit the basis naturally would be whether the materials are magnetic or non-magnetic. Similarly in the study of *dielectric circuits*, materials may be classified as *dielectrics* and *conductors*. All dielectrics are also insulators but the terms are not synonomous as they relate to distinctly different physical properties of the materials. The chief function of the dielectric in electric systems is to provide insulation between conductors so as to confine as much of the electric current as possible to desired paths and thus establish definite electric circuits. In this respect a good dielectric is simply a very poor conductor. In most dielectrics the resistivity is very high or the conductivity is extremely low as compared to that of metals, and hence electric currents can be confined within the conductors with very little leakage through the insulation. Dielectrics also possess another characteristic of special importance in relation to dielectric circuits, namely, the ability to store electric energy in the form of a dielectric field, similar, in many respects, to the magnetic field. Glass, porcelain, mica, impregnated paper and fabrics, insulating compounds, oil, and air are dielectrics generally used on commercial electric power and communication systems, machinery, and appliances.

Condensers.—An electric condenser consists of two conductors, as for example, plates of metal or layers of tinfoil, separated by a dielectric. Condensers consisting of two layers of tinfoil separated by thin paraffined paper are used extensively in telephone systems. The Leyden jar is merely a glass jar with both the inside and the outside surfaces coated with tinfoil, the glass being the dielectric. Any two conductors in transmission or distribution systems combined with the air in between the spacing form a condenser.

Dielectric Flux. Lines or Tubes of Force.—When the two conductor surfaces of a condenser are connected to some source of electromotive force the dielectric in between will be under

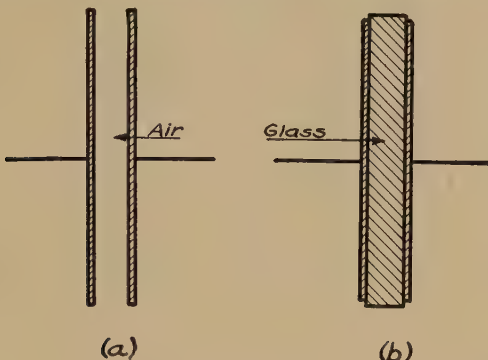


FIG. 1.—Condensers showing dielectrics and conductors.

electric pressure. The difference of potential may be considered as an electric stress which produces more or less strain in the dielectric depending on the material used. The electric stress

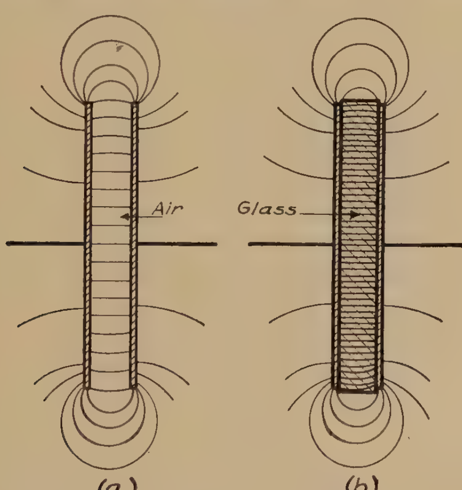


FIG. 2.—Condensers showing lines of force.

and strain are visualized or pictured as represented by, or consisting of *dielectric flux tubes, dielectric flux lines* or *dielectric lines of force.* Figure 2 shows a cross-section of the dielectric field, represented by lines of force between two metal discs with air and glass, respectively, as the dielectric.

In order to represent the physical nature of the dielectric field the dielectric lines of force are endowed with definite properties similar to those of magnetic lines of force. The single important difference is that while magnetic lines of force are closed upon themselves, dielectric lines of force begin and end on the surface of conductors. Specifically the following properties are ascribed to dielectric lines of force:

(*a*) All dielectric lines of force are continuous but terminate on the surface of conductors.

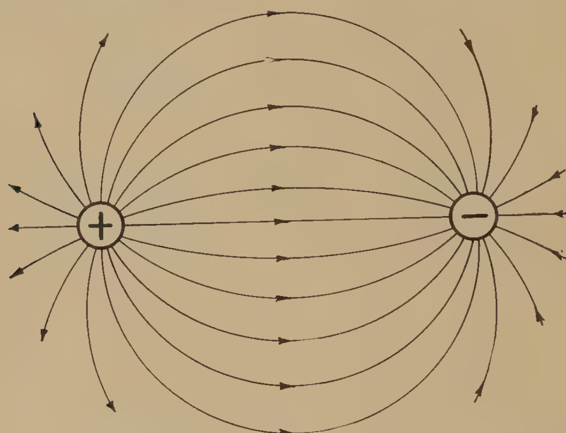


Fig. 3.—Condenser showing dielectric field between two wires.

(*b*) The direction of a dielectric line of force at any point follows the resultant of the dielectric forces acting on a conductor of positive potential or a positive electronic charge at the point.

(*c*) Dielectric lines of force in the same direction repel and in the opposite direction attract one another.

(*d*) The strength of a dielectric field is measured by the density of the dielectric lines of force.

The lines of force in the dielectric field, between the wires of a transmission line are shown in Fig. 3. The direction of the lines, indicated by arrow heads, is in the same direction as the impressed voltage.

The practical unit for dielectric flux is the *dielectric line of force.* The basic definition of the dielectric flux unit may be found under the electrostatic c.g.s. system under System of Units, Chap. VII. The symbol for dielectric flux is the Greek letter ψ or Ψ.

Condensance, Elastance.—The *condensance* of a condenser in a dielectric circuit corresponds to the conductance of the conductor in an electric circuit. *Condensance, capacitance, permittance* and *capacity* are equivalent terms in electrical literature. Permittivity (condensivity) corresponds to conductivity of materials in the electric circuit or permeability in the magnetic circuit. Permittivity is the specific condensance or specific capacitance of the given material; that is, the condensance per $\overline{\text{cm.}}^3$. The permittivity of empty space is unity and very nearly unity for all gases, vegetable fibers, etc., and zero for all good conductors. The range of permittivity in insulator materials is very narrow, in fact less than from zero to ten for dielectrics used in commercial conductors, as may be seen from Table IV. The symbol for permittivity is the Greek letter kappa, κ.

TABLE IV

Material	Permittivity κ	Material	Permittivity κ
Air and other gases	1.0	Mica	4.5 to 7.5
Alcohol, amyl	15.0	Micarta	4.1
Alcohol, ethyl	24.3 to 27.4	Olive oil	3.0 to 3.2
Alcohol, methyl	32.7	Paper with turpentine	2.4
Asphalt	2.7 to 4.1	Paper or jute impregnated	4.3
Bakelite	6.6 to 16.0	Paraffin	1.9 to 2.3
Benzine	1.9	Paraffin oil	1.9
Benzol	2.2 to 2.4	Petroleum	2.0
Cloth varnished	3.5 to 5.5	Porcelain	5.3
Condensite	6.6 to 16.0	Rubber	2.0 to 3.0
Ebonite	1.9 to 3.5	Rubber vulcanized	2.5 to 3.5
Fiber	2.0	Shellac	2.7 to 4.1
Fullerboard	7.5 at 100 C.	Silk	1.6
Fullerboard varnished	2.9 at 25 C.	Slate	6.0 to 7.5
Glass (easily fusible)	2.0 to 5.0	Sulphur	4.0
Glass (difficult to fuse)	5.0 to 10.0	Turpentine	2.2
Gutta percha	3.0 to 5.0	Transformer oil	2.5
Ice	3.0	Vacuum	0.9994
Marble	6.0	Varnish	2.0 to 4.1

Elastance is the reciprocal of condensance. The symbol for elastance is S and for condensance C.

$$S = \frac{1}{C} \qquad (1)$$

Elastance of the dielectric circuit corresponds to the reluctance of the magnetic circuit or the resistance of the electric circuit. The elastance of any given circuit depends on the dimensions of the dielectric and follows laws similar to those for the resistance of the electric circuits; that is, it varies directly as the length l and inversely as the cross-section A, and the permittivity, κ.

$$S \; \alpha \; \frac{l}{\kappa A} \tag{2}$$

Hence, the condensance C varies directly as the permittivity k and the cross-sectional area A and inversely as the length l.

$$C \; \alpha \; \frac{\kappa A}{l} \tag{3}$$

The unit of condensance is the *farad* and the corresponding unit for elastance, the *daraf*.

If l is expressed in centimeters, A in $\overline{\text{cm.}}^2$, and κ, the permittivity of the dielectric, the elastance S is given in darafs by equation (4) and the condensance C in farads in equation (5).

$$S = \frac{4\pi v^2 l}{\kappa A 10^9} = \frac{11.31 \cdot 10^{12} l}{\kappa A} \text{ darafs} \tag{4}$$

$$C = \frac{\kappa A 10^9}{4\pi v^2 l} = \frac{\kappa A}{11.31 \cdot 10^{12} l} \text{ farads} \tag{5}$$

The *farad* is too large a unit for practical purposes and hence in most commercial problems involving dielectric circuits the condensance is measured in *microfarads.*

$$1 \text{ farad} = 10^6 \text{ microfarads} \tag{6}$$

Hence, from equations (5) and (6), the condensance of a circuit is expressed in microfarads by equation (7).

$$C(\text{microfarads}) = 0.8842 \frac{\kappa A (\overline{\text{cm.}}^2)}{l(\text{cm.})} 10^{-7} \tag{7}$$

The constants 4π, v and 10^9 in equations (4), (5), and (7) result from the basic assumptions of unit quantities and definitions in the electrostatic and absolute electromagnetic systems as explained in Chap. VII.

The factor 4π results from the definition of the unit dielectric line of force (unit charge produces one line of force per cm.^{2} on the surface of a sphere of 1 cm. radius). The second factor v is the velocity of propagation of an electromagnetic-dielectric field in space; that is, the ratio of the electrostatic unit to the electromagnetic unit of electric charge.

$$\nu = 3 \cdot 10^{10} \text{ cm./sec.} \tag{8}$$

The third factor 10^9 results from the ratio of the units of current and voltage in the absolute electromagnetic system to the ampere and volt respectively (See Table VIII, Chap. VII).

In Chap. IX it is shown that the farad may also be defined in terms of impressed voltage and resultant current flowing in the dielectric circuit, as follows:

If the voltage impressed on a condenser changes uniformly at the rate of 1 *volt per second and the current produced is* 1 *amp., the condensance of the dielectric circuit is* 1 *farad.* This very important relation is expressed in mathematical form by equation (9) in which ${}_ci$ and e are the instantaneous values of the current and voltage, respectively, and C the condensance.

$${}_ci = C\frac{de}{dt} \qquad (9)$$

In most commercial problems the condensance of a circuit is obtained more readily and with greater accuracy on the basis of equation (9) than equation (5), as it is usually difficult to determine the length and cross-section of the dielectric in the circuit.

Ohm's Law Applied to Dielectric Circuits.—The relation between the voltage, dielectric flux, and elastance or condensance in the dielectric circuit is the same as for the voltage, current, and resistance or conductance of the electric circuit and hence may be stated in the same form, that is, by Ohm's law.

$$\text{Dielectric flux } \alpha \ \frac{\text{voltage}}{\text{elastance}}; \text{ or } \psi \ \alpha \ \frac{E}{S} \qquad (10)$$

If the voltage is given in volts E, the elastance in darafs S, and the dielectric flux ψ, in lines of force (4π and $3 \cdot 10^9$, constants, introduced by definition of line of force, Chap. VII), Ohm's law is expressed by equations (11), (12), or (13).

$$\psi = 4\pi \cdot 3 \cdot 10^9\frac{E}{S} = 3.77 \cdot 10^{10}\frac{E}{S} \qquad (11)$$

$$E = \frac{\psi S}{3.77 \cdot 10^{10}} \qquad (12)$$

$$S = 3.77 \cdot 10^{10}\frac{E}{\psi} \qquad (13)$$

Since the condensance is the reciprocal of the elastance,

$$\psi = 4\pi \cdot 3 \cdot 10^9 CE = 3.77 \cdot 10^{10} CE \qquad (14)$$

$$C = \frac{\psi}{3.77 \cdot 10^{10}E} \qquad (15)$$

Combining equations (5) and (14) gives equation (16) for the lines of dielectric flux in the dimensions of the circuit and permittivity of the dielectric.

$$\psi(\text{lines of force}) = \frac{\kappa A(\overline{\text{cm.}}^2)E(\text{volts})}{300l(\text{cm.})} \tag{16}$$

Kirchhoff's Laws Applied to the Dielectric Circuit.—In the dielectric circuit the *elastance drop* corresponds to the resistance drop in the electric circuit. Kirchhoff's first law applies at any point in the dielectric of a dielectric circuit but not at the surface of conductors, as dielectric flux lines terminate on conductors. Kirchhoff's second law may be applied to dielectric circuits in the same way as to electric circuits by considering the elastance drops as counter-electromotive forces in the circuit.

Elastance or Conductance of Dielectric Circuits.—Since the relation between the dielectric flux, voltage, and elastance or

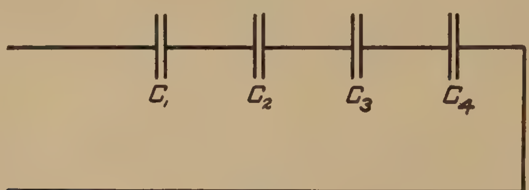


FIG. 4.—Condensers in series.

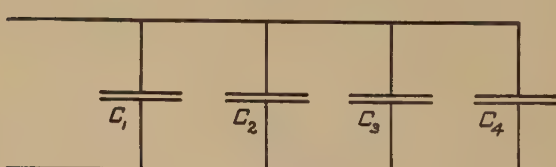


FIG. 5.—Condensers in parallel.

condensance may be expressed by Ohm's law (equations (11) and (14)), it follows that the total elastance or condensance may be found for series and parallel arrangements in the same manner as the total resistance and conductance is found for similar electric circuits.

Therefore, in Fig. 4, the total elastance is the sum of the several elastances in series,

For series circuits:

$$S = S_1 + S_2 + S_3 + S_4 \tag{17}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}. \tag{18}$$

Likewise for parallel circuits as illustrated by Fig. 5 the sum of the several condensances is the total condensance.
For parallel circuits:

$$C = C_1 + C_2 + C_3 + C_4 \tag{19}$$

$$\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \frac{1}{S_4} \tag{20}$$

The distribution of the voltage in series circuits and the current in parallel circuits is therefore the same for the corresponding circuits.

Dielectric Leakage Flux.—The permittivity of dielectric materials is represented by less than 10 as compared to unity for the surrounding space and therefore it is not possible to confine dielectric flux to definite paths of considerable length, as may readily be done with electric currents in electric circuits. The permittivity of the best commercial dielectric materials is only five or six times that of the surrounding air. One might compare the conditions existing in dielectric circuits to an electric circuit consisting of bare copper wire immersed in a salt-water solution. If voltage were impressed on the terminals of the copper wire the resulting current would not be entirely confined to the metal conductor as a considerable leakage current would flow through the salt water. In many condensers the *percentage of leakage flux* is small, however, due to the dimensions of the dielectric circuit; the length of path (for example, the thickness of impregnated paper in telephone condensers) being very short and the cross-section of the dielectric very large. Hence the leakage on the edges of the dielectric forms only a small percentage of the total flux.

Dielectric, Magnetic, and Electric Circuits.—The close analogy existing between magnetic, dielectric, and electric circuits can be shown to advantage by arranging the corresponding quantities in tabular form as in Table V. The three circuits are interdependent and, in general, exist simultaneously in all electromagnetic-dielectric phenomena. The directions of the three factors: (*a*) the electric current, (*b*) the magnetic lines of force, and (*c*) the dielectric lines of force are in space at right angles to each other and exist wherever electric energy is transmitted either by currents in wires or other conductors or in electromagnetic radio waves.

TABLE V

Electric current: $i = Ge = \frac{e}{R}$ amperes	*Dielectric flux (dielectric current):* $\psi = 4\pi \cdot 3 \cdot 10^9 ce = 3.77 \cdot 10^{10} Ce = 3.77 \cdot 10^{10} \frac{E}{S}$ lines of dielectric force	*Magnetic flux (magnetic current):* $\phi = Li10^{-8}$ lines of magnetic force, maxwells, webers One line = 1 maxwell One weber = 10^8 maxwells
Electromotive force, voltage e, volts	*Electromotive force:* e, volts 1 stat volt = 300 volts	*Magnetomotive force:* $\mathfrak{F} = 0.4\pi NI$ gilberts
Conductance: $G = \frac{i}{e}$, mhos.	Condensance, capacitance, permittance or capacity $C = \frac{\psi}{3 \cdot 77 \cdot 10^{10} e}$ farads	*Inductance:* $L = \frac{\phi}{i} 10^{-8} = \frac{N\phi}{I} 10^{-8}$ henrys
Resistance: $R = \frac{e}{i}$, ohms	*Elastance:* $S = \frac{1}{C} = \frac{3.77 \cdot 10^{10} e}{\psi}$ darafs	*Reluctance:* $\mathcal{R} = \frac{\mathfrak{F}}{\phi}$ oersteds
Electric power: $P = ei = Ri^2 = Ge^2$ watts	*Dielectric energy:* $W = \frac{\psi e}{7.54 \cdot 10^{10}} = \frac{Ce^2}{2}$ joules	*Magnetic energy:* $W = \frac{\phi i}{2} 10^{-8} = \frac{Li^2}{2}$ joules
Electric-current density $I_d = \frac{i}{A}$ amperes per $\overline{\text{cm}}^2$	*Dielectric-flux density:* $D = \frac{\psi}{A} = \kappa K$ lines per centimeter	*Magnetic-flux density:* $\mathcal{B} = \frac{\phi}{A}$ lines per $\overline{\text{cm.}}^2$ gausses One maxwell per $\overline{\text{cm.}}^2$ = 1 gauss
Electric gradient: $G' = \frac{e}{l}$ volts per centimeter	*Dielectric gradient:* $G^1 = \frac{e}{l}$ volts per centimeter	*Magnetic gradient:* $\mathfrak{F}' = \frac{\mathfrak{F}}{l}$ ampere-turns per centimeter
Conductivity: $y = \frac{I_d}{G'}$ ohms per $\overline{\text{cm}}^3$	*Condensivity, permittivity, or specific capacity* $\kappa = \frac{D}{K}$	*Permeability:* $\mu = \frac{B}{A}$
Resistivity: $\rho = \frac{1}{Y} = \frac{G'}{I_d}$ ohms per centimeter	*Elastivity:* $\mathfrak{F} = \frac{1}{K} = \frac{K}{D}$	*Reluctivity:* $\nu = \frac{\mathfrak{F}^1}{\mathcal{B}}$

The relative magnitude of the energy in the three circuits may vary greatly and in many cases one or even two may be negligibly

small, but intrinsically all three are present. For a unidirectional constant current no energy changes occur in the magnetic circuit, and similarly for a unidirectional constant voltage the dielectric field is constant. With both current and voltage unidirectional and constant the electric circuit alone transfers energy and hence is the only circuit that enters into the problem. Other aspects of the dielectric field are discussed in Chap. IX.

PROBLEMS

1. A single unit, variable plate, air condenser, as used in radio sets, has an air space between plates of 1/8 in., when set so as to have maximum capacity. The plates are half-sections of a circle of 1½ in. radius. Assuming no leakage of dielectric flux around the edges find the condensance at maximum setting.

2. A multiple-plate, variable condenser has 20 positive plates and 19 negative plates which are spaced 3/32 in. from negative to positive plates when set at maximum value. The plates are half-sections of a circle whose diameter is 4 in. Find maximum condensance, assuming that there is no

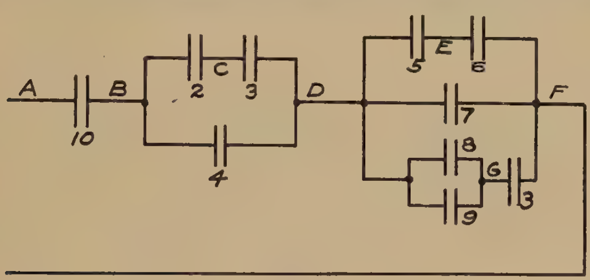


FIG. 6.

leakage flux. If the whole condenser were immersed in methyl alcohol what would be the condensance?

3. A telephone condenser is made up of alternate sheets of paraffin paper and tinfoil. The paper is 0.003 in. thick and the tinfoil sheets are 4 by 8 in. There are 200 sheets of paper. Alternate sheets of tinfoil are connected together to form one terminal and the remaining sheets form the other terminal. Find the condensance of the condenser. Permittivity of paraffin paper is 2.1.

4. Five condensers have a value of 2.6, 3.4, 5.8, 7.1 and 6.3 microfarads respectively. (*a*) Find the total condensance when all five condensers are connected in series. (*b*) Connected in parallel.

5. If the numbers on the diagram in Fig. 6 denote the condensances in microfarads of the respective condensers find the total condensance from *A* to *F*.

CHAPTER V

GENERATION OF VOLTAGE, CURRENT AND FORCE

Sources of Electromotive Force.—The generation of electromotive force and the resulting electric current, if in a closed circuit, implies the conversion of energy of some kind into the electric form. The principal sources are *chemical action* and *electromagnetic induction*, but electromotive force may be produced in other ways as from heat, light, mechanical friction and pressure, and by electrostatic induction.

By heating certain crystals, as tourmaline, a slight difference of potential is developed between the two ends. This is termed *pyro-electricity* and while of interest as a scientific fact has not proved to be of practical value.

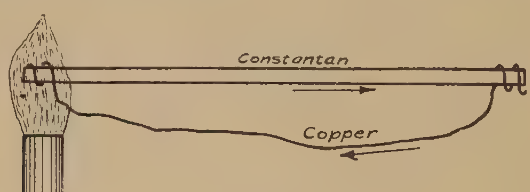


Fig. 1.—Thermocouple.

By keeping one junction of two metal conductors at a higher temperature than the other ends, as illustrated in Fig. 1, voltage is produced which will cause an electric current to flow if the other ends of the conductors are joined so as to form an electric circuit. This phenomenon of transforming heat directly into electricity is called *thermoelectricity* and was discovered by Lubeck in 1821. The junction of the two conductors is called a *thermocouple* (Fig. 1). Several thermocouples joined in series form a *thermopile*, as illustrated in Fig. 2. Every other junction only is heated. The total voltage between the terminals of the thermopile is the sum of the voltages generated at the several junctions connected in series. Bismuth and antimony give the highest voltage per junction: about 0.000,057 volt per degree centigrade difference in temperature. A junction of copper and

constantan (60 per cent copper and 40 per cent nickel) gives 0.00004 volt per degree centigrade.

The principal use of thermocouple is in thermometers for measuring high temperatures as in ovens or furnaces or for measuring temperature at inaccessible places.

Electromotive force is generated directly from mechanical energy by *friction* as in rubbing a glass with silk, or the friction produced between running belts and the surrounding air, etc. Certain crystals develop slight difference of potential if subjected to *pressure.* The *cleavage* of mica and other materials may produce a difference of potential between the parts formed.

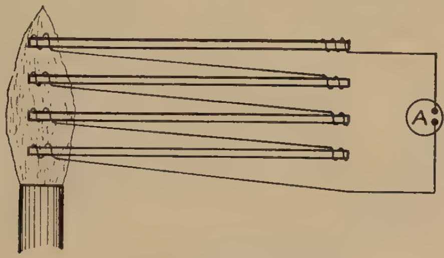


FIG. 2.—Thermopile.

Electric influence machines develop voltage by electrostatic induction. The voltage generated may be thousands of volts although the amount of energy involved usually is small. In principle, influence machines or *electrostatic-induction* generators are very interesting but their practical application is limited. Electrostatic induction is an important factor in lightning phenomena, "static" and other high-voltage disturbances, and can be studied to better advantage after first acquiring a mastery of the basic laws of direct and alternating currents.

The generation of voltage by *chemical action* is of considerable practical importance as both primary and storage batteries have wide application in many important fields. Electrical appliances depending on voltage produced by chemical action are, however, generally accessories to the machinery and electric systems, based on the production of voltage by *electromagnetic induction.* A discussion of the principles involved for the production of voltage by chemical action and their application to industrial requirements is given in Chap. XIX.

Electromagnetic Induction.—When magnetic lines of force are cut by an electrical conductor an electromotive force is

generated in the conductor. This fundamental principle is called *electromagnetic induction* and was discovered in 1831 independently by Joseph Henry in America and Michael Faraday in England. The voltage produced is directly proportional to the length of the conductor, the intensity of the magnetic field and the speed of the motion perpendicular to the magnetic field and the direction of the conductor, or the *voltage generated is proportional to the rate of cutting lines of force.*

$$e \propto \frac{d\phi}{dt} \tag{1}$$

To produce 1 volt of electromotive force 10^8 lines of the magnetic flux must be cut, at a uniform rate, per second.

$$\begin{aligned} 1 \text{ volt} &= 10^8 \text{ lines of force per second} \\ &= 1 \text{ weber per second} \end{aligned} \tag{2}$$

Faraday proved experimentally that the *relative motion* of the field with respect to the conductor was the determining factor. In direct-current generators the field is usually stationary while the armature conductors cut the lines of force, thus generating voltage. In large alternators more economical designs are obtained by placing the fields on the rotating spider while the armature conductors are stationary, surrounding the moving field. In transformers the magnetic lines of force move, due to the periodic increase and decrease of the current, while both the primary and secondary conductors are stationary. In all cases the relative motion is the determining factor; that is, the time rate at which the conductor cuts magnetic lines of force.

Direction of Generated Voltage.—In order to determine readily the direction of the voltage generated by the cutting of lines of force, that is, by electromagnetic induction, several rules or devices have been suggested. Electricians frequently use Fleming's rule although a better and a more easily applied method is to visualize the direction of the lines of force in connection with the motion of the conductor with respect to the magnetic flux.

Fleming's Rule.—(*a*) *Right-hand Rule: Generator Action.*—If the thumb, forefinger and middle finger of the right hand are all set perpendicularly to each other, as shown in (*a*) Fig. 3, so as to represent the three coordinates in space, the thumb pointed in the direction of the motion of the conductor relative to the magnetic field, and the forefinger in the direction of the magnetic

lines of force, then the middle finger will point in the direction in which the generated or induced e.m.f. tends to send the current of electricity.

(*b*) *Left-hand Rule: Motor Action.*—In the same manner, using the left hand, if the forefinger points in the direction of the magnetic lines of force, the middle finger in the direction of flow of the current, then the thumb will point in the direction of the mechanical force on the conductor as shown in (*b*) Fig. 3.

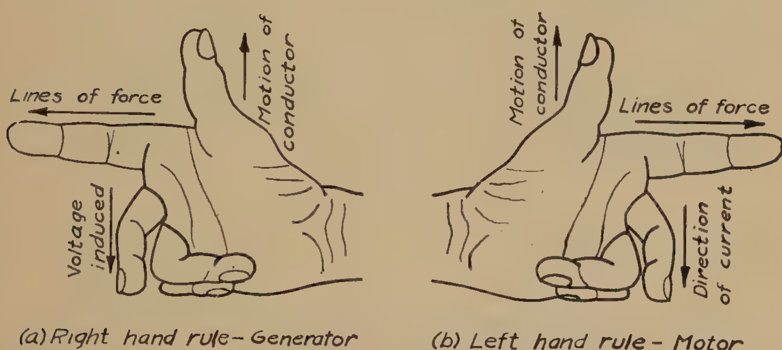


FIG. 3.—Illustrating Fleming's rule.

An easily remembered and readily used method is found in visualizing the relative motion of the conductor and the magnetic field and keeping in mind the properties already ascribed to the lines of force. In Fig. 4, let the conductor, relative to the magnetic lines of force, assume successively the first to the fifth positions. When the conductor is moved across the lines of force these bend and form small circles around the conductor as indicated in the figure. Given the direction of the magnetic field, it is evident that the conductor in the second and following positions encircled by lines of force whose direction indicates that the voltage generated tends to cause the current to flow downward; that is, away from the observer. It is evident that if either the direction of the field flux lines or the relative direction of the motion of the conductors and the flux were reversed, the circular lines of force around the conductor would also be in the opposite direction, and hence indicate a voltage tending to send the current upward; that is, toward the observer.

The lines of force around a current flowing in a conductor expand and increase in number or contract and decrease in number as the current increases or decreases in magnitude.

In Fig. 5 let A and B represent parallel conductors, and let a current of increasing strength be sent through A, in direction towards the observer. The direction of the arrows on the lines of force around B show that the induced voltage in B is in the opposite direction to the voltage in A. With decreasing current in A the lines of force contract in size and move toward the conductor. As a consequence lines of force would cut conductor B and leave lines that encircle B in the opposite direction, indi-

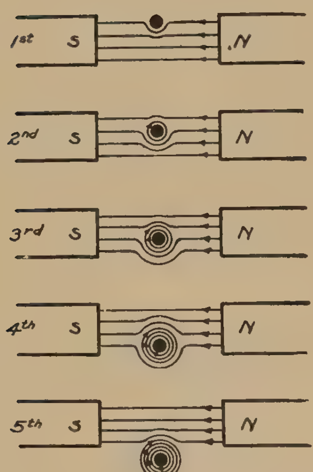


Fig. 4.—Conductor cutting lines of force.

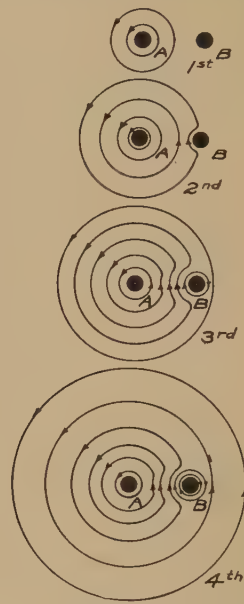


Fig. 5.—Magnetic field cutting conductor.

cating an induced voltage in B in the same direction as the voltage in A. By visualizing the direction and relative motion of the lines of force and the conductors, the direction of the induced voltage can readily be determined.

Lenz's Law.—If the current in a circuit changes in magnitude, the self-induced voltage produced by the change in the field flux around the circuit always tends to send a current in such a direction as to oppose the change in flux which produces it. This simply states that in the electrical field, as in mechanics, action and reaction are equal but opposite in direction.

Instantaneous Values, Wave Forms.—The electromotive force generated at any instant depends solely on the *rate of*

cutting lines of force. This simple relation or single factor thus determines the voltage-time curve or the voltage wave shape. The rate of cutting will depend on the distribution and direction of the magnetic flux lines, the arrangement and the number of conductors in series and the speed. From equation (1), it is evident that while a conductor moves at a uniform speed across a uniform field the voltage generated will be constant. If the

Fig. 6.

field is reversed in direction for half the revolution as in Fig. 6, the voltage also reverses. In Fig. 7 is shown the voltage time curve or voltage wave produced by a single conductor moving at right angles to the field illustrated in Fig. 6, for the conductor between rings *a* and *b*.

Evidently the voltage wave between rings *a* and *c* will have the same rectangular shape but of double amplitude, as shown in Fig. 8, since the two conductors are in symmetrical positions

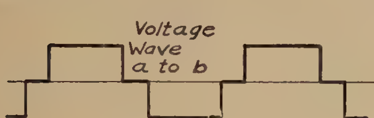


Fig. 7.—Single conductor.

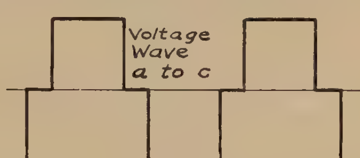


Fig. 8.—Two conductors in series.

on the rotating armature and connected in series. In the portion not covered by the poles no lines are present and hence no voltage is generated.

If the distribution of the magnetic flux be not uniform but concentrated as in Fig. 9, the resulting voltage wave for the winding shown in the diagram becomes peaked as illustrated by the wave in Fig. 10.

The shape of the voltage wave is affected by the number and spacing of the conductors between the points measured as well

as by the density and direction of the lines of force. In a uniform field, as in Fig. 11, the rate of cutting magnetic flux lines by a single conductor will depend on the speed and the angular position of the conductor. If the conductor moves at a uniform

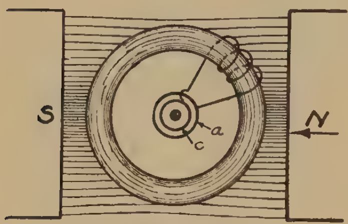


Fig. 9.

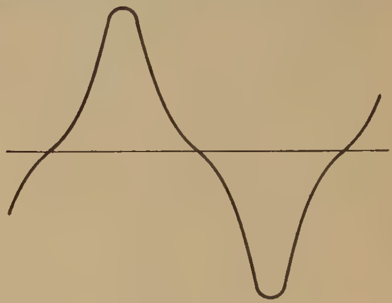


Fig. 10.

speed the voltage wave, generated under these conditions, will be a simple sine wave as shown in Fig. 12. The relation between the maximum and instantaneous voltage values is given by equation (3).

$$e = {}^{M}E \sin 2\pi f(t - t_1) \tag{3}$$

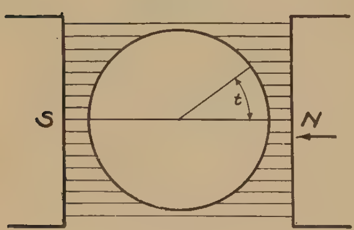


Fig. 11.

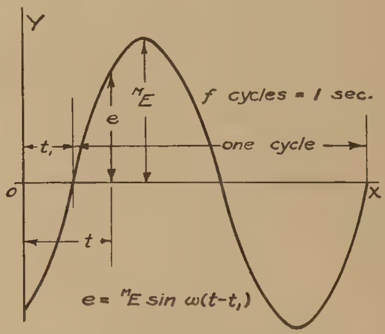


Fig. 12.

In equation (3) as indicated on the diagram in Fig. 12,

e = instantaneous voltage;

${}^{M}E$ = maximum voltage;

f = number of cycles per second, or the frequency of the wave;

t_1 = epoch or phase of the voltage wave; and

t = time in seconds.

In the design of alternators the distribution of the magnetic flux lines and the spacing and number of conductors connected

in series in the armature must be so arranged that the resulting generated voltage will be approximately a sine wave as expressed by equation (3).

In direct-current generators the spacing and number of conductors connected in series must be designed so as to give a resultant constant voltage between the brushes. In any single

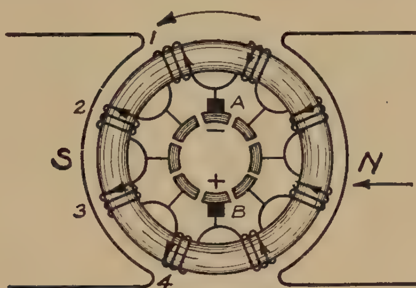


FIG. 13.—Diagram of two-pole direct-current generator.

armature conductor, as in Fig. 13, the generated voltage must necessarily be reversed in direction when moving under a north pole, as compared to the direction when passing under either the preceding or following south pole of the magnetic field. The voltage between the stationary brushes A and B (Fig. 13), however, making contact with segments of the commutator to

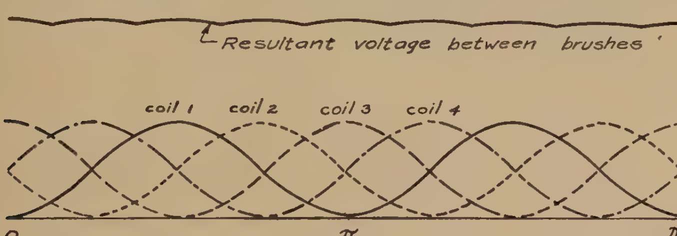


FIG. 14.—Voltage waves between segments and between brushes for Fig. 13.

which the conductors are connected in series, is continuously in the same direction and of constant magnitude. In Fig. 14 are shown the voltage generated in each armature conductor of the generator in Fig. 13, and the resultant nearly constant voltage between the brushes. The generation of voltage by the conductors on a revolving armature cutting magnetic flux lines combined with a continuous coordination in direction of the voltage in the several conductors by means of stationary collecting brushes,

in sliding contact with a revolving commutator, is the basic principle of direct-current generators.

Electric Current.—The generation of voltage precedes the production of an electric current. Voltage generated in a conductor forming a closed circuit will cause an electric current to flow through the circuit in accord with Ohm's law, as explained in Chap. II.

Mechanical Force on Conductors.—In 1820, eleven years preceding the discovery of electromagnetic induction, Oersted observed that if a conductor carrying an electric current be placed in a magnetic field a mechanical force would be exerted on the current and conductor. In the same year Ampere extended the investigation so as to include the reaction or mechanical force between two or more conductors carrying electric currents.

Oersted determined the quantitative relations of the factors involved and showed that the mechanical force or reaction between the field and conductor is directly proport onal to the magnetic field strength $\mathcal{B}$, to the length l of the conductor, to the magnitude of the current I flowing in the conductor, and to the sine of the angular direction θ of the conductor with respect to the direction of the field flux, as expressed by equation (4).

$$F \propto \mathcal{B} l I \sin \theta \tag{4}$$

If the direction of the conductor is at right angles to the direction of the field lines of force $\theta = 90$ deg.; and the direction of the resulting mechanical force would be at right angles both to the field flux and to the direction of the conductor carrying the electric current. In order to obtain the numerical values of the factors in equation (4) a consistent set of units must be used. Thus the relation is expressed quantitatively in equation (5) for the stated units:

$$F = \frac{\mathcal{B} l I \sin \theta}{10} \tag{5}$$

F = force on the conductor in dynes.
$\mathcal{B}$ = magnetic field flux density in gausses.
l = length of conductor in centimeters.
I = current in amperes.
θ = angular inclination of conductor to field flux in degrees.

All dynamos operate on the basic relation expressed by equation (5). For generators the prime mover exerts mechanical force necessary to move the armature conductors in cutting the field

flux, thereby generating voltage which in turn causes currents to flow in the electric circuit. For motors the impressed voltage causes currents to flow in the armature conductors, which in turn react with the field flux producing the mechanical torque required to carry the load.

PROBLEMS

1. A conductor is 20 in. long and is moving at the rate of 100 ft. per second at right angles through a magnetic field whose intensity is 10,000 lines per square centimeter. What voltage is being generated in the conductor?

2. In Fig. 6 the ratio of pole span to pole pitch is 0.8. The conductor length is 30 cm. The diameter of the armature is 20 cm. The flux density is assumed uniform around the armature at 50,000 lines per square inch. If the r.p.m. of the armature is 1,800, what is the maximum voltage between the slip rings *a* and *c*? Draw by rectangular coordinates the voltage time wave indicating the values and positions on the sketch.

3. Given an elementary alternator, as indicated in Fig. 6 (*a*), consisting of two conductors *ab* and *bc* rotating in a uniform field flux as shown in Fig. 11. The speed of the armature is 1,800 r.p.m. and the flux density is 10,000 per square centimeter. The length of each conductor is 32 cm. and the diameter of the armature is 24 cm. Find the maximum value of voltage generated. What is the average value of voltage generated for one half revolution?

How many cycles per second?

If time is being measured 1/1,000 sec. after the wave passes through zero in a positive direction write the equation for the voltage wave.

Plot the voltage time wave in rectangular coordinates.

4. The earth inductor compass as used on airplanes consists of a small armature with many conductors, rotating in space cutting the earth's field. By swinging the armature around, which can be rotated in every possible direction, a position may be obtained where no flux is cut. This position will naturally be where the axis of the armature coincides with the direction of the earth's field. If the plane under these conditions is then headed in the correct direction toward its destination the pilot simply notes that the galvanometer on the instrument board at all times reads zero. Any deviation of the plane from the correct course will become apparent by a deflection of the galvanometer due to the cutting of lines of the earth's field.

Let it be assumed that the earth's field is formed of parallel lines between the two points where a flight is to be undertaken and that it has a density of 0.6 gauss with a dip of 30 deg. with the horizontal position. The armature is 2 in. in diameter and 3 in. long with 800 conductors wound on it which are in series between the terminals leading to the galvanometer. The average speed of rotation of the armature is 2,000 r.p.m. If a contemplated trip is 2,100 miles (San Francisco to Honolulu), what voltage must the galvanometer be visibly deflected by in order to give indications sufficiently accurate for a landing within 100 miles of either side of the destination?

CHAPTER VI

ELECTRIC POWER. ELECTRIC ENERGY

Electric Power.—In the discussion of the electric circuit, Chap. II, the flow of the electric current in a conductor was likened to the flow of water in a pipe. This analogy may be extended to advantage and used as a means for gaining clear concepts of the factors involved in electric power and energy. Consider a pipe in which water is flowing. Let the pressure, at a given point in the line, required to make the water flow in the pipe be F lbs. and let the rate of flow at the given point be V ft. per second. It follows that the mechanical power expended in causing the water to flow through the pipe would be the product of the force F and the rate of flow or velocity V.

$$\begin{aligned}\text{Mechanical power} &= FV = \text{pounds} \times \text{feet per second} \\ &= FV, \text{ foot-pounds per second} \qquad (1)\end{aligned}$$

Similarly, electric power P in an electric circuit is determined by two factors; the voltage or electric pressure E and the current I. The product of the voltage and current represents *electric power*.

$$\text{Electric power, } P = EI \qquad (2)$$

The practical units for electric power are the *watt* and *kilowatt*. If the voltage E is expressed in volts and the current I in amperes, the power P will be given in watts.

$$P = EI \text{ watts} \qquad (3)$$

$$1{,}000 \text{ watts} = 1 \text{ kilowatt} = 1 \text{ kw.} \qquad (4)$$

In alternating currents the *volt-ampere* and *kilovolt-ampere* are also used as practical units. A volt-ampere is equivalent to the watt and the kilovolt-ampere (kva.) to the kilowatt (kw.) in magnitude. The reason for using both kilowatt and kilovolt-ampere is that in alternating-current systems part of the energy delivered by the alternator to the distribution system returns later in the cycle to the generator and hence only part of the energy flowing past any point in the system is left in the load.

The amount of energy flowing past any given point, without regard to its direction of flow during the alternating-current cycle, is measured in volt-amperes or kilovolt-amperes, while the power represented by the energy that does not return to the generator is measured in watts and kilowatts.

In direct-current circuits under steady or constant conditions all of the electric energy delivered by the dynamo is consumed in the system; that is, no part returns to the dynamo and hence the watt and kilowatt are the proper units for electric power. The relation between the horsepower (hp.) in mechanical energy and the electrical units is given in equation (5).

$$1 \text{ hp.} = 746 \text{ watts} = 0.746 \text{ kw.} \tag{5}$$

Electric Energy.—The product of mechanical power P and the time t during which it acts represents mechanical work or mechanical energy W.

$$\text{Mechanical energy } W = Pt = FVt \tag{6}$$

For the same reason the product of electric power P and the time t during which it flows represents the electric energy W involved.

$$\text{Electric energy } W = Pt = EIt \tag{7}$$

The practical units for electric energy are the *watt-second* or *joule*, and the *kilowatt-hour* (kw.-hr.).

If power P is measured in watts and time t in seconds,

$$W = Pt = \text{watt-seconds or joules} \tag{8}$$

If power P is measured in kilowatts and time t in hours,

$$W = Pt = \text{kilowatt-hours (kw.-hr.)} \tag{9}$$

Joule's Law.—The quantitative relation between the units for measuring energy in its various forms have been determined experimentally with great accuracy. In 1841 Joule proved that the electric energy converted into heat when an electric current flows in a conductor is directly proportional to the resistance of the conductor, to the square of the current and to the length of time the current is flowing. Stated in mathematical terms *Joule's law* for direct currents is expressed, by equation (10):

$$\text{Heat} = RI^2t \tag{10}$$

If the heat H is expressed in gram-calories, the resistance R in ohms, the current I in amperes, and the time t in seconds, the relation is given quantitatively by equation (11):

$$H = 0.24RI^2t \text{ gram-calories} \tag{11}$$

In heating appliances, as electric ranges, flat irons, etc., all of the electric energy is transformed into heat as expressed by equation (11). Since all of the voltage is consumed by the resistance drop the expression for energy in equation (7) may be written as in equation (12). If R is given in ohms, I in amperes, and t in seconds the energy W will be expressed in watt-seconds or joules.

$$W = Pt = EIt = RI^2t \text{ joules} \tag{12}$$

In other apparatus like incandescent lamps, electric motors, etc., only part of the energy is converted into heat, while the remaining portion is transformed into light, mechanical work, or some other form of energy. For example, in a circuit supplying power to a certain electric motor, 87 per cent of the energy was converted into mechanical work, while 15 per cent appeared as heat chiefly due to the RI^2 losses in the circuit. Under these conditions equation (13) would express the energy relation if W represents the electric energy input, R the equivalent resistance for the heat losses, I the current, and P the mechanical power.

$$W = Pt + RI^2t \tag{13}$$

Since the purpose of an electric motor is to convert electric energy into mechanical work the efficiency of the above motor would be 87 per cent and the losses 13 per cent. In heating appliances (resistor types) all of the electric energy is transformed into heat and in this respect resistance-heating appliances are 100 per cent efficient.

The following list of energy equivalents are given for convenience in solving problems:

1 joule = 1 watt-sec. = 10^7 ergs = 0.7376 ft.-lb.
= 0.2389 g.-cal. = 0.102 kg.-m. = 0.000948 B.t.u.
1 ft.-lb. = 1.356 joules = 0.3239 g.-cal. = 0.1383 kg.-m.
= 0.001285 B.t.u. = 0.0003766 watt-hr.
1 B.t.u. = 1,054.8 joules = 778.1 ft.-lb. = 252 g.-cal.
= 0.2930 watt.-hr.

Generator and Motor Action.—The conversion of mechanical work into electric energy is the province of generators. In Fig. 1 is shown a diagrammatical representation of the basic interaction between magnetic fluxes in the field and armature of direct-current generators. The poles pieces and coils marked F represent the magnetic fields, the circle A the rotating armature, while the brushes through which the current is collected from

the armature are lettered B and D and the direction of the voltage E generated in the armature is indicated by the plus and minus signs.

If the brushes be connected to some outside circuit having a resistance R, a current will flow in accord with Ohm's law. When a current flows in the armature conductors a mechanical force in excess of that required to overcome friction must be applied to make the armature rotate in order to overcome the counter-force produced between the field flux and the lines of force encircling the conductors carrying the armature current. The applied

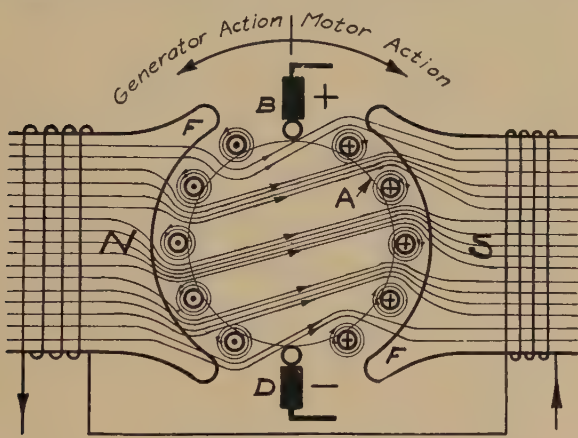


FIG. 1.—Diagram of direct-current generator and motor, illustrating basic relations in transforming mechanical energy to electrical energy (generator action) and the reverse process (motor action).

mechanical force multiplied by the distance over which it acts represents mechanical energy or work. The opposing force, represented by the repulsion between the field magnetic flux lines and the armature current lines of force, multiplied by the distance over which the armature conductor moves represents the energy transformed into the electrical form. The *generator* is therefore a machine which transforms mechanical energy into electrical energy. The complex magnetic field interactions producing this transformation may be pictured or visualized by means of Faraday's simple system of lines of force.

The reverse process—of transforming electric energy into mechanical energy—is accomplished by the *electric motor*. In principle the direct-current motor is simply a generator operated backwards. Evidently, if voltage from an outside source be

applied to the brushes of a generator a current would flow through the armature conductors. The lines of force encircling the current in each conductor would react with the field flux lines repelling on one side and attracting on the other, as indicated in Fig. 1. Thus the lines of force provide a ready means for gaining insight into the action of the electric motor—a machine which transforms electric energy into mechanical work.

PROBLEMS

1. An electric generator delivers a current of 400 amp. at 250 volts. What is the horsepower rating of the driving engine if the efficiency of the generator at the above load is 93 per cent?

2. A 15-hp. motor operates from a 125-volt supply at an efficiency of 86 per cent when under full load. Find the full load current of the motor.

3. An electric water heater is to be designed so that it will raise the temperature of a tank of water from 20 to 100°C. in 10 min. The tank is cylindrical in form, 1 ft. in diameter, and is filled with water to a height of 6 ft. What value of resistance should the heating coil have, assuming that 10 per cent of the electrical input obtained from 110-volt mains will be lost by radiation?

4. Power to a factory is supplied by a transmission line at 440 volts. The load current is 500 amp. The resistance of each of the two lead-in wires connecting the transmission line to the factor is 0.015 ohm. How many kilowatt-hours will the meter register at the load end of the line if the load stays constant during 8 hrs. a day for 30 days? If electrical energy costs 2 cts. per kilowatt-hour what should be the charge?

5. In Problem 4, how much energy is supplied by the transmission line to the lead-in wire, how much is received in the factory and how much is lost as heat in the lead-in wires? At 2 cts. per kilowatt-hour, what would be the value of the energy lost in the lead-in wires?

6. A 30-watt lamp burns continuously for 24 hrs. How high must a 10,000-lb. weight be raised to possess an equal amount of potential energy?

7. A flat iron has a resistance of 25 ohms and the voltage of the supply circuit is 122 volts. How much heat is generated per hour in B.T.U.? Find the cost per B.T.U. if the cost of the electric energy is 5½ cts. per kilowatt-hour.

8. A factory load is 50 kw. at the load point. The voltage of the distribution line is 2,300 volts. The resistance of the conductors connecting the distribution line to the load is 0.275 ohm each. Find: (*a*) the current required for the given load; (*b*) the voltage at the load.

9. Given 5 lb. of 0.0201 by ¼ in. nichrome resistance ribbon which when operating at a current density of 204 cir. mils per ampere (corresponding to a temperature of 500°C.) has a resistivity of 720 ohms per circular mil-foot. The wire weighs 17.6 lb. per 1,000 ft. Design the largest heater possible, using the above amount of wire and operating at the given current density on 110-volt mains. Find the current and power taken by the heater.

CHAPTER VII

SYSTEMS OF UNITS. SYMBOLS

The elemental units in mechanics for length, mass and time, as defined in the metric system, are used in systems of units for electric quantities. The *centimeter* is the unit of *length*, the *gram* the unit of *mass*, and the *second* the unit of *time*, and for this reason the metric system is frequently referred to as the centimeter-gram-second system or the c.g.s. system. The decimal plan of the c.g.s. system is also adhered to in forming secondary electric units. Larger or smaller secondary units are obtained by multiplying the fundamental unit in the system by the factor 10^x, in which x may be any positive or negative integer. The names for the secondary units are obtained by using prefixes that indicate the relative magnitude of the unit with respect to the standard for the system.

The more generally used prefixes are given in Table VI.

Table VI

micro	1/1,000,000	10^{-6}
milli	1/1,000	10^{-3}
centi	1/100	10^{-2}
deci	1/10	10^{-1}
deka	10	10
hecto	100	10^{2}
kilo	1,000	10^{3}
myria	10,000	10^{4}
mega	1,000,000	10^{6}

To illustrate, 1/1,000 meter is a millimeter and 1,000 grams a kilogram, 100 liters a hecto-liter and 1,000,000 ohms a megaohm or megohm.

The basic units in mechanics for both the c.g.s. and English systems used in direct-current engineering are listed with conversion factors in Table VII.

Several systems of units for electric quantities have been developed and are more or less in general use. Three are of

TABLE VII.—FUNDAMENTAL AND MECHANICAL UNITS. CONVERSION FACTORS.

Quantity	Symbol	Defining equation	Metric system c.g.s. units	Abbreviation	English system foot-pound-second units	Abbreviation	Equivalent values
Length	l	Fundamental	Centimeter	cm.	Foot Inch	ft. in.	1 foot = 30.48 centimeters 1 inch = 2.54 centimeters
Mass	m	Fundamental	Gram	g.	Pound	lb.	1 pound = 453.59 grams
Time	t	Fundamental	Second	sec.	Minute Second	min. sec.	1 minute = 60 seconds
Area	A	$A = l_1 l_2$	Square centimeter	sq. cm. $\overline{\text{cm.}}^2$	Square foot Square inch	sq. ft. sq. in.	1 square foot = 929.03 $\overline{\text{cm.}}^2$ 1 square inch = 6.45 $\overline{\text{cm.}}^2$
Volume	V	$V = l_1 l_2 l_2$	Cubic centimeter	cu. cm. $\overline{\text{cm.}}^3$	Cubic foot Cubic inch	cu. ft. cu. in.	1 cubic foot = 28,317 $\overline{\text{cm.}}^3$ 1 cubic inch = 16.39 $\overline{\text{cm.}}^3$
Angle	α β γ	$\alpha = \frac{jl_1}{l_2}$	Radian Degree Minute Second	rad. deg.; ° min.; ′ sec.; ″	.... Degree Minute Second	.... deg.; ° min.; ′ sec.; ″	1 radian = 57° 17.1′ 1 degree = 60 minutes 1 minute = 60 seconds 1° = 60′ 1′ = 60″
Velocity (linear)	v	$v = \frac{dl}{dt}$	Centimeter per second	cm. per sec.	Feet per minute Feet per second	ft. per min. ft. per sec.	1 foot per minute = 1828.8 centimeters per second 1 foot per second = 30.48 centimeters per second
Velocity (angular)	ω	$\omega = \frac{d\alpha}{dt}$	Radians per second	rad. per sec.			
Acceleration	a	$a = \frac{dv}{dt}$	Centimeter per second per second	cm. per sec. per sec.	Feet per sec. per sec.	ft. per sec. per sec.	1 foot per · second per second = 30.48 centimeters per second., per second
Force	F	$F = ma$	Dyne	dyne	Pound (av.)	lb.	1 pound = 4.448 × 10^5 dyne
Torque	T	$T = \frac{W}{\alpha}$	Dyne perpendicular-centimeter	dyne-perp.-cm.	Pound perp. foot	lb. perp. ft.	1 pound perp. foot = 1.356 × 10^7 dynes perp. centimeters
Power	P	$P = Fv$ $= EI$	Erg per second Watt	erg per sec. watt	Foot pound per minute	ft. lb. per min.	1 foot pound per minute = 230.4 grams centimeters per second 1 foot pound per minute = 81.42 watts per second 1 watt = 10^7 ergs per second
Energy	W	$W = FVt = Fl$ $= EIt$ $= Pt$	Erg Joule	erg joule	Foot-pound	ft. lb.	1 joule = 10^7 ergs 1 foot-pound = 1.357 × 10^7 ergs.
Modulus of elasticity	....	$\frac{F}{A}$	Dyne per square centimeter	Dyne-per cm.	Pound per square inch	lb. per sq. in.	1 pound per square inch = 2.8697 × 10^6 dynes per cm.
Temperature	$T°$	....	Centigrade	C. cent.	Farenheit scale	F.	1°F. = $\frac{5}{9}$°C.

outstanding importance—the *electrostatic or stat* system, the *electromagnetic or absolute or ab* system, and the *international or practical* system.

Electrostatic or Stat System.—The electrostatic system of units is the oldest and, while the units are to some extent still used, it is chiefly of interest as the first systematic coordination of quantitative measurement of electric quantities.

The system centers on the properties of the electric charge and the structure is based on:—

(*a*) Coulomb's law of attraction and repulsion between electrically charged bodies.

(*b*) The definition of a unit electric charge.

(*c*) The assumption that the unit charge produces 4π lines of dielectric flux.

(*d*) That the permittivity of space is unity (essentially unity for air under atmospheric conditions).

The dimensions of the system are the *centimeter, gram, second,* and the *permittivity* factor.

Coulomb's Law: In 1785, Coulomb, a French engineer, determined experimentally, the law of repulsion and attraction between electrically charged bodies. By means of a simple torsion balance he measured the force existing between pairs of small bodies electrically charged and found that: *The force of repulsion between two charges of the same kind of electricity is directly proportional to the product of the magnitude of the charges and inversely proportional to the square of the distance between the charged bodies.* If Q and Q' represent the magnitude of the charges, d the distance, and F the force of repulsion, *Coulomb's law* is expressed by equation (1).—

$$F \propto \frac{QQ'}{d^2} \tag{1}$$

If the charges Q and Q' are of the same magnitude, the distance d, 1 cm. and the force F, 1 dyne, then each charge would be of unit value. The *unit electrostatic charge* is that quantity of electricity with which a small body must be charged, so that if placed in air at a distance of 1 cm. from a similar body charged with an equal amount of the same kind of electricity, it produces an electrostatic force of repulsion of 1 *dyne* between the two bodies.

It is also assumed that each electric charge produces or consists of 4π lines of dielectric flux; that is, one line of force per $\overline{\text{cm}}.^2$

on the surface of the sphere of 1 cm. radius surrounding the unit charge. The unit electrostatic charge may also be defined in terms of the *electronic charge,* a natural quantitative unit. One electronic charge equals $4.774 \cdot 10^{-10}$ units of electrostatic charge; or the unit electrostatic charge equals $2.09342 \cdot 10^{9}$ electronic charges.

Coulomb's investigations preceded by 35 years Ampere's discovery (1820) of the magnetic field surrounding electric currents; and the basis of quantitative units in the dielectric field was definitely established before electromagnetic phenomena were known to exist. The electrostatic system was later extended to include quantities relating to electric currents and magnetic fields. The quantitative relations of units in the electrostatic, electromagnetic, and international or practical systems are given in Table VII.

It is somewhat difficult, if not impossible, to gain clear concepts of the quantitative relations existing between voltage, permeance, and electrostatic lines of force in the dielectric circuit from the basic definition that a unit electric charge on a sphere of unit radius produces one line of force per $\overline{\text{cm}}.^2$ of surface, that is, 4π lines of force. This important concept can be gained with little effort however, by the same method as is generally used for finding the similar relations between voltage, resistance, and current in the electric circuit. Consider two plane metal surfaces each 1 $\overline{\text{cm}}.^2$ placed parallel to each other, 1 cm. apart and let the space between them be either empty or filled with some material whose permittivity is unity. To establish an *electrostatic field represented by one dielectric line of force between the two surfaces passing through the* $\overline{\text{cm}}.^3$ *between the plates requires* 1 *statvolt or* 300 *volts.*

If the $\overline{\text{cm}}.^3$ volume between the two plates were filled with some material, as glass for example, whose permittivity is not unity but 4.8 (for the sample used), then 1 statvolt, that is 300 volts, would cause 4.8 dielectric lines of force to pass between the plates through the glass cube.

Electromagnetic Absolute or Ab System.—The electromagnetic or ab system is centered on the properties of the magnetic pole and was developed independently of the electrostatic system. The quantitative ratio between the units in the two systems, that is, the number of electrostatic units in one electromagnetic unit of electric charge, as shown in Table VII, has been proved to

be the velocity v of an electromagnetic-dielectric field in free space, which is the same as the velocity of light.

$$v = 3 \cdot 10^{10} \text{ cm. per second} \tag{2}$$

The electromagnetic system is based on,

(*a*) The law of repulsion or attraction between magnetic fields.

(*b*) The definition of the unit magnetic pole.

(*c*) The assumption that the unit pole has 4π lines of magnetic flux.

(*d*) The assumption that the permeability for space is unity (practically unity for air under atmospheric conditions.)

The dimensions of the system are the *centimeter, gram, second,* and the *permeability factor.*

Consider two long slender magnets placed on the same straight line with the north poles of strength m and m' a distance d apart. If the magnets be of sufficient length the south poles would be so far apart that any force produced by them would be negligible in comparison to the force between the two adjacent north poles. Under these conditions the *force of repulsion between the north poles would be directly proportional to the product of the strength of each pole and inversely proportional to the square of the distance between them, as expressed by equation* (3).

$$F \propto \frac{mm'}{d^2} \tag{3}$$

The law for the interaction between magnetic poles is therefore of the same form as Coulomb's law for the forces existing between electrically charged bodies.

Each of the above north poles would have unit strength or would be defined as *unit poles* if m equals m', the distance d, 1 cm., and the force of repulsion F, 1 *dyne.* It is also assumed that the unit magnetic pole has 4π lines of magnetic flux; that is, one line of force passes through or emanates from each $\overline{\text{cm.}}^2$ on the surface of a sphere of 1 cm. radius surrounding the unit pole.

On the basis of the definitions of a unit pole and the units of length, mass, time, and permeability in combination with the established laws of electromagnetic phenomena, the electromagnetic system of units is constructed. To make the process clear it may be well to state briefly the derivation of units for (*a*) *magnetic field strength,* (*b*) *electric current,* and (*c*) *electromotive force.*

(*a*) *Unit of Field Strength.*—Any region in which a magnetic pole, if placed in the given space, would be acted upon by a magnetic force is a magnetic field. From the definition of a unit magnetic pole and the law of inverse squares, equation (3), the number of lines of force per $\overline{\text{cm.}}^2$ or field strength H is at a distance d in air equal to m^1/d^2. Substituting in equation (3), the mechanical force acting between the magnetic field and the magnetic pole is directly proportional to strength of the field H and the strength of the magnetic pole m.

$$F = Hm \tag{4}$$

If the magnetic field exerts a force of 1 dyne on a unit magnetic pole the field has unit strength.

(*b*) *Unit of Current.*—Electric currents are surrounded by magnetic fields. The strength of the magnetic field at any given point produced by a current in an element of the conductor is directly proportional to the magnitude of the current and to the length of conductor element and inversely proportional to the square of the distance of the point from the conductor element.

$$H \propto \frac{Il}{r^2} \tag{5}$$

Let the conductor be bent into an arc of 1 cm. radius. The current flowing in the conductor will be of unit value if for each centimeter of conductor length a magnetic field of unit strength is produced at the center of the arc; that is, the unit current flowing through the conductor forming a complete circumference of the circle of unit radius produces a field strength of 2π units at the center of the circle and in direction at right angles to the plane of the circle.

The unit of current as specified in the preceding sentences is the unit of current in the electromagnetic or ab system and is called the *ab-ampere.*

$$1 \text{ ab-amp.} = 10 \text{ amp.} \tag{6}$$

(*c*) *Electromotive Force.*—When an electric conductor moves relatively to a magnetic field the voltage generated in the conductor is directly proportional to the time rate of cutting lines of force, or, to the rate of change in the number of interlinkages of the magnetic flux lines with the electric current.

$$e \propto \frac{d\phi}{dt} \tag{7}$$

If the lines of force are cut at the rate of one line per second the electromotive force generated is 1 *ab-volt*, the unit of voltage in the electromagnetic system.

$$1 \text{ ab-volt} = 10^{-8} \text{ volts} \tag{8}$$

The quantitative relations of the electromagnetic units to the corresponding units of the electrostatic and the international practical units are given in Table VIII.

International or Practical System.[1]—The definition for the unit electric charge and the unit magnetic pole of the electrostatic and electromagnetic systems, respectively, may be stated concisely, but it is not a simple matter to accurately determine experimentally the values specified. Moreover, most of the units in both systems are of inconvenient size for use in everyday electrical measurements. As a consequence a third system, the *international*, was developed and the electric units generally used are based on standards defined by international electrical congresses and in the United States legalized by Act of Congress in 1894. The *international* or *practical* system has the *international ohm, international ampere, centimeter* and *second* as fundamental units. The unit magnetic pole and the unit electric charge are relegated to subordinate positions, corresponding to their unimportance in practical work.

1. The *international ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice, 14.4521 g. in mass, of a constant cross-sectional area and of a length of 106.3 cm. The ohm equals 10^9 units of resistance in the electromagnetic system.

2. The *international ampere* is the unvarying electric current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0.0011180 g. per second. The ampere equals 10^{-1} units of current in the electromagnetic system.

[1] For more extended information on systems of units the student is referred to the following bulletins of the U. S. Bur. Standards:

WOLFF, F. A., "The So-called International Units." *Bull.* 1, Vol. 1, 1904.

WOLFF, F. A., "Selection and Definition of the Fundamental Electrical Units to be Proposed for International Adoption." *Bull.* 2, Vol. V, 1908.

STRATTON, S. W., "Announcement of a Change in the Value of the International Volt." *Circ.* 29, 1910.

DELLINGER, J. H., "International System of Electric and Magnetic Units." Scientific Paper 292, 1916.

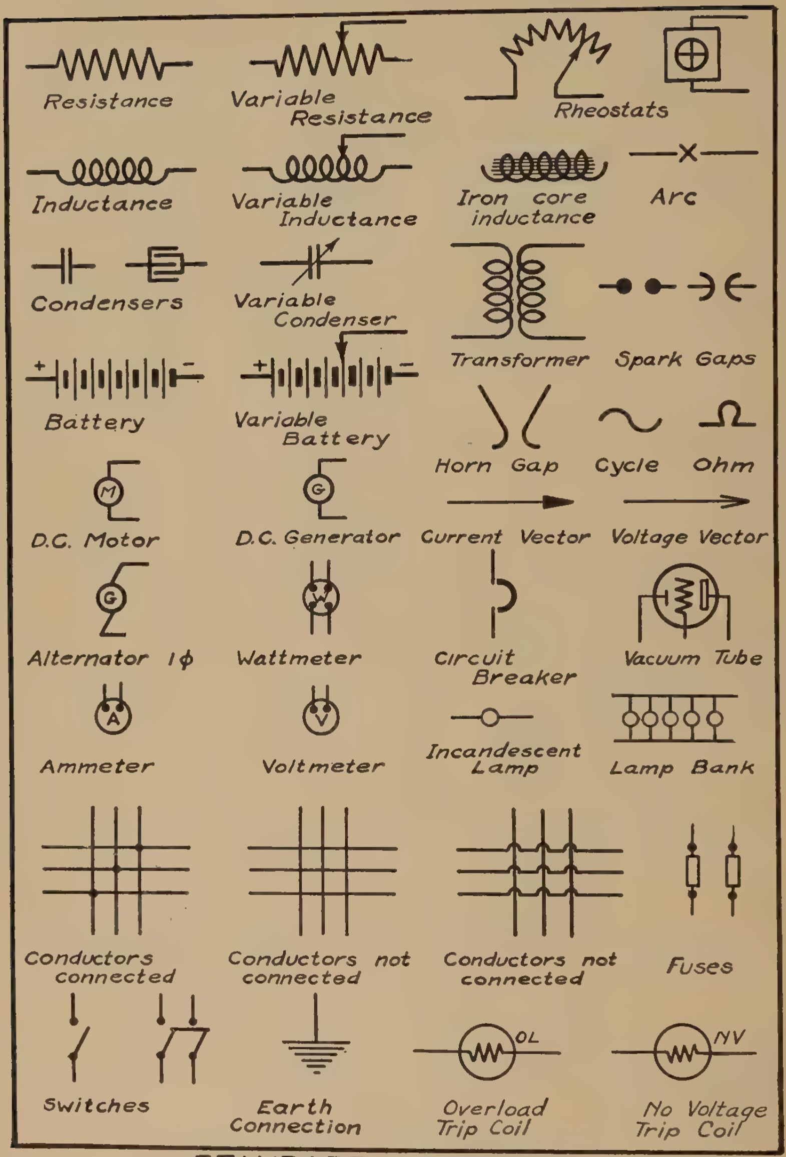
TABLE VIII.—ELECTRICAL UNITS, CONVERSION FACTORS

International c.g.s. system of practical units				Conversion factors $V = 3 \cdot 10^{10}$			Quantitative relations
Quantity	Symbol	Name of unit	Equations	Practical to electromagnetic	Electromagnetic to electrostatic	Practical to electrostatic	The International c.g.s. system of practical units; The electromagnetic absolute c.g.s. system of ab units; The electrostatic absolute c.g.s. system of stat units
Capacitance, Condensance	C	farad	$C\text{ (farads)} = \frac{1}{S(\text{darafs})} = \frac{Q(\text{coulombs})}{E(\text{volts})} = \frac{kA(\text{cm.}^2)\,10^9}{4\pi V^2(\text{cm.}^2)\,l(\text{cm.})} = \frac{kA(\text{cm.}^2)}{113.1 \cdot 10'' \, l(\text{cm.})}$	10^{-9}	v^2	$9 \cdot 10^{11}$	1 farad = 10^{-9} abfarads = $9{\cdot}10^{11}$ statfarads
Conductance	G	mho	$G(\text{mhos}) = \frac{1}{R(\text{ohms})} = \frac{I(\text{amperes})}{E(\text{volts})}$	10^{-9}	v^2	$9 \cdot 10^{11}$	1 mho = 10^{-9} abmhos = $9 \cdot 10^{11}$ statmhos
Conductivity	γ	....	$\gamma = \frac{1}{\rho}$				
Current	i, I	ampere	$I(\text{amperes}) = \frac{E(\text{volts})}{R(\text{ohms})}$	10^{-1}	v	$3 \cdot 10^9$	1 ampere = 10^{-1} abamperes = $3 \cdot 10^9$ stat-amperes.
Dielectric flux	ψ, Ψ	line of force	$\psi(\text{line of force}) = 4\pi Q'(\text{stat coulombs}) = 3.77 \cdot 10^{10}\, Q(\text{coulombs}) = 3.77 \cdot 10^{10}\, C(\text{farads})\; E(\text{volts}) = \frac{kA(\text{cm.}^2)E(\text{volts})}{300\, l(\text{cm.})}$				
Elastance	S	daraf	$S(\text{darafs}) = \frac{1}{C(\text{farads})} = \frac{E(\text{volts})}{Q(\text{coulombs})} = \frac{4\pi V^2\, l(\text{cm.})}{kA(\text{cm.}^2)\, 10^9} = \frac{113.1 \cdot 10^{11}\, l(\text{cm.})}{kA(\text{cm.}^2)}$	10^9	$\frac{1}{v^2}$	$\frac{1}{9 \cdot 10^{11}}$	1 daraf = 10^9 abdarafs = $\frac{1}{9 \cdot 10^{11}}$ statdarafs
Elasticity	σ	....	$\sigma = \frac{1}{k}$				
Electromotive force, Electric potential	e, E	volt	$E(\text{volts}) = R(\text{ohms})\, I(\text{amperes}) = \frac{Q(\text{coulombs})}{C(\text{farads})}$	10^8	......	$\frac{1}{300}$	1 volt = 10^8 abvolts = $\frac{1}{300}$ statvolts

TABLE VIII.—(*Continued*)

Energy	W	joule	$= \frac{d\phi(\text{maxwells})10^{-8}}{dt(\text{seconds})}$ $W(\text{joules}) = E(\text{volts})I(\text{amperes})t(\text{seconds})$ 1 joule = 1 watt second = 10^7 ergs = 0.000948 B.T.U. = 0.2389 gram calories = 0.102 kg. meters = 0.7376 ft.-lb.	10^7	......	10^7	1 joule = 10^7 abjoules = 10^7 statjoules
Inductance	L	henry	$L(\text{henrys}) = \frac{4\pi\mu N^2(\text{turns})A(\overline{\text{cm.}}^2)}{10^9\, l(\text{cm.})}$	10^9	$\frac{1}{v^2}$	$\frac{1}{9 \cdot 10^{11}}$	1 henry = 10^9 abhenry = $\frac{1}{9 \cdot 10^{11}}$ statthenry
Magnetic flux	ϕ, Φ	line of force; tube of force, line of induction, maxwell; weber	1 maxwell = 1 line of force = 1 line of induction. 1 weber = 10^8 maxwells. $\phi(\text{maxwells}) = \frac{\mathfrak{F}(\text{gilberts})}{\mathfrak{R}(\text{oersteds})} = \frac{0.4\pi N(\text{turns})I(\text{amperes})}{\frac{l(\text{cm.})}{\mu A(\overline{\text{cm.}}^2)}}$ $\phi(\text{webers}) = \frac{L(\text{henry})\ I(\text{ampere})}{N(\text{turns})}$				
Magnetic flux density	$\mathfrak{B}$	gauss	1 gauss = 1 maxwell per $\overline{\text{cm.}}^2$ $B(\text{gausses}) = \frac{\phi(\text{maxwells})}{A(\overline{\text{cm.}}^2)} =$ $= \mu H(\text{gilberts per cm.})$				
Magnetic intensity	H	gilberts per centimeter	$H(\text{gilberts per cm.}) = \frac{B(\text{gausses})}{\mu}$				
Magnetomotive force	$\mathfrak{F}$ m.m.f.	ampere-turn gilbert	1 gilbert $= \frac{1}{0.4\pi}$ ampere-turns $\mathfrak{F}(\text{gilberts}) = 0.4\pi N(\text{turns})\ I(\text{amperes})$ $= \mathfrak{R}(\text{oersteds})\ \phi(\text{maxwells})$				
Permeability	μ	a number	$\mu = \frac{1}{v}$ $= \frac{\mathfrak{B}(\text{gausses})}{H(\text{gilberts per cm.})}$				
Permittivity	k	a number	$\kappa = \frac{1}{\sigma}$				
Power	p, P	watt	$P(\text{watts}) = E(\text{volts})\ I(\text{amperes})$ $= R(\text{ohms})I^2(\text{amperes})$	10^7	......	10^7	1 watt = 10^7 abwatts = 10^7 statwatts

TABLE IX.—STANDARD ELECTRICAL SYMBOLS



STANDARD SYMBOLS

TABLE X.—SYMBOLS DEVELOPED BY AMERICAN ENGINEERING STANDARD COMMITTEE

STANDARD SYMBOLS FOR ELECTRICAL EQUIPMENT OF BUILDINGS

Ceiling Outlet	
Ceiling Outlet (Gas and Electric)	
Ceiling Lamp Receptacle—Specification to Describe Type such as Key, Keyless or Pull Chain	Ⓡ
Ceiling Outlet for Extensions	Ⓔ
Ceiling Fan Outlet	
Floor Outlet	
Drop Cord	Ⓓ
Wall Bracket	
Wall Bracket (Gas and Electric)	
Wall Outlet for Extensions	Ⓔ
Wall Fan Outlet	
Wall Lamp Receptacle—Specification to Describe Type such as Key, Keyless or Pull Chain.	Ⓡ
Single Convenience Outlet	
Double Convenience Outlet	$_2$
Junction Box	Ⓙ
Special Purpose Outlet—Lighting, Heating and Power as Described in Specification	
Special Purpose Outlet—Lighting, Heating and Power as Described in Specification	
Special Purpose Outlet—Lighting, Heating and Power as Described in Specification	
Exit Light	
Floor Elbow	O^E
Floor Tee	O^T
Pull Switch	P.S.
Local Switch—Single Pole	S^1
Local Switch—Double Pole	S^2
Local Switch—3 Way	S^3
Local Switch—4 Way	S^4
Automatic Door Switch	S^D
Key Push Button Switch	S^K
Electrolier Switch	S^E
Push Button Switch and Pilot	S^P
Remote Control Push Button Switch	S^R
Tank Switch	T.S.
Motor	
Motor Controller	M.C.
Lighting Panel	
Power Panel	
Heating Panel	
Pull Box	
Cable Supporting Box	
Meter	
Transformer	
Branch Circuit, Run Concealed under Floor Above	
Branch Circuit, Run Exposed	
Branch Circuit, Run Concealed Under Floor	
Feeder Run, Concealed under Floor Above	
Feeder Run, Exposed	
Feeder Run, Concealed under Floor	
Pole Line	
Push Button	
Buzzer	
Bell	
Annunciator	
Interior Telephone	
Public Telephone	
Local Fire Alarm Gong	F
City Fire Alarm Station	
Local Fire Alarm Station	F
Fire Alarm Central Station	F.A.
Speaking Tube	
Nurse's Signal Plug	N
Maid's Plug	M
Horn Outlet	
District Messenger Call	
Clock (Secondary)	
Clock (Master)	
Time Stamp	
Electric Door Opener	
Watchman Station	W
Watchman Central Station Detector	W
Public Telephone—P. B. X. Switchboard	P.B.X.
Interior Telephone Central Switchboard	I.X.
Interconnection Cabinet	
Telephone Cabinet	
Telegraph Cabinet	
Special Outlet for Signal System as Described in Specification.	
Battery	
Signal Wires in Conduit Concealed Under Floor	
Signal Wires in Conduit Concealed under Floor Above	
This Character Marked on Tap Circuits Indicates 2 No. 14 Conductors in ½-in. Conduit (see note)	
3 No. 14 Conductors in ½-in. Conduit	
4 No. 14 Conductors in ¾-in. Conduit Unless Marked ½-in.	
5 No. 14 Conductors in ¾-in. Conduit	
6 No. 14 Conductors in 1-in. Conduit Unless Marked ¾-in.	
7 No. 14 Conductors in 1-in. Conduit	
8 No. 14 Conductors in 1-in. Conduit	

NOTE.—If larger conductors than Number 14 are used, use the same symbols and mark the conductor and conduit size on the run.

3. *The international volt* is the electrical pressure which, when steadily applied to a conductor the resistance of which is 1 international ohm, will produce a current of 1 international ampere. The volt equals 10^8 units of electromotive force in the electromagnetic system. A more convenient subsidiary standard for the volt is found in the *Weston cadmium sulphate cell.* Under conditions that can readily be met the electromotive force of the Weston cell is 1.0183 volts.

4. The *international watt* is the energy expended per second by an unvarying electric current of 1 international ampere under an electric pressure of 1 international volt. The watt equals 10^7 units of power in the electromagnetic system.

Multiplying Factors.—In Table VIII are given the quantitative relations of the units in the *electrostatic or stat* system, the *electromagnetic* or *ab* system, and the *international practical* system. The multiplying factors are:

(*a*) 10^x, with x a positive or negative integer.

(*b*) 4π, the surface of a sphere of unit radius.

(*c*) $v = 3 \cdot 10^{10}$ cm. per second, the velocity of propagation in cm/sec. of an electromagnetic-dielectric field in free space—the same as the velocity of light.

(*d*) Ratio of time units used: seconds, minutes, hours, etc.

Other Systems.—Several other systems of electric units have been proposed and, in a few cases, used to a limited extent. The chief defect in the international system of units is the introduction of the 4π factor, due to the illogical assumption of one line of force per $\overline{\text{cm}}^2$. on the surface of a sphere of unit radius, instead of letting a single line represent the flux from a unit pole or a unit charge. In the Heaviside system of units this defect is eliminated at its source, while in some of the other systems the correction is made by including the 4π factor in the definitions for permeability and permittivity. However, the international or practical system, based upon the electromagnetic or absolute system, provides satisfactory practical units for the measurement of electric and magnetic quantities and is in general use.

Symbols.—In order to more readily follow discussions of electrical phenomena, machinery and appliances, an extensive system of symbolic representation has been developed. Not merely letters are used, as for the units involved in Tables VII and VIII, but other figures in the form of compact drawings or stereotyped sketches, as illustrated in Tables IX and X, are also in general use.

CHAPTER VIII

INDICATING INSTRUMENTS

The basic principle on which most direct-current indicating instruments operate is to balance the force produced by the interaction between the magnetic field of a coil carrying an electric current with the field of a permanent magnet against the force required to twist a fiber or a hairspring. If a direct current flows in the coil represented in Fig. 1, a magnetic field will be found in the surrounding space. If the current flows in at the terminal marked A and leaves at B the direction of the lines of force will be as indicated by the arrows in the diagram. It is evident that the coil carrying the current is equivalent to a magnet, the left side being a north pole N and the right side a south pole S. If the direction of the current in the coil be reversed the magnetic field produced by the current would also be reversed, and the north and south poles exchange space positions. Let the coil

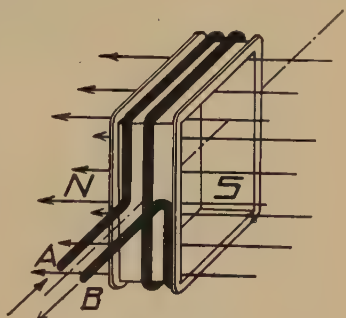


Fig. 1.—Magnetic field of a coil carrying an electric current.

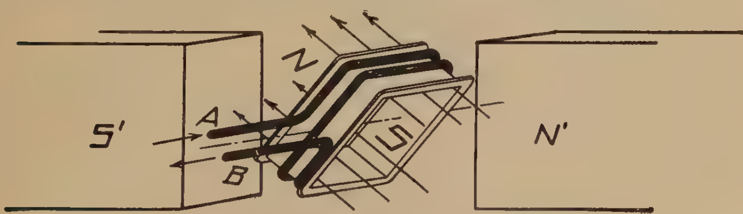


Fig. 2.—Coil carrying current in a stationary field.

AB in Fig. 1 be placed between the poles of a permanent magnet having a north pole N' and a south pole S' as in Fig. 2. If the coil is free to turn on an axis in the plane of the coil and at right angles to the lines of force between the poles N', S' of the per-

manent magnet the interaction between the two fields will cause the coil to move into the position shown in Fig. 3. That is, the coil is turned until the pole N comes as near as possible to pole S' and likewise pole S directly in line with N'. If the current is reversed in direction the coil AB would turn 180 deg. so as to again bring the unlike poles near together and the like poles as

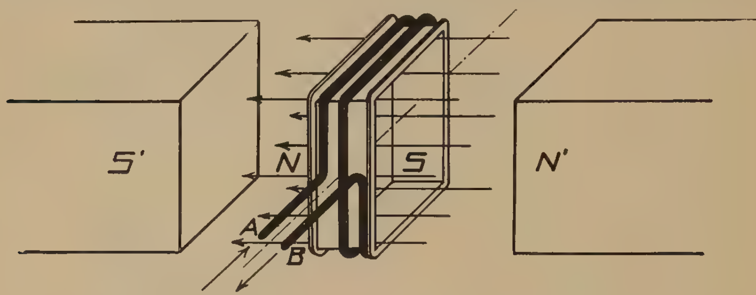


FIG. 3.—Movable coil in stationary magnetic field.

far apart as possible under the given conditions. If the coil is attached to a suspension fiber, the upper end of which is fastened to a stationary support, the turning of the coil on the axis passing through the supporting fiber would produce a twist in the fiber and cause a torque which would tend to bring the coil back into the neutral or zero position as shown in Fig. 4; that is, the plane

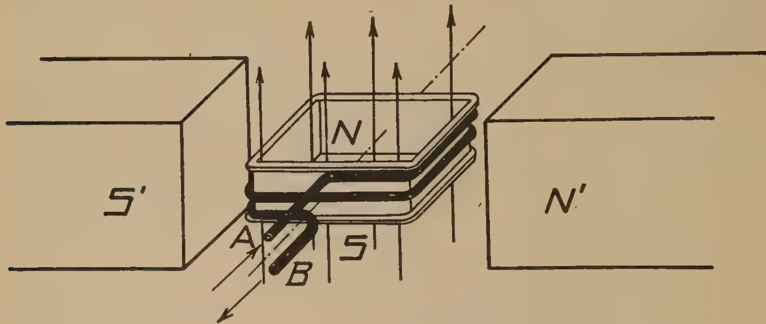


FIG. 4.—Movable coil in neutral position.

of the coil is parallel to the lines of force in the permanent field between the poles N', S'.

If a direct current is made to flow in the coil supported as specified above and as indicated in Fig. 4, the interaction of the two magnetic fields will tend to turn the coil into a position at right angles to the lines of force in the field of the permanent

magnet as in Fig. 3 while the twist of the supporting fiber will produce a torque in the opposite direction, tending to keep the coil in the original position parallel to the lines of force between

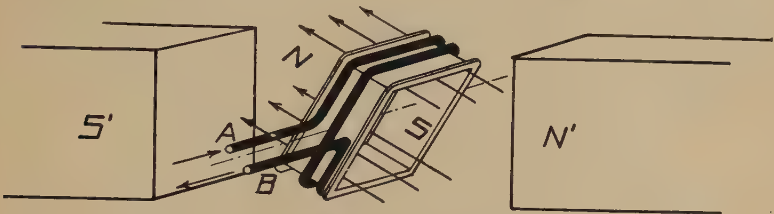


FIG. 5.—Coil carrying current in stationary magnetic field.

$N'S'$ as in Fig. 4. As a consequence, the coil assumes a position in between the two extremes as illustrated in Fig. 5. The angular deflection from the zero position depends on the relative magnitude of the two conflicting forces.

The D'Arsonval Galvanometer. The galvanometer is an instrument for detecting and measuring small electric currents. A large variety of forms and designs of galvanometers have been constructed and used for various purposes. The D'Arsonval galvanometer is the type in general use, however, as this instrument is simple in design, can be made rugged in construction, and is largely unaffected by stray magnetic fields. A circuit diagram of the D'Arsonval galvanometer is shown in Fig. 6 and a photograph of a wall-type instrument in Fig. 7. The diagram in Fig. 6 shows the electric and magnetic circuits. A coil of insulated wire called the armature, is held suspended by means of a phosphor-bronze fiber between the poles of a permanent magnet. The suspension fiber is firmly fixed at its upper

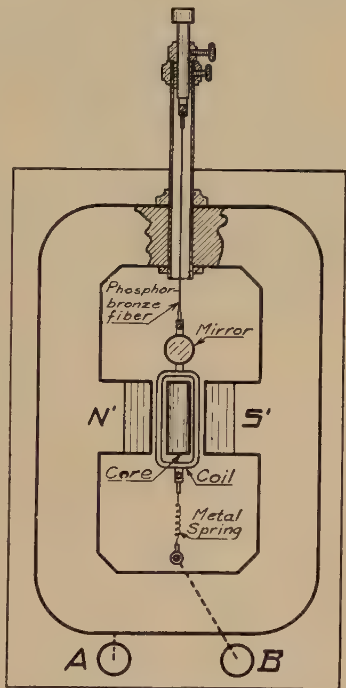


FIG. 6.—Circuit diagram of D'Arsonval galvanometer.

end to the frame of the instrument but the coil or armature may turn around a vertical axis; that is, the turning of the armature twists the supporting fiber. The lower end of the armature is attached to the base of the instrument by means of a very flexible metal spring. A small mirror is attached to the supporting fiber at the upper end of the moving element. The mirror is fixed

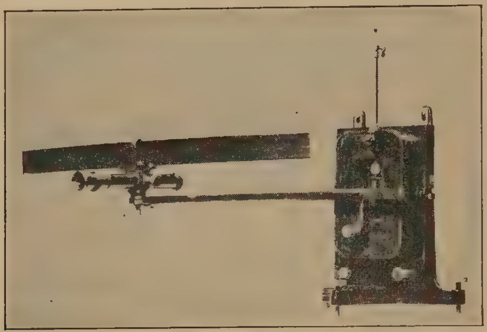


FIG. 7.—D'Arsonval galvanometer. Wall-type.

in a plane essentially parallel to the armature coil and the attachment of the suspension fiber to the frame of the instrument is so adjusted that when the electric circuit is open the mirror reflects the central point of the scale (shown in front of the instrument Fig. 7) to the cross-hairs of the telescope through which observations are taken. The iron core in the armature reduces the

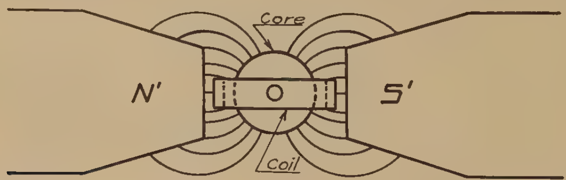


FIG. 8.—Effect of iron core on permanent magnetic field of a galvanometer.

reluctance of the magnetic circuit and gives a radial direction to the lines of force, (Fig. 8) from the permanent magnet when passing through the armature. The suspension fiber, armature coil, and spring are connected in series so as to form an electric circuit. When the galvanometer is used this electric circuit is closed and the current to be measured enters either at the terminal A, passes through the suspension fiber, armature coil, and spring and leaves the instrument by the terminal B, or in the reverse direction. When the current flows through the armature a

magnetic field is produced with lines of force at right angles to the plane of the coil. Hence when the coil is in the zero position the field produced by the current is in direction at right angles to the lines of force of the permanent magnet $N'S'$. The reaction between the two magnetic fields produces a force which turns the armature and thereby twists the suspension fiber until the torque produced in the fiber is equal in magnitude and opposite in direction to the moment of the force produced by the magnetic reaction.

In some forms of D'Arsonval galvanometers, the telescope is omitted. The mirror on the moving element is in that case concave and of such focal length as to focus a beam of light, coming from a lamp, on a ground-glass scale placed a short distance in front of the instrument. On the ground glass is etched or marked a scale and by observing the position of the spot of light on the translucent glass the deflection of the galvanometer may be measured.


Fig. 9.—Portable galvanometer.

The D'Arsonval galvanometer is so designed that the angular deflections of the armature are practically proportional to the magnitude of the current. If the scale, Fig. 7, is curved so as to coincide with the circumference of a circle having the mirror at its center, the readings of the uniformly spaced divisions on the scale will be porportional to the magnitudes of the currents passing through the instrument. A portable type of galvanometer is shown in Fig. 9. The working elements of this instrument consist of a permanent magnet, a double pivoted coil or armature held in position by two hair springs, and a pointer rigidly attached to the armature. This instrument is supersensitive and in many cases it is more convenient to use a portable instrument than the delicate, suspended coil type of galvanometer.

In operating the instrument, it is very desirable that the armature and hence the mirror should come to rest quickly after any given change in the current measured, in order that the readings may be taken with dispatch. This advantage is gained by damping the wiring of the suspended armature by means of an air vane;

by eddy currents in the metallic parts induced by the motion of the armature in the stationary magnetic field; by use of the Ayrton shunt; or by other means that produce a drag on the motion of the armature.

Galvanometer Shunts.—If the currents to be measured are so large that if passed through the galvanometer the deflection would go beyond the length of the scale, or the reaction would be so violent as to cause damage to the instrument, a shunt must be used. A *shunt* is merely a low-resistance conductor which

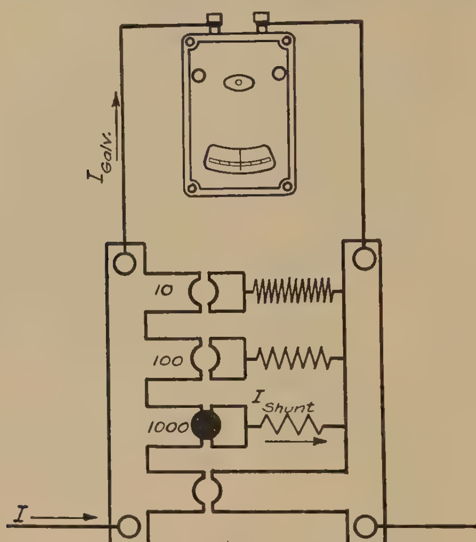


Fig. 10.—Circuit diagram of galvanometer with shunt.

by-passes a known proportion of the current so that only a definite portion of the total current flows through the galvanometer. Two general types of shunts, as illustrated in Figs. 10 and 11, are in general use. The first type consists of four conductors so arranged, as shown in Fig. 11, that any one of these conductors can be connected across the terminals of the galvanometer by inserting a plug in the corresponding hole. The resistances in the four parallel paths in the shunt are so adjusted in amount with respect to the resistance in the galvanometer circuit that a definite fraction of the total current will flow through the galvanometer. The commonly used ratios are such that 10 per cent, 1 per cent, or 0.1 per cent of the total current will flow through the galvanometer. Hence, if the galvanometer has been

graduated to indicate in amperes the magnitude of the current passing through it, to obtain the value of the total current measured when using a shunt, the readings must be multiplied by the factors 10, 100, and 1,000 respectively depending on which shunt circuit was in use.

The Ayrton Shunt.—A circuit diagram of the Ayrton shunt is shown in Fig. 11. A constant resistance is permanently connected across the terminals *A*, *B* of the galvanometer. One line terminal is also connected permanently to *A*, but the other line terminal *C* is movable and connects to definite points on the resistance *A*-*B* by a plug contact. The points on the resistance marked 0.001, 0.01, 0.1, and 1.0 represent division points so that the amount of resistance from *A* to the point marked 0.001 is one thousandth part of the total resistance *A*-*B*; similarly from *A* to 0.01, one hundredth part; from *A* to 0.1, one tenth of the total resistance.

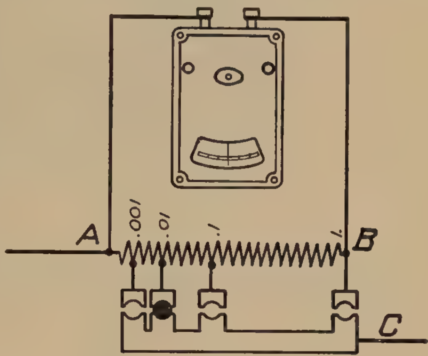


Fig. 11.—The Ayrton shunt.

It should be noted:

(*a*) That the caibration of the Ayrton shunt is not affected by changes in the galvanometer resistance and hence may be used with any galvanometer.

(*b*) That a constant resistance shunted across the terminals of the galvanometer is a frequently used device for damping the oscillation of the galvanometer armature.

Voltmeters.—Most direct-current voltmeters operate on the same basic principles as the D'Arsonval galvanometer. In fact a voltmeter is a galvanometer; that is, an instrument that measures the voltage or potential difference between any two given points of an electric circuit. In general, the design of portable voltmeters differs from the D'Arsonval galvanometer as follows:

(*a*) The movable element is supported on two steel pivots resting in jewel (sapphire) bearings.

(*b*) The torque required to oppose the moment of the magnetic fields is supplied by two hair springs.

(*c*) The armature has many turns of fine insulated wire and the coil swings through a larger arc.

(*d*) The coil and pointer comprise the movable element. The soft iron core inside the coil is stationary with respect to the permanent magnet.

Fig. 12.—Portable voltmeter.

(*e*) A large resistance (several thousand ohms for a voltmeter with scale graduated from 0 to 150 volts) is placed inside the cover of the instrument and connected in series with the moving element.

Fig. 13.—Inside view of voltmeter in Fig. 12.

Voltmeters used for measuring small voltages are usually called millivoltmeters to indicate that the scale is graduated in millivolts, that is, in thousandth's of volts. Thus the full-scale

reading of millivoltmeters graduated from 0 to 150 would indicate a difference of potential of 0.15 volts and not 150 volts as would be the case in the ordinary voltmeter.

Voltmeters can be used for measuring differences of potential greater than the rated capacity of the instrument by connecting additional resistance in series as indicated in Fig. 14.

It is evident from the circuit diagram in Fig. 14 that the voltmeter would indicate the difference of potential from A to B. Likewise from Ohm's law the ratio of the voltage AB to the total voltage AC is the ratio of the resistance inside the voltmeter R_V to the total resistance $R_V + R_M$.

Therefore,

$$E_{AC} = \frac{R_V + R_M}{R_V} E_{AB} \qquad (1)$$

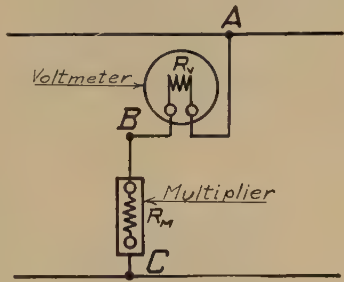


FIG. 14.—Circuit diagram for voltmeter used with multiplier.

In order to give a convenient numerical value to the voltage ratio in equation (1), special resistance boxes, called *multipliers*, are provided which have resistances that are exact multiples of the resistance in the voltmeter itself. If the multiplier has a resistance nine times as great as the instrument the correct value of the voltage measured is obtained by multiplying the readings on the voltmeter by 10. The more commonly used voltage ratios are: 2:1; 5:1; 10:1; 20:1; 100:1; and 1,000:1.

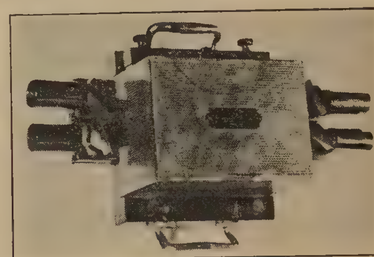
Ammeters and Ammeter Shunts.—An ammeter is an instrument for measuring the current flowing in an electric circuit. The basic principle for the generally used direct-current ammeters is the same as for the voltmeters or the D'Arsonval galvanometer. In the portable types the armature is supported on two hardened steel pivots resting in sapphire jewels. The duty on the jewels and pivots is extremely light so that with proper care high-grade instruments may be used a lifetime with little, if any, increase in the friction.

Portable ammeters are constructed on the same basic principles as portable voltmeters. The instruments are direct reading, compact, and serviceable. The scale is provided with a mirror which, combined with the knife-edge pointer, makes it possible to take accurate readings with ease and rapidity. As only small currents can be passed through the movable element

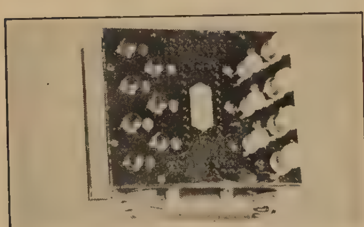
the ammeter has a self-contained shunt which carries the greater part of the current.

Ammeters with external shunts are essentially millivoltmeters that measure the voltage drop across the resistance of the shunt. In fact, millivoltmeters in combination with carefully calibrated external shunts become the most desirable portable ammeters.

100 to 500 ampere shunt.



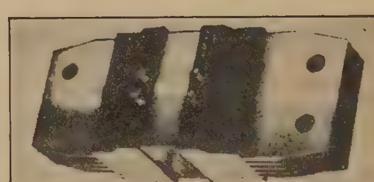
1,000 ampere shunt.



1 to 100 ampere shunt.

1.5 to 150 ampere shunt.

1,000 ampere switchboard shunt.



6,000 ampere switchboard shunt.

Fig. 15.—Typical ammeter shunts.

This combination results in an ammeter by which both large and small currents can be measured with a high degree of accuracy. The same millivoltmeter will serve for currents of all values, and to extend the scale, it is merely necessary to provide the required number and range of shunts. A set of precision shunts for use with a millivoltmeter in measuring currents ranging from 1 to 6,000 amp. is shown in Fig. 15. To protect ammeters

from excessive currents or transient high voltages, provision should be made for short circuiting the instrument when not taking readings.

Wattmeters.—The wattmeter measures the power that passes the given point in an electrical circuit. Wattmeters of the electrodynamometer type are generally used for measuring power both in direct-current and alternating-current circuits. In direct currents higher accuracy can be secured in measuring the watts by taking the product of the current and voltage as obtained by the use of a direct-current ammeter and voltmeter instead of using a wattmeter. The magnetic field of the wattmeter is

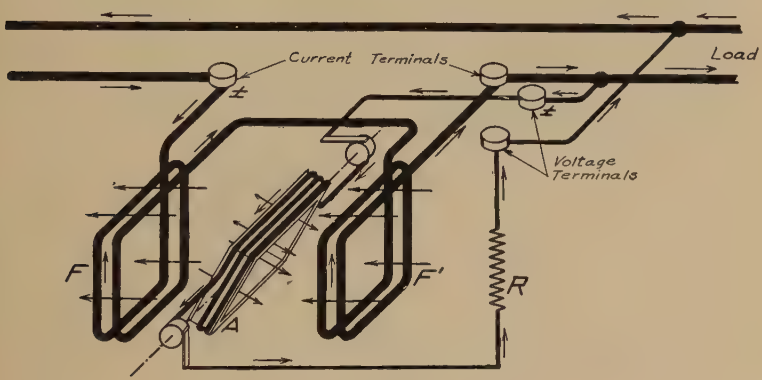


Fig. 16.—Wattmeter circuit diagram.

produced by the current flowing through a stationary coil of few turns FF' in Fig. 17 connected in series with the power circuit.

The movable element or armature A, in Fig. 16, is a coil consisting of many turns of fine wire connected, through a high resistance R, across the two leads in the power circuit. The current flowing through the movable element is therefore proportional to the voltage of the power circuit. Hence the moment of the force or reaction between the field produced by the currents in the stationary or movable coils is proportional to the product of the current and the voltage, that is, to the watts in the power circuit. The turning moment produced by the two magnetic fields is balanced by the torque of spiral springs attached to the ends of the armature, not shown in Fig. 17.

In Fig. 17 it will be noted that one terminal of the current coil and likewise of the potential coil are marked with a $\pm$ sign. This marking is for the purpose of making sure when connecting

the wattmeter into a circuit that there will not be a dangerous potential between the current and voltage coils. When connected as shown in Fig. 17, there will be practically no difference in potential between the coils. If the connections to the voltage

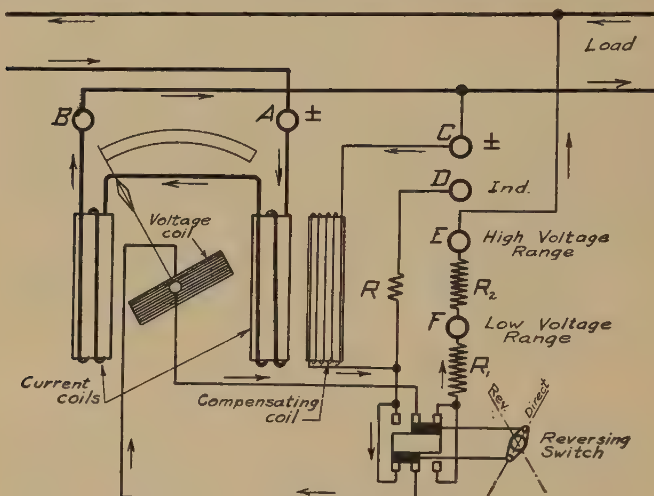


FIG. 17.—One type of compensated wattmeter circuit diagram.

coil are reversed or if the resistance R had been on the opposite end of the potential circuit, however, then nearly full-line voltage would exist between the stationary and movable coils. Aside

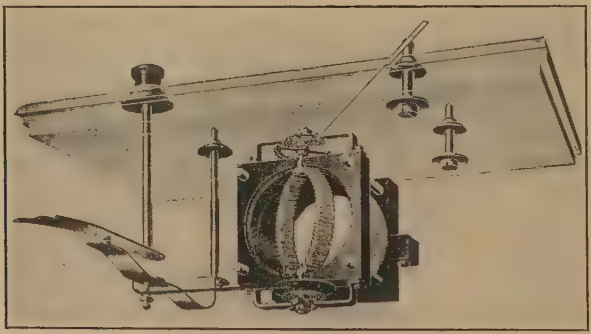


FIG. 18.—Elements of wattmeter.

from the fact that this high voltage may cause arc over and consequent destruction of the meter there is furthermore the chance of error in the deflections of the meter due to static forces acting between the two coils.

In Fig. 17 it will be noted that the current through the current coil is not only the load current but includes the voltage-coil current as well. The deflection of the meter is thus too great by an amount equal to the power used by the potential coil. In order to correct for this error, a few turns known as a "compensating coil" are placed in the potential circuit and act in opposition to the field set up by the current coil as shown in Fig. 17. In a compensated wattmeter it is very essential that the current coil as well as the voltage coil be correctly connected into the circuit because otherwise the error would not only still exist but it would be actually doubled. The current and voltage coils will always be correctly connected if the instantaneous direction of the current entering the ± terminals, of the coils are the same. This is indicated by the arrows in Fig. 17.

FIG. 19.—Wattmeter: Electrodynamometer type. (*Weston Elec. Inst. Co.*)

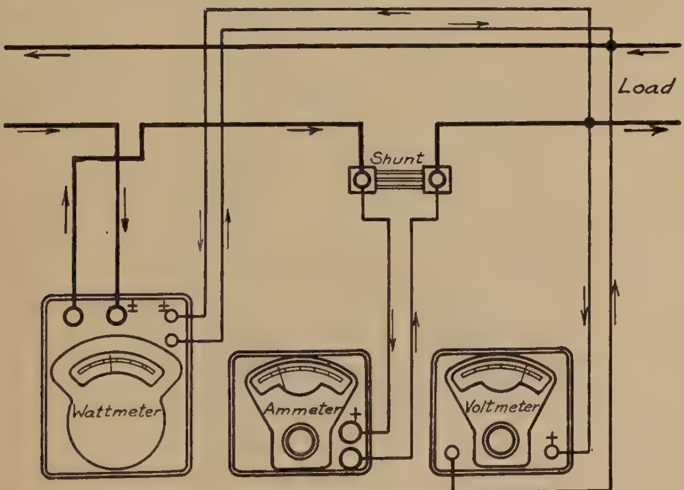


FIG. 20.—Circuit diagram showing connections for ammeter (millivoltmeter and shunt) voltmeter and wattmeter.

When a wattmeter is to be calibrated by using separate sources of supply for the voltage and current coils, the independent ter-

minal D is used which by-passes the compensating coil. The resistance of the voltage-coil circuit remains the same because resistance R in the independent circuit is equal to the compensating coil resistance.

In alternating-current circuits a wattmeter will often tend to read backwards and some provision must be made for changing the direction of current through one of the two coils. Fig. 17 shows how this is taken care of in the potential coil circuit by a reversing switch.

Hot-wire Ammeters and Voltmeters.—In the hot-wire type of instruments, the expansion of a wire due to heat produced

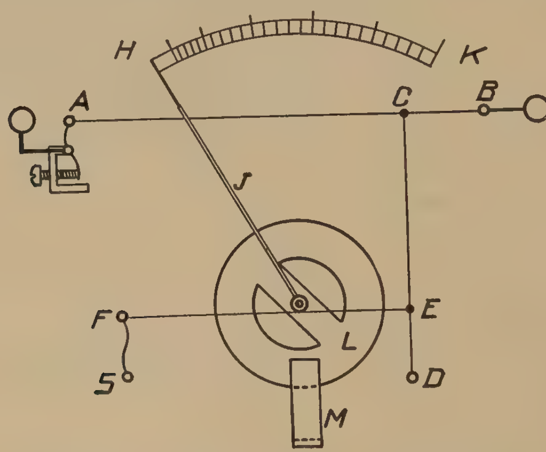


FIG. 21.—Circuit diagram of hot-wire ammeter.

when a current passed through it is utilized to produce the moment required to move a pointer over a graduated scale. In Fig. 21 is shown the circuit diagram of a hot-wire ammeter. When the current flows through the wire A-B it expands and the tension on the fine bronze wire CD produced by the spring SF through the silk cord EF pulls the point C downwards and causes the points E and F to move in the left direction. As the silk cord EF is wound around the pivot of the pointer the movement of EF will turn the pointer in the clockwise direction and, therefore, if the instrument has been properly calibrated, the pointer indicates on the scale H-K the magnitude of the current. Adjustments of the instrument to keep the pointer at the zero point on the scale when the circuit is open is made by the screw at A. In order to make the instrument "dead beat" an aluminum

disc L is fastened to the pointer shaft and rotates between the poles of a small permanent magnet M. Eddy currents set up

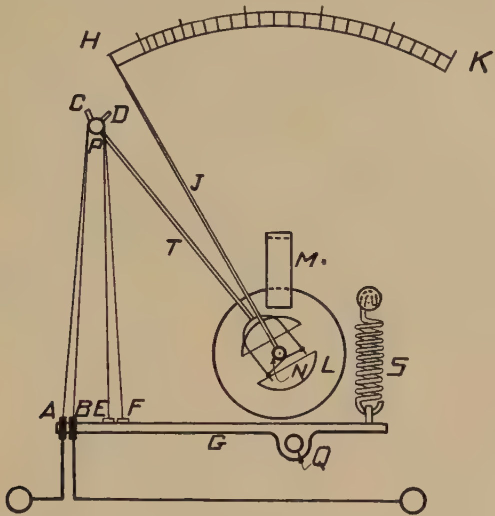


FIG. 22.—Circuit diagram of hot-wire ammeter.

in the disc produce a reaction between the disc and magnet tending to retard its motion.

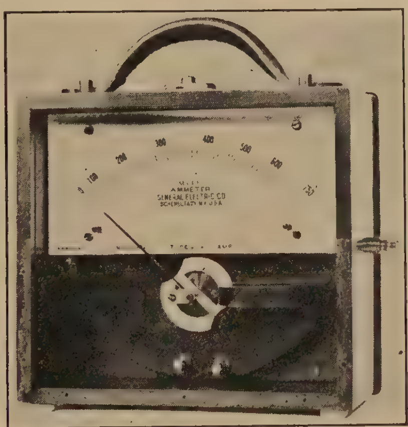


FIG. 23.—Hot-wire milliammeter.

In Fig. 22 is shown the circuit diagram of another type of hot-wire ammeter differing considerably from that shown in Fig. 21. A heavy arm G is pivoted at Q and pulled at one end by a heavy

spring S. The pull of the spring is balanced at the other end of G by two wires which loop over pins on the shaft P. That is, the wire A-B loops over the pin C while the wire E-F loops over pin D. The ends of the wire A-B are insulated where they pass through the arm G. The current to be measured thus passes through the wire A-B only. The expansion of A-B causes the shaft P to turn clockwise swinging the forked arm T with it. Across the ends of the forked end of T a thread is connected looping around the shaft N to which the pointer J is attached. The movement of J is thus produced when T moves. The small change in length of A-B is in this way magnified many times. The dampening device is the same as that previously described.

The hot-wire voltmeter has the same circuit diagram as the ammeter except that a large resistance R is inserted in series with the wire A-B. The hot-wire type of instrument is sluggish in operation as it takes an appreciable length of time for the element A-B to reach the temperature corresponding to the current passing through it. This type of instrument is often used as a transfer apparatus, when determining the direct-current equivalent of alternating currents having distorted wave forms. The hot-wire instruments are of special importance in radio for the measurement of high-frequency alternating currents as the indications are independent of the frequency if used without a shunt.

The Wheatstone Bridge.—The Wheatstone bridge has very wide application in electrical measurements and is essentially an instrument for measuring resistance. The basic principle of this important instrument can readily be grasped from the elementary circuit diagram in Fig. 24.

Current from the storage battery B flows through a divided circuit from A to C. It is evident that if the four resistances R_1, R_2, R_3, and R_4 in the four branches are equal the points E and D will be at the same potentials and hence no current will flow through the galvanometer bridging these points. It is also evident that if the resistances are in the proportion $R_1:R_2$ as $R_3:R_4$ no current will flow through the galvanometer as the points E and D will be at the same potential. In general, if the values of R_1, R_2, and R_3 are fixed and of known value while the resistance R_4 can be varied and adjusted until the points E and D will be at the same potential, the value of R_4 when no current flows through the galvanometer can therefore be obtained by the

application of Ohm's and Kirchhoff's laws as expressed in equation (2).

$$R_4 = \frac{R_1 R_3}{R_2} \tag{2}$$

A large variety of types and forms of Wheatstone bridges are in use but all operate on the same basic principle. A convenient

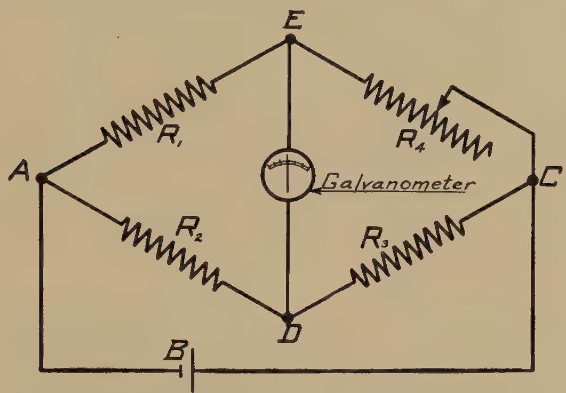


FIG. 24.—Circuit diagram of Wheatstone bridge.

portable type is shown in Fig. 26 and the corresponding circuit diagram in Fig. 27. The set is a dial Wheatstone bridge, complete with galvanometer, battery, and operating switches. To protect the moving system against severe vibration during transportation a simple but effective coil clamp is provided. The principle

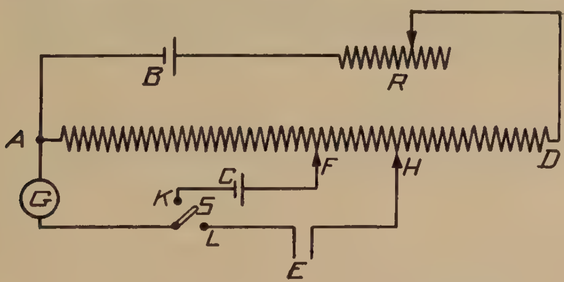


FIG. 25.—Circuit diagram of a simple potentiometer.

of the Wheatstone bridge is widely used as the null method eliminates the need for calibrating the moving element and also permits the use of extremely sensitive galvanometers.

Potentiometers.—The potentiometer is an instrument for measuring voltage with a high degree of accuracy. It may also be used indirectly for measuring small currents by the application

of Ohm's law. The basic principle of the potentiometer is to balance the electromotive force of a standard cell against a fractional part of the voltage to be measured. In Fig. 25 is shown the circuit diagram of a simple potentiometer, and the relative positions of the essential parts are indicated by letters as follows:

C = a standard Weston cadmium cell (Chap. XIX).
G = a galvanometer of high sensitivity.
AD = an accurately calibrated constant resistance.
B = storage battery.
R = resistance regulating the current flowing through AD.
E = the voltage to be measured.
F = contact point of standard Weston cell at point 1.0183 volts on the resistance AD.
H = contact point of the circuit to be measured with the resistance AD.

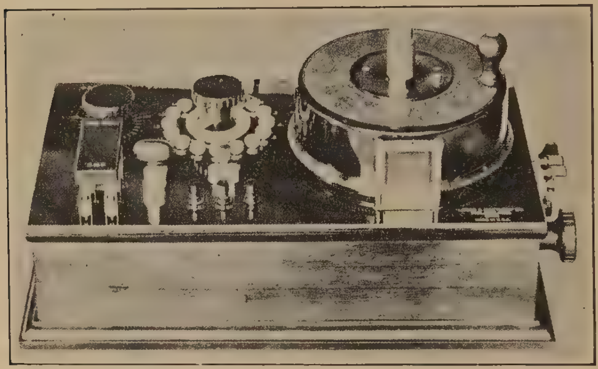


FIG. 26.—Potentiometer. Type K. *(Leeds and Northrup Co.)*

For measuring voltage less than 1.6 volts let the resistance AD be 16 ohms consisting of 16 units in series of 1 ohm each. A current from the storage battery B regulated by the resistance R is adjusted until 0.1 amp. flows from A to B. This is accomplished when the voltage between the points A and F exactly balances the voltage of the standard cell C. That is, no current flows through the galvanometer G when the switch S closes the circuit at K. The switch S is opened. The point H is then adjusted until no current flows through the galvanometer when the switch S closes the E voltage circuit at L. When correct balance is obtained the scale reading at H is to that at F as the voltage E is to the electromotive force of the standard cell C.

Therefore,

$$E = \frac{H}{F} 1.0183 \text{ volts} \tag{3}$$

It is important to obtain an accurate balance in the voltage at both F and H. If any current flows from the standard cell the terminal voltage will differ from the total electromotive force as the Weston cell has an internal resistance of about 200 ohms. The standard cell must be used with great care in order to keep the voltage at the terminals constant.

Commercial potentiometers are more complicated in design than the simple basic circuit illustrated in Fig. 25. The various arrangements make it possible for the operator to take accurate

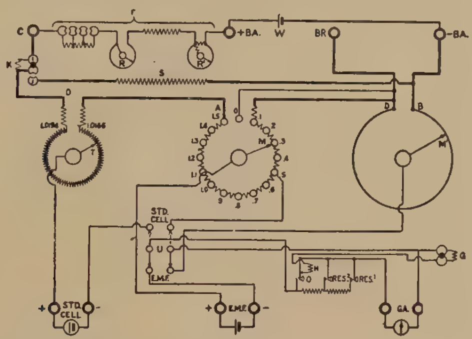


FIG. 27.—Circuit diagram for potentiometer Fig. 26. (*Leeds and Northrup Co.*)

readings rapidly without causing voltage variations or deterioration in the standard cell or changes in the circuit constants.

Oscillographs and Oscillograms.—The oscillograph is a simple but extremely serviceable instrument for indicating and recording instantaneous values of currents and voltages in electric circuits. The essential element of oscillographs of the galvanometer type generally used in commercial work, is an insulated loop of fine wire, placed in a magnetic field. A small mirror is attached to both wires as shown in Fig. 28. The current measured flows through the loop of wire and hence the reaction between the permanent field and the fields around the two sides of the loop will be in opposite direction. This produces a turning movement on the mirror proportional to the strength of the current. If the current is reversed the mirror will be turned in the opposite direction. The deflection, therefore, in angular position, of a beam of light thrown on the small mirror will vary with the

instantaneous magnitude and direction of the current flowing through the loop.

In comparison to other forms of galvanometers the moving element in the oscillograph is extremely light, consisting of a mirror of pin-head size and very short lengths of a fine wire. For very rapid changes in current or voltage this type of oscillograph can not be used, as the moving element is too heavy and the instrument will not respond quickly enough. For vibrations greater than 8,000 cycles per second some form of cathode ray oscillograph, in which the moving element is a stream of electrons, must be used.

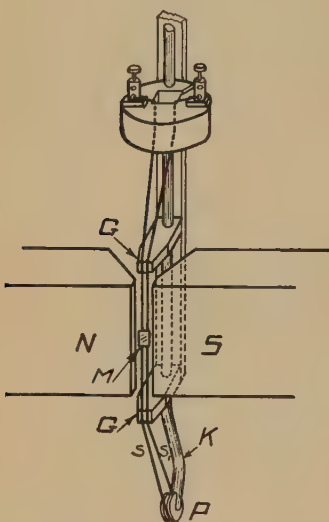


Fig. 28.—Vibrating element of oscillograph.

Vibrator Type of Oscillograph.—The essential elements of this instrument are:

(*a*) A modified moving-coil galvanometer consisting of a loop of wire to which is attached a mirror free to vibrate in a plane perpendicular to the loop.

(*b*) A strong magnetic field.

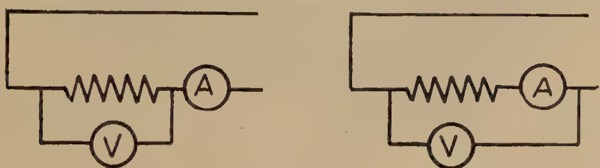
(*c*) A moving photographic film or falling photographic plate. The galvanometer part of the instrument is shown in Fig. 28.

In a narrow gap between the poles N, S of a powerful magnet are stretched two parallel conductors s, s', formed by bending a thin strip of phosphor bronze back on itself over an ivory pulley P. A spiral spring attached to the shaft K serves to keep a uniform tension on the strips, and the guide pieces G limits the length of the vibrating portion to the part actually in the magnetic field. A small mirror M bridges across the two strips as shown in the figure. Passing a current through such a vibrator causes one of the strips to advance while the other recedes, and the mirror is thus turned about a vertical axis.

The whole of the vibrator, as this part of the instrument is called, is immersed in an oil bath, the object of the oil being to damp the movement of the strips, and make the instrument dead-beat.

The beam of light, coming from an arc lamp, and reflected from the mirror M, is received on a photographic film. With constant strength of the magnetic field the instantaneous value of the current is proportional to the linear displacement of the spot of light focused on the film. With alternating currents the spot of light oscillates to and fro as the current varies and with the film stationary would thus trace a straight line. To obtain an image of the wave form, the photographic film is moved in a direction at right angles to the direction of the movement of the spot of light. Or a second mirror can be interposed in the path of the beam of light, and this mirror caused to vibrate or rotate so as to impart to the beam of light a uniform motion about an axis at right angles to the zero position of the beam and also in the initial plane of vibration. The spot of light will then trace on a stationary screen or plate the time curve of the vibration of the current flowing in the vibrator loop. In place of the record mirror a revolving drum on which is placed a photographic film is in general use. By letting the beam of light from the mirror on the vibrator loop be focused on the film attached to the drum and by letting the drum rotation be at right angles to the oscillatory motion of the beam of light, the oscillogram is formed on the moving film.

Megohmer.—Resistance measurements are generally made by the indirect method of taking current and voltage readings



(a) (b)

FIG. 29.—Circuit connections for resistance measurements.

and computing the resistance by the application of Ohm's law. In order to eliminate the errors introduced by the instruments themselves, corrections must be made for the current flowing through the voltmeter and for the voltage drop in the ammeter. The use of high-resistance voltmeters and low-resistance ammeters as well as the circuit connections of the instruments are important factors in securing accurate results. Thus for measuring a fairly high resistance the voltmeter and ammeter should be connected in circuit as shown by the diagram in Fig. 29 (*b*);

while for measuring low resistance the connections shown in the circuit diagram in Fig. 30 (a) should be used.

For more accurate resistance measurements Wheatstone or Kelvin bridge circuits are generally used. If the null method is used no current flows through the galvanometer when the bridge is balanced. This permits the use of galvanometers of high sensibility and, moreover, does not require calibration of the galvanometer scale.

FIG. 30.—The "2 in 1" Megohmer. (*W. H. Sticht and Company.*)

For measuring high resistance, as normally obtains in the insulation of dynamos, cables, or distribution networks, the megohmer, illustrated in Fig. 30, and with circuit diagram, as in Fig. 31, is a convenient and serviceable instrument. In the "2 in 1" megohmer the restraining torque is not produced by a hair spring as in most moving coil instruments but by a coil (Fig. 31) mounted on the same axis as the main coil a. The insulation to be measured is placed in circuit by being connected to terminals 1 and 3. The torque produced by the potential coil b varies as the impressed voltage E, produced by the direct-current

generator in the instrument; while the torque produced by coil a is proportional to the current I, which flows through the insulation resistance. The currents in the two coils a and b flow in opposite direction and hence the deflection of the pointer indicates the quotient of the voltage E and the current I; that is, directly proportional to the resistance of the insulation measured. Since the current I, in coil a, is proportional to the voltage E across the terminals of coil b the readings are not affected by variations in the voltage E, supplied by the hand-operated direct-current generator; provided the resistance measured is not affected by the impressed voltage. The scale of the instrument is calibrated in mega ohms, hence the name, "megohmer."

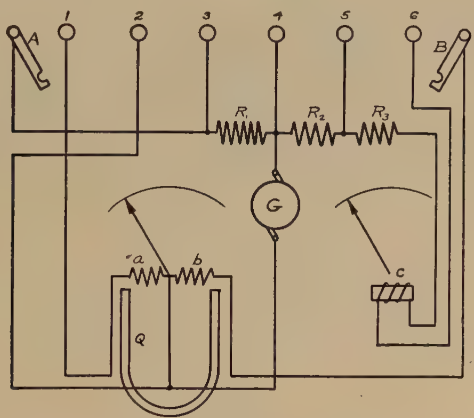


FIG. 31.—Circuit diagram of Megohmer in Fig. 30.

Permeameters.—The permeameter is an instrument for measuring the inherent magnetic properties of materials. Several forms of magnetic measuring instruments have been developed, among which the Koepsel permeameter and the Fahy Simplex permeameter are leading types.

The Fahy Simplex permeameter with accessory equipment consisting of a control box, current rheostats, ballistic galvanometer with lamp and scale, are shown in Fig. 32; and the corresponding circuit diagrams in Fig. 33. The permeameter consists of a laminated silicon steel core of U-shape, comprising a yoke portion and two core arms C and C' having slotted extensions. Upon the yoke portion of the core a magnetizing coil is wound, the terminals of which are connected to the posts M of the terminal block. Between the ends of the core arms is a test coil B,

within which the specimen X to be tested is inserted so as to bridge the polar faces of the core arms and the slotted extensions. This winding is used for measuring the flux induced in the test

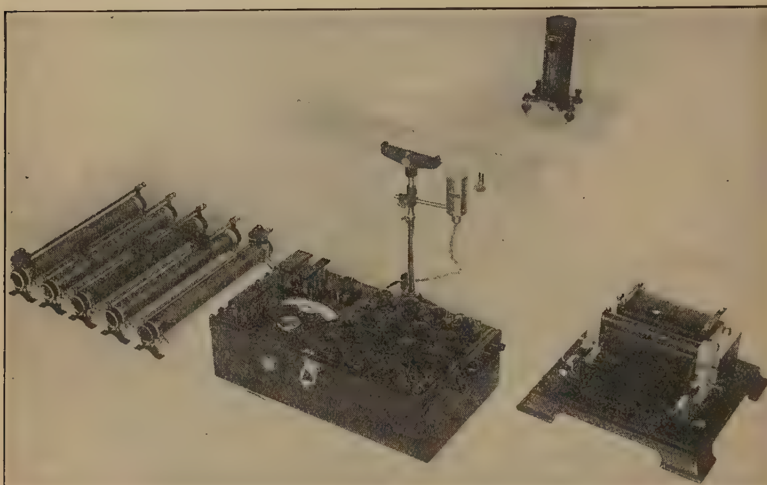


FIG. 32.—Fahy Simplex permeameter and accessory equipment. (*F. P. Fahy.*)

specimen. Its terminals are connected to the binding posts marked B of the inner terminal block. The magnetic potential coil H for measuring the magnetizing force acting on the test

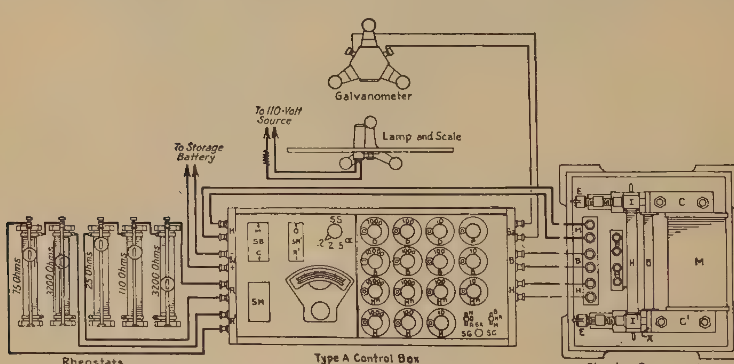


FIG. 33.—Circuit diagram of permeameter assembly in Fig. 32. (*F. P. Fahy.*)

specimen is placed between the upper ends of the contact shoes I and I'. The terminals of this coil are connected to posts H of the inner terminal block. The inner pairs of B and H posts are cross-connected to the B and H binding posts on the outer

terminal block. Clamp screws E hold the ends of the test specimen firmly in place between the contact shoes I and I' and the polar faces of the core arms. This instrument accommodates test specimens up to 2¾ in. in section, and of 10 in. or more in length. The control box comprises an arrangement of switches, galvanometer adjusting resistances, ammeter and calibrating inductance. The ammeter is arranged with the switch SS (Fig. 33) to read 0.2, 2.0, 5.0 and ∞ amp. full-scale deflections, respectively. The calibrating inductance is located inside the control box and has a value of 20 mh.

In principle, the operation consists of taking direct measurements of the magnetizing force and the flux produced in the test specimen. The test specimen is magnetized by sending a current through the magnetizing winding of the yoke. By reversing the magnetizing current, an e.m.f. is induced in the test winding encircling the test specimen. The magnitude of this indirect voltage is proportional to the flux density $\mathfrak{B}$ in the specimen and is measured by the deflection of the galvanometer. Likewise, the reversal of the magnetizing current induces an e.m.f. in the potential coil winding which is a measure of the magnetizing force H acting on the specimen. This method of measuring permeability is therefore analogous to the determining of resistance by taking voltmeter and ammeter reading; that is, by the drop of potential method.

PROBLEMS

1. The resistance of a galvanometer is 550 ohms.

(*a*) Compute the resistances of shunts for the galvanometer in order that $\frac{1}{10}$, $\frac{1}{100}$, and 1/1,000 of the total line current, respectively, shall flow through the galvanometer.

2. A galvanometer having a resistance of 1,600 ohms is to be used with an Ayrton shunt having a total resistance from A to B (Fig. 12) of 9,600 ohms.

(*a*) If the line current is 5 milliamperes how much current is passing through the galvanometer when the line terminal is at the 0.01 point on AB? On the 0.001 point?

(*b*) Compare the results of part (*a*) with the current passing through the galvanometer when the line terminal is at B (Fig. 12).

(*c*) By what ratio is the sensitivity of the galvanometer reduced due to the addition of the above Ayrton shunt?

3. Repeat Problem 2 except that the galvanometer resistance is 400 ohms. Note the difference in results from Problem 2.

4. A 50-scale millivoltmeter has a resistance of 4.2 ohms. Find the resistances of shunts for this meter so as to measure 50, 100, and 500 amp., respectively, at full-scale deflection.

5. The resistance of the moving element of a voltmeter is 75 ohms. Full-scale deflection is produced by 0.66 volts. What resistances must be placed in series with the moving coil in order that the meter shall have full-scale range of 10, 75, 150, and 1,000 volts, respectively?

6. It is desired to measure the insulation resistance between the armature winding and the armature core of a generator. A 750-scale voltmeter having a resistance of 150,000 ohms reads 500 volts across a direct-current circuit. One terminal of the 500-volt circuit is then connected to the frame of the machine and the other to the commutator segments with the same voltage impressed. The voltmeter when placed in series with this insulation circuit reads 2.5 volts. What is the insulation resistance in megohms of the armature winding to the frame?

7. In Fig. 34 is shown the circuit diagram of the Varley loop method of finding a cable fault which is grounded. A balance is obtained for both positions a and b of the switch S by adjusting the variable resistances M, N,

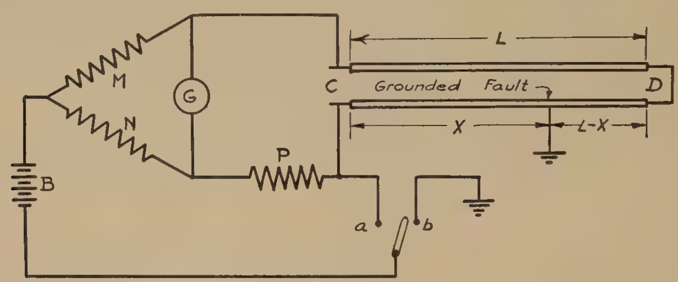


FIG. 34.—Varley loop circuit diagram.

and P. The far ends of the two cables are connected together for a test as indicated in the diagram. When the switch is thrown to a the resistance of the total cable (out and return if the cable is not broken in two) can be computed, from which r, the resistance per foot, may be obtained if the total length of cable is known. If r, per foot, is already known, this reading is not necessary, as the reading for the switch at b would be sufficient. Assuming that r is known, derive the formula for X, the distance to the fault, using the symbols in Fig. 34 when the switch is in position b.

8. One of the wires of a pair of No. 19 telephone wires is known to be grounded. A Varley loop test (see Problem 7) is to be used in locating the fault. The distance between stations is 2,000 ft. With $M = N = 100$ ohms a balance is obtained for the switch in position a by adjusting P until it is equal to 34.24 ohms. With the switch at b a balance is obtained when P is equal to 16.2 ohms. M and N remain the same as before. Find the distance to the fault.

9. A 150-scale voltmeter and a 1-amp. scale ammeter are used in finding the resistance of a field winding. They have resistances of 10,000 and 0.03 ohms respectively. If the meter readings are 125 volts and 0.29 amp. when the ammeter is connected so as to include the voltmeter current, what are the apparent and actual resistances of the field winding? What are the apparent and actual resistances of the field winding if the same readings

were obtained when the voltmeter is connected so as to include the ammeter voltage drop?

10. A 3-scale voltmeter has a resistance of 400 ohms and is used together with a 300-scale ammeter having a resistance of 0.0006 ohms in finding the resistance of an armature winding. With the voltmeter so connected as to include the voltage of the ammeter the meters read 275 amp. and 2.65 volts. What are the apparent and actual resistances of the armature winding? What are the apparent and actual resistances of the field winding if the same readings were obtained when the ammeter is so connected as to include the current of the voltmeter?

CHAPTER IX

THE DIELECTRIC FIELD

In Chap. IV the characteristic features of dielectric phenomena are discussed from a dynamic point of view. Emphasis is placed on the similarity of dielectric flux, permittance, and voltage in the dielectric circuit with magnetic flux, permeability, and magnetomotive force in the magnetic circuit and likewise with current, resistance, and electromotive force in the electric circuit. The approach to the problems involved in the dielectric field is generally made from an entirely different direction. Many phases of dielectric phenomena are essentially static in nature and can, to good advantage, be studied on the basis of electric charges or quanta of electricity having specific properties. It should be clearly kept in mind, however, that dielectric phenomena form an integral part of electromagnetic or dynamic electricity and that dielectric flux lines are closely interwoven or interlinked with magnetic lines of force and electric currents.

Dielectric Induction.—On the basis of the electron theory an electric charge consists of either an excess of negative electronic quanta or electrons, or the corresponding positive electronic charges on ions or protons. To the negative and positive electronic charges are ascribed such properties as have been found necessary to account for observed dielectric phenomena. In order to visualize or picture the physical conditions in the space surrounding the charged bodies some of the properties of the electronic charges are expressed by dielectric lines of force as explained in Chap. IV.

The experimentally observed facts that charges of unlike signs attract and charges of like signs repel each other, combined with the possibility of insulating both positive and negative charges, form the basis of electrostatic induction. If in Fig. 1 a body A carrying a positive charge $+Q_A$ is brought near to another conductor B, the negative electronic charges $-Q_B$ or electrons will be attracted and move to the end nearest to A, while at the same time the positive charge $+Q_A$ on B will be repelled by the

positive charge $+Q_B$ on body A and hence move away from A. If the body B is connected to ground or some body on which the $+Q_B$ charge can move farther away from $+Q_A$ then only $-Q_B$ which is held in place by $+Q_A$ will stay on the body B. Let the connection to ground be broken and let the body A be removed. Then B will be found charged with negative electricity $-Q_B$, or B has been given a negative charge by *electrostatic induction.* A positive charge will induce a negative charge on bodies near it, and, conversely, a negative charge will induce positive charges on conductors in its vicinity.

FIG. 1.—Electrostatic induction.

Condensers may be charged to high potentials by electrostatic induction machines, in which the above described cycle is rapidly repeated while the successive charges are collected by means of brushes.

In any condenser of constant condensance the quantity of electricity in the charge would be directly proportional to the voltage.

$$Q \alpha CE \tag{1}$$

$$Q(\text{coulombs}) = C(\text{farads})\ E(\text{volts}) \tag{2}$$

The quantity of electricity Q is measured in *coulombs* or ampere-seconds. One *coulomb* is the quantity of electricity carried by 1 amp. of current flowing for 1 sec.

$$1 \text{ coulomb} = 1 \text{ amp.-sec.} \tag{3}$$

Hence the current is the rate at which electricity passes at any given cross-section of the conductor.

$$_cI = \frac{Q}{T} \text{ or } _ci = \frac{dQ}{dt} \tag{4}$$

Energy Stored in Dielectric Field.—When a difference of potential exists between conductors an electric stress is exerted on the insulating materials in the intervening space. This stress produces the equivalent of a strain in the electrically elastic

dielectrics. The product of stress and strain represents energy. At potentials less than the rupturing voltage, that is, within the elastic limit of the dielectric, the strain is directly proportional to the stress. Hence, the energy stored must be proportional to the square of the impressed voltage. An equation expressing the energy stored in the dielectric field in terms of the impressed voltage and condensance of the circuit may be derived from relations established in Chap. IV.

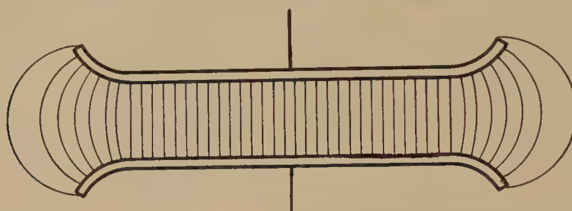


FIG. 2.—Condenser and dielectric field.

From equation (10) Chap. IV,

$$\psi \;\alpha\; Ce \tag{5}$$

$$\psi \text{ (lines of force)} = 3.77 \cdot 10^{10} C(\text{farads}) e(\text{volts}) \tag{6}$$

While the dielectric field is formed a current ${}_ci$ flows into the condenser, which is directly proportional to the time rate of change in the dielectric flux.

$${}_ci \;\alpha\; \frac{d\psi}{dt} \tag{7}$$

$${}_ci \text{ (amperes)} = \frac{1}{3.77 \cdot 10^{10}} \frac{d\psi}{dt} \text{ (lines per second)} \tag{8}$$

From equations (2) and (4),

$${}_ci = C\frac{de}{dt} \text{ amperes} \tag{9}$$

The power P or the rate at which energy W is being stored in the dielectric field is the product of the voltage and the current.

$$P = e\,{}_ci \text{ watts} \tag{10}$$

$$dw = Pdt \tag{11}$$

From equations (9), (10), and (11),

$$dw = e\,{}_ci_c\,dt = Cede \tag{12}$$

$$\int_o^w dw = C\int_o^e ede \tag{13}$$

$$W(\text{joules}) = \frac{C(\text{farads})e^2(\text{volts})}{2} \tag{14}$$

The same relation is expressed in equation (15) with C in microfarads, the more generally used unit for condensance.

$$W(\text{joules}) = \frac{C(\text{microfarads})e^2(\text{volts})}{2 \cdot 10^6} \tag{15}$$

The energy stored in a condenser is thus shown to be proportional to the square of the voltage. If the voltage is increased the electric circuit supplies more energy to the dielectric field. For voltages within the elastic limit of the dielectric the process is reversible. When the voltage is decreased the energy in the dielectric field is returned to the electric circuit, for if e and therefore ψ decrease then, de/dt and hence ${}_ci$ are negative which means that energy flows from the dielectric field back into the electric circuit.

Mechanical Force between Condenser Plates.—Coulomb established the first quantitative relation between electric quantities by measuring the mechanical force of repulsion and attraction between two electrically charged bodies (Chap. VII). In a condenser one plate is charged with negative and the other with positive electricity and a mechanical force is produced, tending to force the plates together by the attraction between the two electric charges.

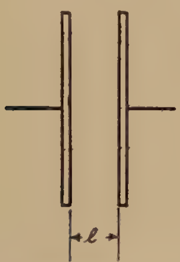


FIG. 3.—Condenser with movable plates.

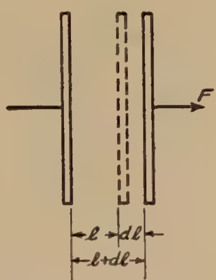


FIG. 4.—Condenser with movable plates.

The condenser represented in Fig. 3 is so constructed that the two parallel plates may be moved nearer or farther apart, thereby varying the length but not the cross-sectional area of the dielectric (air) between them.

First Stage:

Condensance $= C_1$, farads

Impressed voltage $= E_1$, volts

Under these conditions the quantity of electricity and the energy in the condenser are given by equations (16) and (17).

$$Q = C_1E_1 \text{ coulombs} \tag{16}$$

$$W = \frac{C_1E_1^2}{2} \text{ joules} \tag{17}$$

Let the distance between plates be l cm.

Second Stage:

It is assumed that there is no leakage in the condenser and that the source of e.m.f. is disconnected so that the number of electronic charges represented by Q, stored in the first stage is not changed. Let plates be moved apart by a mechanical force F, a distance dl, so that the length of the dielectric circuit between the two plates is changed from l in Fig. 4 to $l + dl$ in Fig. 4. Since the only change in the dielectric between the two plates was to increase its length from l to $l + dl$ the condensance C_2 is found by equation (18).

$$C_2 = \frac{l}{l + dl}C_1 \tag{18}$$

Since no leakage of electronic charges is assumed the quantity of electricity Q remains unchanged and is expressed by equation (19) for the second stage.

$$Q = C_2E_2 \tag{19}$$

The relation between E_2 and E_1 is found from equations (17), (18), and (19).

$$E_2 = \frac{l + dl}{l}E_1 \tag{20}$$

The electric energy stored in the dielectric field in the second stage is given by equation (21).

$$W_2 = \frac{C_2E_2^2}{2} \tag{21}$$

The increase in energy in the dielectric from the first to the second stage is therefore expressed by equation (22).

$$dw = W_2 - W_1 = \frac{C_1E_1^2dl}{2l} \text{ joules} \tag{22}$$

$$dw = \frac{C_1E_1^2dl10^7}{2l} \text{ ergs} \tag{23}$$

The mechanical work in moving the plates further apart against the force of attraction between the electric charges on the plates is expressed by equation (24).

$$dw \propto F dl \qquad (24)$$

$$dw(\text{ergs}) = F(dynes)dl(\text{cm.}) \qquad (25)$$

The increase of energy in the condenser must be equal to the mechanical energy required to make the change in distance between the condenser plates. Hence from equations (23) and (25) the mechanical work,

$$Fdl = \frac{C_1 E_1{}^2 dl 10^7}{2l} \text{ ergs} \qquad (26)$$

Hence:

$$F \text{ (dynes)} = \frac{C(\text{farads})E_1{}^2(\text{volts})10^7}{2l(\text{cm.})}$$

$$= \frac{10C^1(\text{microfarads})E_1{}^2(\text{volts})}{2l(\text{cm.})} \qquad (27)$$

The expression for the condensance in terms of the permittivity and geometric dimensions of the condenser was derived in Chap. IV, equation (7).

$$C(\text{farads}) = \frac{\kappa A 10^9}{4\pi v^2 l}; \text{ or } C(\text{microfarads}) = 0.8842\frac{kA(\overline{\text{cm.}}^2)}{l(\text{cm.})10^7} \qquad (28)$$

Hence from (27) and (28),

$$F(\text{dynes}) = 0.4421\frac{\kappa A(\overline{\text{cm.}}^2)E_1{}^2(\text{volts})}{l^2(\text{cm.})10^6} \qquad (29)$$

From equation (29) it is evident that for ordinary distribution voltages the mechanical forces due to electrostatic repulsion or attraction are negligibly small. In high-tension systems, however, the force may often be of considerable magnitude, as it increases with the square of the voltage. Thus the force between two plates each 1 m. sq. and 1 cm. apart in air and at a potential of 100 volts,

$$F = 0.4421\frac{1 \cdot \overline{100}^2 \cdot \overline{100}^2}{1^2 \cdot 10^6} = 44.21 \text{ dynes} = 0.045 \text{ g.} \qquad (30)$$

If the voltage is increased to 20,000 volts, however, the force would be increased in proportion of $\overline{100}^2$ to $\overline{20{,}000}^2$ or 40,000 times the value under the first set of conditions. For the higher voltage the force F would be 1,800 g. or nearly 4 lb.

Discharging a Condenser through Resistance.—Given a condenser C charged by a direct-current voltage E and the circuit

arranged as shown in Fig. 5, so that the energy stored can be dissipated as heat when the condenser is discharged through the resistance R after the switch S is opened. In the diagram in Fig. 5, v_2 represents the vibrator element of an oscillograph (Chap. VIII). The beam of light coming from the mirror on the moving element falls on a photographic film placed on the rotating drum. The ordinates, at each instant for the curve v_2, on the oscillogram in Fig. 5 indicate the angle the mirror was turned from the neutral position denoted by the base line $P.Q.$

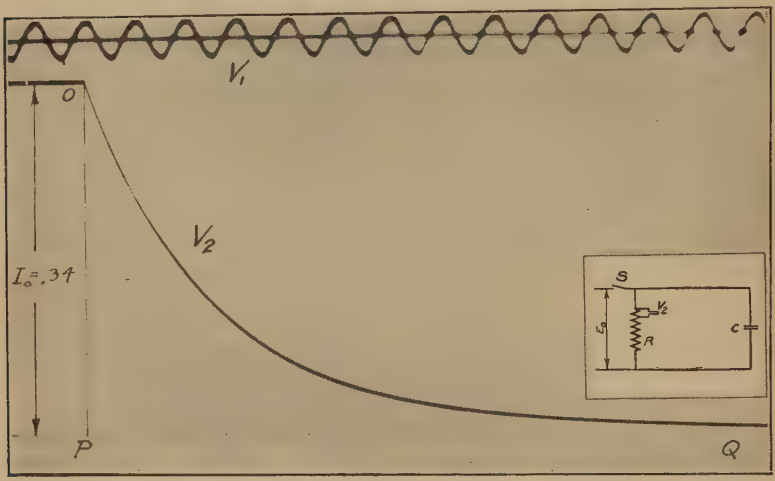


FIG. 5.—Discharging condenser through resistance. Oscillogram. $C = 69.5$ mf.; $E = 105$ volts; $R = 308$ ohms; $I_0 = 0.34$ amp.; $V_1 = 100 \sim$.

The abscissæ represent time. The alternating current shown by the curve v_1 measures the time elapsed, as each complete cycle represents $1/100$ sec. With switch S closed and the condenser charged, no current flows in the conductors leading to the condenser itself, but the current passing through the resistance R and, therefore, the vibrator V_2 is expressed by equation (31).

$$I_0 = \frac{E_0}{R} \tag{31}$$

This current I_0 is represented by the ordinates of the v_2 curve up to the instant marked P at which time the switch S was opened. Opening the switch cuts off the direct-current supply voltage E_0 and hence the dielectric field of the condenser is the only source of energy in the circuit to which the oscillograph is connected. At the instant the switch is opened the voltage

across the condenser terminals is E_0 and hence the initial current flowing through the resistance R must be equal to I_0 as represented by the ordinate PO on the oscillogram.

From Ohm's law the voltage ${}_Re$ at any instant across the resistance R while the condenser discharges is given by equation (32).

$$ {}_Re = Ri \tag{32} $$

From equation (9)

$$ i = C\frac{d_ce}{dt} \tag{33} $$

Hence, solving for the voltage across the terminals of the condenser,

$$ d_ce = \frac{idt}{C} \tag{34} $$

$$ {}_ce = \frac{1}{C}\int idt \tag{35} $$

Applying Kirchhoff's law to the circuit gives equations (36) and from which in combination with equation (32) and (35) equation (37) is derived. By solving the differential equation (37), an expression for the current-time relation during the discharge period is obtained as shown by equation (43).

$$ Re + {}_ce = 0 \tag{36} $$

$$ Ri + \frac{1}{C}\int idt = 0 \tag{37} $$

$$ R\frac{di}{dt} + \frac{i}{C} = 0 \tag{38} $$

$$ \frac{di}{i} = -\frac{dt}{CR} \tag{39} $$

$$ \int_{I_0}^{i}\frac{di}{i} = -\frac{1}{CR}\int_0^t dt \tag{40} $$

$$ \log_\epsilon \frac{i}{I_0} = -\frac{t}{CR} \tag{41} $$

$$ \frac{i}{I_0} = \epsilon^{-\frac{t}{CR}} \tag{42} $$

$$ i = I_0\epsilon^{-\frac{t}{CR}} \tag{43} $$

Since at any instant ${}_Re = Ri$, the instantaneous voltages across the terminals of the condenser during the discharge period is given by equation (44).

$$ e = E_0\epsilon^{-\frac{t}{CR}} \tag{44} $$

The instantaneous values for both the current and the voltage may be obtained, therefore, directly from the V_2 curve on the oscillogram by merely using the corresponding scale of ordinates. The scale values are determined by initial values of the current I_0 and voltage E_0 at the instant the switch S is opened.

Charging a Condenser, Resistance in Series.—When the dielectric field is formed or a condenser charged, energy is stored in the dielectric. The current-time and voltage-time relations are expressed by the same equation both for the charging as the dis-

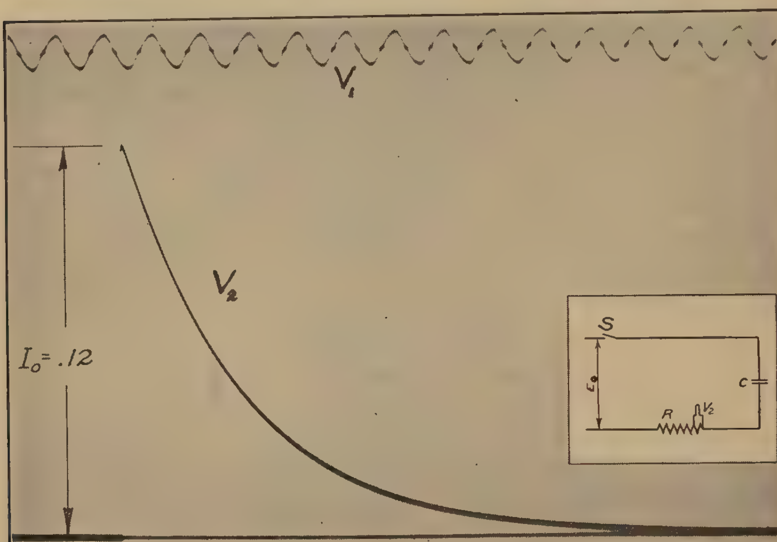


FIG. 6.—Charging a condenser. Oscillogram. C = 28.0 mf.; E_0 = 98 volts; R = 820 ohms; I_0 = 0.12 amp.; V_1 = 100 ~.

charging periods. In Fig. 6 is shown an oscillogram of the current-time curve during the charging period, with circuit diagrams showing the point in the circuit at which the vibrator of the oscillograph was located. The expression for the current-time, or V_2 curve in Fig. 6 is identical in form to that for V_2 in Fig. 5, and is given by equation (43). The initial value I_0 is determined in each case by the values for E_0 and R in the given circuit.

Disruptive Strength, Piercing Pressure.—When the voltage between two conductors is gradually increased the stress on the intervening dielectric likewise increases until the pressure is suddenly equalized by a discharge or formation of a short circuit through the dielectric. If sufficient energy is supplied to the

conductors, the current continues to flow in the form of an arc at a comparatively low voltage, depending upon the resistance of the breakdown path. Instead of stating the disruptive strength of an insulating material in terms of the density of dielectric lines of force, it is customary to give the voltage at which a given thickness of the specified material will be punctured. This is called the *disruptive strength* or *piercing pressure*, and its value depends upon many factors. These may be grouped in two divisions: (*a*) factors affecting the specific dielectric strength of any given material; (*b*) factors affecting the voltage gradient at various points in the dielectric circuits.

Table XI.—Approximate Piercing Pressure for 1 Millimeter Thickness

	Kilovolts
Air at atmospheric pressure (76 cm.)	3
Boiled linseed oil	8
Dry wood	0.4 to 0.6
Electron	25
Fiber vulcanized	8 to 18
Fish paper	10 to 15
Kraft paper	4 to 6
Marble	2 to 4
Maple, oiled or paraffined	3 to 45
Melted paraffin	15 to 20
Mica	25 to 120
Micanite	33
Paraffined paper	40 to 60
Transformer oil	4 to 6
Turpentine	3.5 to 6.5
Varnished cambric	45 to 70
Varnished silk	45 to 70
Vulcanized rubber	9 to 10

Specific Dielectric Strength.—The more important factors are: (1) material; (2) moisture; (3) temperature; (4) thickness; (5) mechanical stresses; (6) length of time the voltage is applied.

1. The specific piercing pressure varies widely for different materials, and bears no direct relation to the insulation resistance. Solids like rubber, glass, mica, porcelain, or fiber have much higher disruptive strengths than liquids and gases. But in a solid a puncture is not readily repaired, while in liquids and gases the path of the disruptive spark is quickly filled and the dielectric automatically regains its normal strength. For definite chemical compounds, the piercing pressure for a given thickness is fairly

constant, but many insulating compositions are mere physical mixtures and a lack of uniformity necessarily produces a great variation in the disruptive strength.

2. The presence of moisture, especially in solids and liquids, decreases the disruptive strength. As most materials are more or less hygroscopic, the moisture content must be carefully determined.

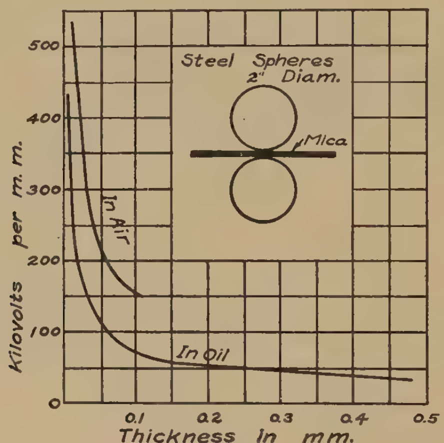


Fig. 7.—Disruptive strength in mica of air and in oil.

3. The temperature within ordinary limits has little effect upon the disruptive strength, provided no chemical changes are produced in the material. For commercial operation the permissible rise in temperature is definitely limited so as to prevent chemical changes in the material.

4. The disruptive strength is not directly proportional to the thickness of the dielectric. Usually the thickness increases faster than the piercing pressure. This is illustrated for mica in Fig. 7. For very thin layers the variation is generally the reverse.

5. In the presence of mechanical stresses the piercing pressure is decreased.

6. The length of time during which the pressure is applied has considerable effect upon the piercing pressure. There seems to be something similar to fatigue or a slightly viscous give in the material, requiring a few seconds or minutes to reach equilibrium. In testing insulation on commercial machines it is customary to apply the pressure for 1 min. when they are warm, as

this provides the necessary time duration and also conforms with working temperature conditions.

In some materials like oiled press-boards it requires several minutes for the fatigue to reach a permanent value at any given voltage. For practical application of the factors affecting the dielectric strength a number of empirical formulæ have been devised. In commercial work the data given in handbooks and the Standardization Rules of the A.I.E.E. are the main guides in determining the permissible limits for electrical designs.

The Voltage Gradient.—The voltage gradients in the space surrounding conductors at a difference of potential depend directly on the relative density of the dielectric lines of force. The factors that determine the distribution of the dielectric flux may be discussed under four divisions.

1. One homogeneous dielectric.

2. Two or more insulating materials forming parallel dielectric circuits.

3. Two or more insulating materials forming series dielectric circuits.

4. Combinations of series and parallel circuits.

One Homogeneous Dielectric.—In the space surrounding two conductors, at a difference of potential, the distribution of the lines of force depends directly upon the geometric form or

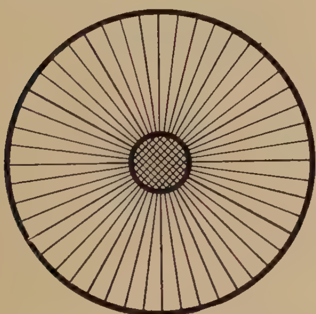


FIG. 8.—Cross-section of cable.

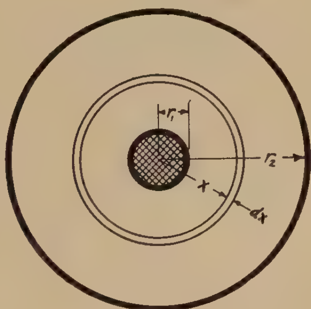


FIG. 9.

the dimensions of the dielectric. In dielectric circuits of simple form the flux density and voltage gradient may be readily calculated.

First Example. Lead-covered Cable.—Given a single-conductor, lead-covered cable with an insulating dielectric of one homogeneous material. When a difference of voltage exists between

the conductor and the lead sheath, the direction of the stress (Fig. 8) and hence the flux, is radial. Since all the lines connect the conductor and the lead sheath, the density is greatest at the surface of the conductor and least at the lead sheath. Concentric cylinders form equipotential surfaces. An expression for the voltage required to pass the flux ψ through the elemental cylinder 1 cm. long, of dx thickness and at x distance from the center may be obtained from equations (6) and (28). In applying

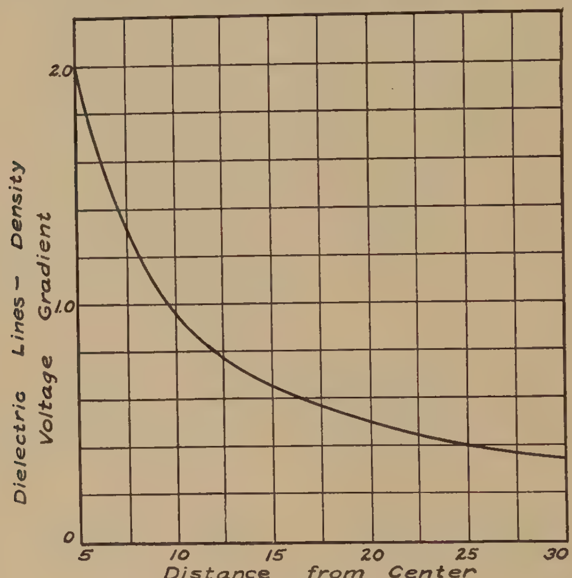


FIG. 10.—Voltage gradient in cables.

the equations to the elemental cylinder $l = dx$; $A = 2\pi x$ and $e = de$. Therefore,

$$de = \frac{300\psi dx}{2\pi\kappa x} \tag{45}$$

Let r_1 = radius of conductor,
r_2 = radius (inside) of lead sheath.

The voltage between the conductor and sheath is, therefore,

$$e = \frac{300\psi}{2\pi\kappa}\int_{r_1}^{r_2}\frac{dx}{x} = \frac{300\psi}{2\pi\kappa}\log_\epsilon\frac{r_2}{r_1} = 20.6\frac{\psi}{k}\log_{10}\frac{r_2}{r_1} \text{ volts} \tag{46}$$

$$\psi = \frac{2\pi\kappa e}{300\log_\epsilon\dfrac{r_2}{r_1}} = \frac{e\kappa}{20.6\log_{10}\dfrac{r_2}{r_1}} \text{ lines} \tag{47}$$

The voltage gradient G at any point within the dielectric is expressed by equation (48).

$$G = \frac{de}{dx} = \frac{300\psi}{2\pi\kappa x} = \frac{e}{x \log_\epsilon \frac{r_2}{r_1}} = \frac{0.4343e}{x \log_{10} \frac{r_2}{r_1}} \text{ volts per centimeter} \quad (48)$$

For constant voltage applied to any given cable the curve for equation (48) is that of an hyperbola referred to its asymptotes. In Fig. 10, $r_1 = 5$, $r_2 = 30$ and the ordinates represent either the voltage gradient or the dielectric flux density, for values of x between 5 and 30.

The maximum voltage gradient is at the surface of the conductor:

$${}^{m}G = \frac{e}{r_1 \log_\epsilon \left(\frac{r_2}{r_1}\right)} = \frac{0.4343e}{r_1 \log_{10} \left(\frac{r_2}{r_1}\right)} \text{ volts per centimeter} \quad (9)$$

Second Example. Two Parallel Plates.—If the plates are of considerable size, the distribution of the flux, in the area between the plates, is uniform except at or near the edges. Near the edges the density is greater, and at points outside, less than in the space between the plates (Fig. 11).

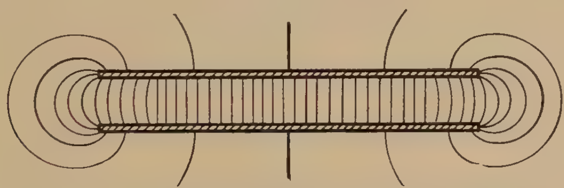


FIG. 11.—Dielectric flux between two parallel plates.

Let x = the distance between the plates in centimeters.

A = the area over which the flux is uniformly distributed in $\overline{\text{cm}}^2$.

The elastance,

$$S = \frac{4\pi v^2 x}{\kappa A 10^9} \text{ darafs.} \quad (50)$$

The condensance,

$$C = \frac{\kappa A 10^9}{4\pi v^2 x} \text{ farads} \quad (51)$$

Dielectric flux,

$$\psi = \frac{3.77 \cdot 10^{10} e}{S} = 3.77 \cdot 10^{10} Ce = \frac{\kappa A e}{300x} \text{ lines} \quad (52)$$

Flux density,

$$D = \frac{\psi}{A} = \frac{\kappa e}{300x} \text{ lines per } \overline{\text{cm.}}^2 \tag{53}$$

Voltage

$$e = \frac{300\psi x}{\kappa A} \text{ volts.} \tag{54}$$

Voltage gradient,

$$G = \frac{de}{dx} = \frac{300\psi}{\kappa A} \text{ volts per centimeter.} \tag{55}$$

Third Example. Two Parallel Round Conductors in Air.—The most important practical example is the transmission line and the discussion deals with the problem under the restrictions

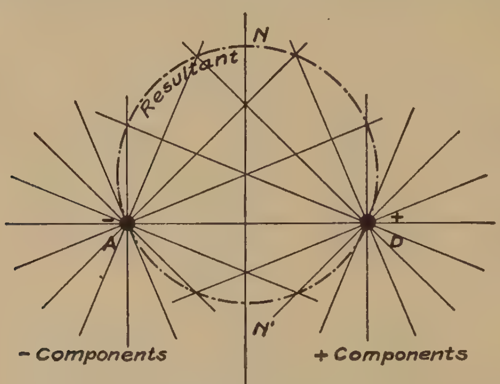


FIG. 12.—Resultant dielectric field on transmission line.

of high-tension transmission-line conditions. It is therefore assumed that:

1. The diameters of the two conductors are equal.
2. The distance between the conductors is large as compared with the diameter of each wire.

Since the two conductors have constant diameters and are parallel in position, equipotential surfaces must be cylinders and hence the distribution of the lines of force is the same in any normal plane. All phases of the problem may therefore be shown on a cross-section perpendicular to the conductors. Consider a neutral plane midway between the conductors and perpendicular to the plane joining their centers, as indicated by the line NN' in Fig. 12. The two wires are equally positive and negative referred to the line NN'. When taken independently,

the fields for each are radial straight lines. The resultant field is the superposition of the separate fields, and the flux lines assume curved paths as shown in Fig. 13. These curved paths are arcs of circles passing through the points A and B and hence have their centers along the line NN^1. This may be shown as in Fig. 14. Take any point P in a normal plane. Let $AP = s$ and $BP = u$. The dielectric stress or field intensity at the point P due to the negative voltage of A is along PA and inversely proportional to the distance s. Likewise the stress due to the positive voltage of B is in the direction BP and inversely proportional to the distance u.

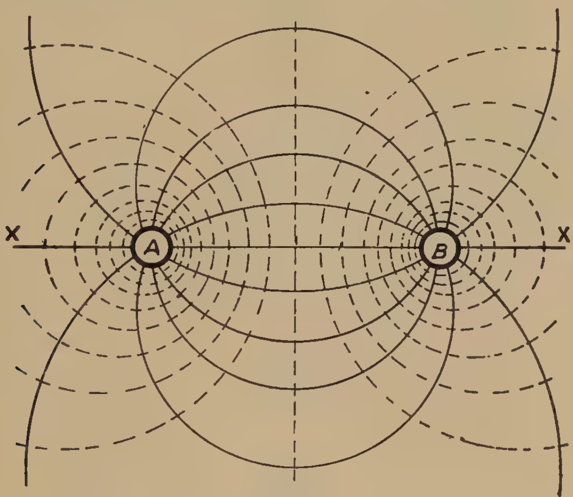


Fig. 13.—Dielectric field for transmission line.

Lay off the lines PN and PH in Fig. 14, so that

$$PN:PH::\frac{1}{u}:\frac{1}{s} \tag{56}$$

Complete the parallelogram $PHKN$. Then the line PK represents, in direction and relative magnitude, the dielectric field intensity at the point P. In the two triangles APB and PHK two sides are proportional and the included angles equal; hence the triangles are similar. Therefore, the angles ABP and HPK are equal. Hence the line PK must be tangent to a circle passing through the three points A, P, and B. Since P is any point in the normal plane, the direction of the dielectric lines of force

between the points A and B is along arcs of circles passing through A and B.

The equipotential surfaces are perpendicular at every point to the direction of the flux lines and therefore must be cylinders with their axes in the plane passing through both conductors, as indicated by the equipotential circles, dotted lines in Fig. 13.

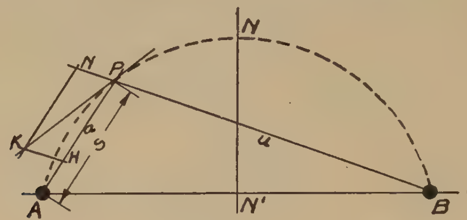


FIG. 14.—Dielectric field intensity.

From the diagram it is evident that the maximum flux density and voltage gradient must be in the plane passing through the centers of the two conductors, or along the line XX in Figs. 13 and 15. Since the maximum values are of special importance in commercial problems, and in order to simplify the expressions, the discussion will be limited to the plane represented by the line XX.

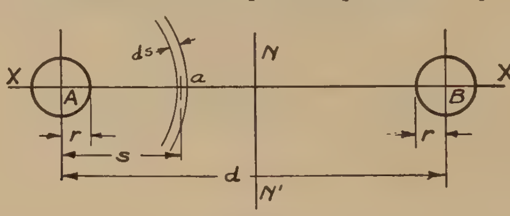


FIG. 15.

Let d = the distance in centimeters, between the centers of the conductors.

r = the radius in centimeters of each conductor.

s = distance in centimeters of any point a from the center of conductor A.

K_a = dielectric field intensity at point a.

D_a = dielectric flux density at point a.

ψ = total flux.

κ = permittivity of air = 1.0

e_n = voltage to neutral.

At the point a the dielectric flux density due to the voltage difference from A to NN' is proportional to $1/s$; and for the voltage difference from NN' to B it is similarly proportional to $1/d-s$;

at any point in the plane of two parallel conductors the total flux density is the scalar sum of the flux density from each conductor. Hence at any point a on a line joining the centers of conductors A and B in Fig. 15 and at a distance s from conductor A the flux density is given by equation (50).

$$D_a = \frac{\psi}{2\pi}\left[\frac{1}{s} + \frac{1}{d-s}\right] \text{lines per } \overline{\text{cm}}^2 \tag{57}$$

The voltage absorbed in any elemental zone ds (for air the permittivity, $\kappa = 1$) is expressed by equation (58).

$$de = 300 D_a ds = \frac{300}{2\pi}\psi\left[\frac{1}{s} + \frac{1}{d-s}\right] ds \text{ volts} \tag{58}$$

The total voltage from A to neutral NN' is, therefore,

$$e_n = \frac{300}{2\pi}\psi \int_r^{\frac{d}{2}}\left[\frac{1}{s} + \frac{1}{d-s}\right] ds \tag{59}$$

$$= \frac{300}{2\pi}\psi \log_\epsilon\left(\frac{d-r}{r}\right) \text{ volts} \tag{60}$$

Therefore,

$$\psi = \frac{2\pi e_n}{300 \log_\epsilon\left(\frac{d-r}{r}\right)} \text{ lines} \tag{61}$$

Voltage gradient,

$$G = \frac{de_n}{ds} = \frac{300}{2\pi}\psi\left[\frac{1}{s} + \frac{1}{d-s}\right] \tag{62}$$

$$= \frac{e_n}{\log_\epsilon\left(\frac{d-r}{r}\right)}\left(\frac{1}{s} + \frac{1}{d-s}\right) \tag{63}$$

$$= \frac{e_n d}{(sd - s^2)\log_\epsilon\left(\frac{d-r}{r}\right)} \text{ volts per centimeter} \tag{64}$$

Maximum voltage gradient,

$$^MG = \frac{e_n d}{(rd - r^2)\log_\epsilon\left(\frac{d-r}{r}\right)} \tag{65}$$

Under the assumption that r is small as compared to d, which is the case in transmission-line problems, the maximum voltage gradient becomes,

$$^MG = \frac{e_n}{r \log_\epsilon\left(\frac{d}{r}\right)} = \frac{0.4343 e_n}{r \log_{10}\left(\frac{d}{r}\right)} \tag{66}$$

For e, the total voltage between conductors, the maximum gradient.

$$^{M}G = \frac{e}{2r \log_\epsilon\left(\frac{d}{r}\right)} = \frac{0.4343e}{2r \log_{10}\left(\frac{d}{r}\right)} \qquad (67)$$

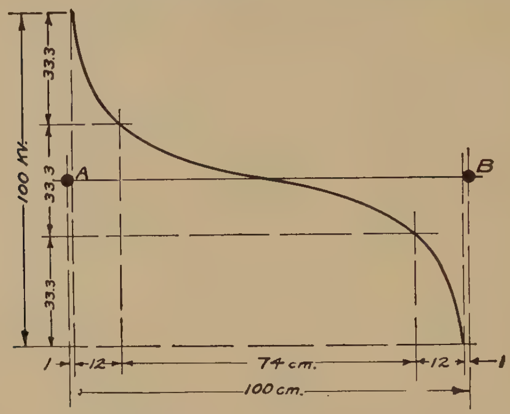


Fig. 16.—Voltage gradient between parallel wires.

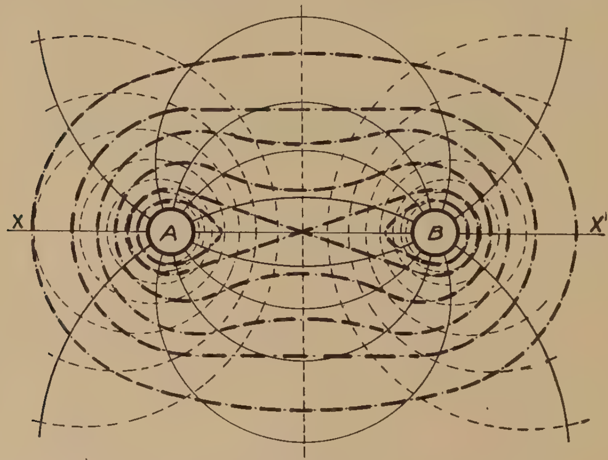


Fig. 17.—Constant potential curves. Parallel wires.

In Fig. 16 the voltages are plotted as ordinates with the distances from the wires as abscissæ in the plane through the wires. The broken lines in Fig. 17 show the equigradient lines in the field around the conductors.

Residual Charge.—Due to the heterogeneous nature of insulation materials and compositions, combined with a slow leakage,

all the energy stored dielectrically is not released immediately upon short circuiting the conductors. This is particularly the case with the use of direct currents when the voltage is applied for long periods in one direction. The energy stored in the dielectric at some distance from the conductor must be transmitted through the intervening layers. This transmission is not all made instantaneously and is often complicated by leakage of various amounts in the several layers, and hence an appreciable length of time is required both to energize and to discharge the dielectric. On this account a single discharge of a cable or other apparatus having considerable condensance may not be sufficient. The two sides should be short circuited for some time or several "groundings" may be necessary to remove the stored energy, and thus prevent any dangerous shock to the workmen.

Two or More Dielectrics in Parallel.—Let the space between two metallic plates be divided into three equal parts, filled with rubber, air, and glass as indicated in Fig. 18. With a difference of voltage existing between the plates, the density of the dielec-

FIG. 18.—Parallel dielectric circuits.

tric flux in the three insulating materials will be directly proportional to their permittivities, or approximately at 3:1:5. By varying the impressed voltage, the flux density also changes, but the relative flux densities are still in the same ratio. The voltage distribution in the space may be represented by a straight line (Fig. 18); and the voltage gradient $G = de/dx$ is a constant. In order to illustrate the elemental principles it is assumed that there is no leakage, *i.e.*, the dielectric flux is confined to a definite path, as is the case with electric circuits. Under actual conditions there are no sharp divisions between good and poor conductors of dielectric flux. A good dielectric like glass has a permittivity only about five times that of air or of a vacuum. Hence, in the above illustration there would be no material avail-

able for separating the rubber and the air, or the glass and the air. Likewise, as the permittivity of air is unity and not zero, the flux cannot be confined to the direct path between the two plates but also passes through the air on the sides.

Two or More Dielectrics in Series.—Let three equal thicknesses of rubber, air, and glass fill the space between two parallel metallic plates, as indicated in Fig. 19; and consider only that portion having uniform distribution of the field flux. The dielectric flux passes through the three materials in series and hence the flux densities in the glass, air, and rubber are the same. The

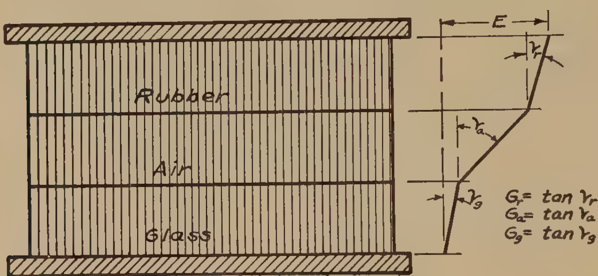


Fig. 19.—Series dielectric circuits.

voltages required to force the flux through equal distances of the three materials are inversely proportional to the permittivities. This may be indicated graphically by plotting the relation between volts and length of flux path as in Fig. 19. For each material the curve is a straight line, but the slope differs in the three cases. The voltage gradients are therefore in the ratio of 1/3:1/1:1/5 or as 5:15:3. Hence while the air occupies only one-third of the space it takes $^{15}/_{23}$ of the total voltage. The voltage gradient in the air is therefore almost twice as great as would be the case if the rubber and glass were removed. If the voltage gradient is near the breakdown strength of air alone between the plates, the insertion of the rubber and glass would so increase the gradient in the air that corona and sparks would appear. For example, let the voltage between the plates be 20 kv. times the distance in centimeters apart. With air alone as the dielectric the voltage gradient would be 20 kv., which is below the rupturing voltage. After inserting the rubber and glass plates the voltage gradient in the air is 39.1 kv., far above its dielectric strength, and the air space is aglow with corona and spark discharges.

Combination of Series and Parallel Circuits.—From the discussion of the simple series and parallel dielectric circuits it is seen that the laws are similar to those of the electric circuits. In principle, therefore, the combination of series and parallel circuit offers no difficulties, and the laws may be easily formulated. The calculation of the numerical values for electrical appliances, however, is in most cases very difficult because of the irregular shape of the dielectric. Electrical conductors are usually of simple geometric forms. Wires, bars, ribbons, cables, etc., whose dimensions are readily determined, form the bulk of resistance problems in electric circuits. In the dielectric circuit the same simple law applies; namely, the elastance varies directly as the length and the specific constant of the material used and inversely as the cross-section, but the shape of the dielectric makes it very difficult to determine the dimensions. In most cases it is manifestly impossible to measure either the length or the cross-section of the dielectric circuit, and as a consequence the exact distribution of the dielectric flux can not be calculated.

The small range in the specific condensance or permittivity of dielectrics makes it impossible to confine the dielectric flux to as definite paths as obtains for the electric current or for the magnetic circuit. In principle, the laws for the dielectric flux distribution can easily be formulated. The calculation of the numerical values is in most cases difficult, however, and frequently impossible because of the irregular shape of the dielectric and the comparatively small difference in the permittivity of the insulating material and the surrounding air.

Fortunately, the relative distribution of the dielectric flux is of little importance unless the voltage gradient at any point approaches or exceeds the disruptive strength of the insulating material. For all low-voltage apparatus calculations for dielectric flux density are not necessary. For high-voltage systems the dielectric-flux distribution is of great importance and apparatus must be so designed that at no point shall the voltage gradient exceed the dielectric strength of the insulation.

PROBLEMS

1. A condenser has a condensance of 250 mf. How much energy is stored in the condenser when 100, 220, and 440 volts are impressed across its terminals? Give results in joules, gram-calories, and foot-pounds.

2. A parallel plate air condenser has an area of 200 cm.^{2} and the plates are 0.5 cm. apart. It is charged to a potential of 300 volts and then the

plates are drawn out to 1.3 cm. apart. Find C_1, C_2, and E_2 and the amount of work done in ergs in drawing out the plates.

3. What would C_3, E_3 and energy stored be if in Problem 2 a glass plate 1.3 cm. thick were placed between the plates? $\kappa = 9.9$ for the glass used.

4. A series circuit is made up of a condenser whose condensance is 4.8 mf., a resistance of 52 ohms, a knife switch and a battery of 102 volts with an internal resistance of 3 ohms.

(*a*) If the switch is closed at what rate will the current be decreasing after 0.007 sec.?

(*b*) At what rate at the instant of closing the switch?

(*c*) If the current decreased continually at a rate equal to the initial rate how long a time would be required to discharge the condenser?

(*d*) Compare this time in seconds with the product of R in ohms and C in farads.

(*e*) What is the value of the current when $t = RC$.

5. If, after the condenser of Problem 4 is charged, the battery is removed and the circuit is again closed find:

(*a*) The initial rate of decrease of current.

(*b*) If the current decreases uniformly at the initial rate how long a time will elapse before the condenser is discharged?

(*c*) Compare the time in (*b*) with $t = RC$.

(*d*) What is the rate of decrease of the current at 0.005 sec. after closing the switch?

(*e*) What is the value of the current in amperes 0.001 sec. after closing the switch?

(*f*) What is the value of the initial current in amperes?

6. A lead-covered cable has a single conductor of ½ in. diameter solid copper. The diameter inside of the lead sheath is 2 in. If 10,000 volts be applied between the conductor and the sheath, find the voltage gradient: (a) at the surface of the conductor and (*b*) at the sheath.

7. If the insulating material in the cable of Problem 6 has a permittivity of 2.2, what would be the condensance per mile of this cable between conductor and sheath? How many dielectric lines of flux per centimeter length of cable?

8. A lead-covered cable having the same dimensions and impressed voltage as given in Problem 6 contains two different materials as insulation between the conductor and sheath. One of the materials has a permittivity of 3.6 and is uniformly molded around the conductor to a thickness of ¼ in. and covered with a layer of tinfoil. The other material, the same kind as used in Problem 7 having a permittivity of 2.2, occupies the space between the inner insulation and the lead sheath.

(*a*) Find the voltage gradient at both the inner and outer surfaces of the first insulation material.

(*b*) Find the voltage gradient at both the inner and outer surfaces of the second insulation material.

(*c*) Find the condensance per mile of this cable.

(*d*) How many dielectric lines of flux are in a section of the above cable 1 cm. long?

9. A parallel plate air condenser has a cross-sectional area of 200 cm. The distance between plates is 3 cm. A voltage of 75,000 volts is impressed.

(*a*) Find condensance of the condenser.

(*b*) What is the voltage gradient?

(*c*) How many dielectric lines of flux are produced?

(*d*) Find flux density.

10. A glass plate 1 cm. thick, having a permittivity of 9.5, is suddenly thrust between the plates of the condenser in Problem 9.

(*a*) What is the voltage gradient in both the air and glass? The 75,000 volts is still impressed.

(*b*) Explain what happens between the condenser plates.

11. A two-wire transmission line, 96 mi. long has a spacing of 108 in. The size of wire is No 000 copper. Find the condensance between the two wires. Find the voltage gradient at the surface of the conductors and midway between the conductors, if the line voltage is 55,000 volts.

CHAPTER X

THE MAGNETIC FIELD

The basic laws of the magnetic circuit and the properties of magnetic lines of force are stated in Chap. III. But magnetic fields possess other properties, some of which have no counterpart in the electric circuit, as for example, the reversible electromagnetic process of storing energy. The discussions in this chapter are limited to magnetic fields in non-magnetic materials, that is, for conditions under which the permeability factor is unity. The magnetic properties of iron and steel and their effects on magnetic fields are treated in Chap. XI.

The north magnetic pole, unit magnet pole or unit pole, as defined in Chap. III, is essentially a theoretical quantity, a useful and convenient concept of a quantitative base in defining the properties of electromagnetic fields, but of little practical value for experimental purposes. From the unit pole 4π lines of force emanate, uniformly distributed in space; that is, one line of force passes through each $\overline{\text{cm.}}^2$ of surface of a sphere of 1 cm. radius surrounding the pole.

As stated in Chap. III the practical units of magnetic flux are the *line of force*, or line of induction, the *maxwell* and the *weber*.

$$1 \text{ maxwell} = 1 \text{ line of magnetic force} \tag{1}$$
$$1 \text{ weber} = 10^8 \text{ maxwells} \tag{2}$$

The practical unit of flux density, or the number of maxwells per unit area of surface at right angles to the direction of the lines of force, is the *gauss*. The symbol for flux density is $\mathcal{B}$ or B.

$$1 \text{ gauss} = 1 \text{ maxwell per } \overline{\text{cm.}}^2 \tag{3}$$

The practical unit for reluctance is the oersted. The oersted is the reluctance per $\overline{\text{cm.}}^3$ of space or air or any non-magnetic material.

$$1 \text{ oersted} = \text{reluctance per } \overline{\text{cm.}}^3 \text{ of space} \tag{4}$$

The practical units of magnetomotive force are the *gilbert* and the *ampere-turn*. A magnetomotive force of 1 gilbert will produce 1 maxwell of flux through an oersted reluctance.

$$\mathfrak{F}(\text{gilberts}) = \phi(\text{maxwells}) \cdot R(\text{oersteds}) \tag{5}$$
$$1 \text{ ampere-turn} = 0.4\pi \text{ gilberts}$$

The reaction between the magnetic field investigated and a unit magnet pole forms the basis for definitions of quantitative values in electromagnetic fields. As the terminology used is somewhat misleading it is important to keep clearly in mind whether the quantity to be measured is the *force* (*field intensity*) existing between the unit pole and the field investigated, the *work* (*magnetic potential*) expended in bringing the unit pole to the point specified or the *work* (*magnetomotive force*) required to carry the unit pole over one complete interlinkage, or the line integral, of the electric circuit. The unit pole is used as the basic unit of measure for determining the *field intensity*, the *magnetic potential*, and the *magnetomotive force* produced by an electric current.

Field Intensity. (*a*) *Magnet Pole.*—Consider two magnet poles, as in Fig. 1, of strength m and m' units respectively, placed a distance x cm. apart. If both m and m' are either north or south poles a force of repulsion exists between them as expressed by equation (6).

$$F \; \alpha \; \frac{mm'}{x^2} \tag{6}$$

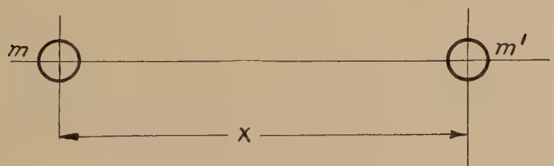


Fig. 1.—Magnet poles.

If m^1 is a unit pole the force exerted by pole m on the unit pole at a distance x cm. is given by equation (7).

$$F^1 = \frac{m}{x^2} \text{ dynes} \tag{7}$$

The force exerted on the unit pole is termed the "field intensity" of the magnetic field at the given point. The symbol for

field intensity is H and the unit is *gilberts per centimeter.* Hence from equation (7),

$$H = \frac{m}{x^2} \text{ gilberts per centimeter} \tag{8}$$

The flux density,

$$B = \mu H \tag{9}$$

For air the permeability $\mu = 1$, hence the flux density at the point a distance x from the pole m, is given by equation (10), (identical in form to the expression for the field intensity H).

$$B = \frac{m}{x^2} \text{ gausses} \tag{10}$$

(*b*) *Field Intensity Around Straight Wire Carrying Current.*—In Fig. 2 let the line AB represent a straight wire carrying a current of 1 amp. and m the position and strength of a magnet pole. An element dy of the conductor is at a distance x from m, while r is the perpendicular distance of the pole from the wire AB.

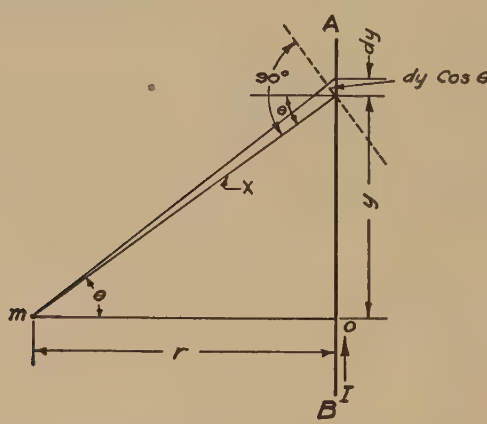


FIG. 2.—Field intensity from current in a straight wire.

Under the above stated conditions the force exerted on the pole m by the field of the current I in the conductor element dy is expressed by equation (11); which is a statement of Biot and Savart's law.

$$dF = \frac{Imdy}{10x^2} \cos \theta \tag{11}$$

If m is a positive or north pole and the current flows from B towards A, as indicated by the arrows, the direction of the force dF will be upwards, at right angles to the plane determined by

the straight wire and point m. If either the direction of the current flow be reversed or the north pole replaced by a south pole the direction of the force on the pole m will be in the reverse direction.

From Fig. 2,

$$\cos \theta = \frac{r}{x} \tag{12}$$

$$x^2 = r^2 + y^2 \tag{13}$$

From equations (11), (12), and (13),

$$dF = \frac{rImdy}{10(r^2 + y^2)^{3/2}} \tag{14}$$

For all the elements in the long straight wire (infinite in length),

$$F = \frac{rIm}{10}\int_{-\infty}^{+\infty} \frac{dy}{(r^2 + y^2)^{3/2}} \tag{15}$$

$$= \frac{2Im}{10r} \tag{16}$$

The field intensity H is measured by the force exerted on a unit pole m and hence:

$$H_r = \frac{2I}{10r} \text{ gilberts per centimeter.} \tag{17}$$

Under the assumption of not having any magnetic materials in the spaces surrounding the wire, the permeability $\mu = 1$. Therefore the flux density at a distance r cm. from the straight wire carrying a current of I amp. is,

$$B_r = \frac{2I}{10r} \text{ gausses} \tag{18}$$

Equations (17) and (18) for H the field intensity, and for B the flux density respectively, may be derived by applying Ohm's law (Chap. III) to the magnetic circuit interlinking the conductor in which a current is flowing. The relation between the magnetic flux, magnetomotive force, and reluctance in the space surrounding a coil of N turns and carrying a current of I amp. is expressed by equation (19).

$$\phi = \frac{\mathfrak{F}(\text{gilberts})}{\mathfrak{R}(\text{oersted})} \text{ maxwells} \tag{19}$$

$$= \frac{4\pi N(\text{turns})I(\text{amp.})}{\dfrac{10\,l(\text{cm.})}{\mu A(\text{cm}^2.)}} = \frac{0.4\pi NI\mu A}{l} \text{ maxwells} \tag{20}$$

For a straight wire in air $N = 1$ and likewise the permeability, $\mu = 1$. The magnetic lines of force interlinking the current in the wire form concentric circles, hence, the length of path $l = 2\pi r$ at a distance r from the center of the wire. Therefore the flux density B_r at a distance of r cm. from the wire is given by equation (21), which is identical in form to equation (18).

$$B_r = \frac{\phi}{A} = \frac{0.4\pi I}{2\pi r} = \frac{0.2I}{r} \text{ gausses} \tag{21}$$

With the permeability unity the flux density B_r in gausses is numerically equal to the field intensity H_r in gilberts per centimeter.

$$H_r = \frac{0.2I}{r} \text{ gilberts per centimeter} \tag{22}$$

If the conductor (wire) carrying the current were surrounded by a magnetic material, permeability μ, the flux density B_r' at a distance of r cm. from the center of the wire is expressed by equation (23).

$$B_r' = \mu H_r = \frac{0.2I\mu}{r} \text{ gausses} \tag{23}$$

(*c*) *Field Intensity on Axis of Circular Coil.*—Given a circular coil of N turns and of radius r cm. carrying a current of I amp., as illustrated by Fig. 3., let a magnet pole of unit strength be placed at the point P at a distance x from 0 and on the axis of the coil. By twisting the leads the magnetic effect of bringing the current to the circular coil may be made very small and considered negligible.

The force between the current I in the element dl and the unit pole at P is given by equation (24) in accord with the law of Biot and Savart.

$$dF_p = \frac{NIdl}{10(r^2 + x^2)} \text{ dynes} \tag{24}$$

The direction of the force dF_p is at right angles to the straight line joining P and D. Let the component of dF_p along the axis OP be denoted by dF_{op}.

$$dF_{op} = \frac{NIdl \sin \theta}{10(r^2 + x^2)} \text{ dynes} \tag{25}$$

$$= \frac{NIrdl}{10(r^2 + x^2)^{3/2}} \text{ dynes} \tag{26}$$

It is evident from the diagram that by taking the total force at P, produced by all the dl elements in the coil, the components at right angles to the axis OP, neutralize each other, while the total force along the axis is the sum or integral of all the elements in the coil. Hence the force on the unit pole at P produced by the current in the circular coil is expressed by equations (27) and (28).

$$\int_o^{F_{op}} dF_{op} = \frac{rNI}{10(r^2 + x^2)^{3/2}} \int_o^{2\pi r} dl \text{ dynes} \qquad (27)$$

$$F_{op} = \frac{2\pi r^2 NI}{10(r^2 + x^2)^{3/2}} \text{ dynes} \qquad (28)$$

If the point P is at the center O of the coil, when $x = O$, the force on the unit pole is given by equation (29).

$$F_o = \frac{2\pi NI}{10r} \text{ dynes} \qquad (29)$$

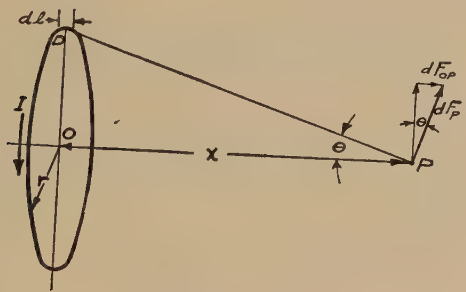


FIG. 3.—Field intensity from current in circular coil.

From equation (29) the unit of current, the ampere, may be defined as that current which, if flowing in a circular coil of 1 turn and of 1 cm. radius, produces on a unit magnetic pole placed at the center of the coil a force of 0.2π dynes.

As the *field intensity* is measured by the force exerted on a unit pole, the value of H for any point on the axis is expressed by equation (30) and at the center of the circular coil by equation (31).

$$H_p = \frac{2\pi r^2 NI}{10(r^2 + x^2)^{3/2}} \text{ gilberts per centimeter} \qquad (30)$$

Field intensity at center of circle,

$$H_o = \frac{2\pi NI}{10r} \text{ gilberts per centimeter} \qquad (31)$$

(*d*) *Field Intensity on Axis of Solenoid.*—Solenoids are used in a large variety of electrical appliances. The dimensions vary over a wide range and due care must be taken in applying the formulæ to keep within the assumptions made in their derivation.

Let Fig. 4 represent a solenoid of N turns uniformly distributed over its length of l cm. and carrying a current of I amp. Let r denote the radius and AB the axis with the middle point at O. It is assumed that the wires are small and wound completely in one layer so that the current in flowing through the solenoid may be considered as forming a thin sheet or layer of negligible thickness. Let a unit magnet pole be placed at P on the axis and a distance D from the center O. Consider an

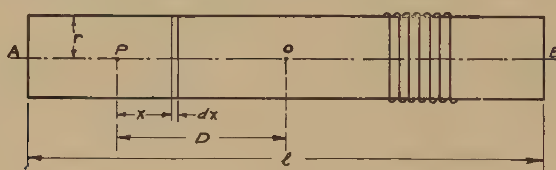


FIG. 4.—Field intensity on axis of solenoid.

element dx of the solenoid a distance x along the axis, from P. Under the given conditions this element may be considered as a circular coil with the line AB as its axis. From equation (31) and the above given dimensions, the field intensity produced by the element dx as a circular coil would be,

$$dH_p = \frac{2\pi r^2 \frac{N}{l} I dx}{10(r^2 + x^2)^{3/2}} \text{ gilberts per centimeter} \tag{32}$$

Hence the total field intensity at the point P, for the solenoid is expressed by equations (33) and (34),

$$H_p = \frac{2\pi r^2 NI}{10l} \int_{\frac{l}{2}+D}^{\frac{l}{2}-D} \frac{dx}{(r^2 + x^2)^{3/2}} \text{ gilberts per centimeter} \tag{33}$$

$$= \frac{2\pi NI}{10l} \left[\frac{\frac{l}{2} - D}{r^2 + \left[\frac{l}{2} - D\right]^2} + \frac{\frac{l}{2} + D}{r^2 + \left[\frac{l}{2} + D\right]^2} \right] \text{ gilberts per centimeter} \tag{34}$$

At the center of the solenoid $D = O$ and the field intensity is given by equation (35).

$$H_o = \frac{2\pi NI}{10\sqrt{r^2 + \frac{l^2}{4}}} \text{ gilberts per centimeter} \tag{35}$$

For solenoids in which l is large as compared with r, the field intensity at the center is very nearly,

$$H_o' = \frac{4\pi NI}{10l} \text{ gilberts per centimeter} \tag{36}$$

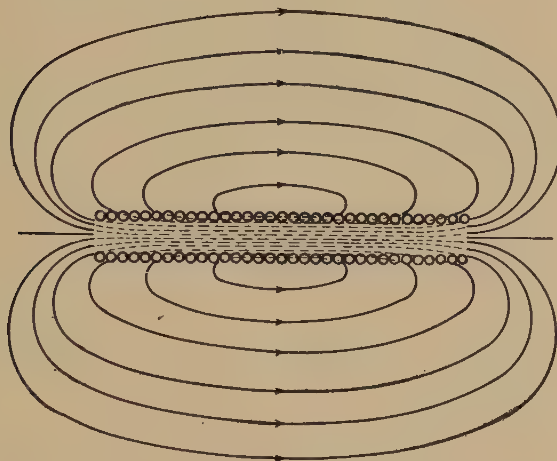


FIG. 5.—Solenoid showing distribution of lines of force.

The field intensity at the ends of the solenoid H is obtained from equation (35) by substituting ½ for D.

$$H_e = \frac{2\pi NI}{10\sqrt{r^2 + l^2}} \text{ gilberts per centimeter} \tag{37}$$

If l is large compared to r the field intensity on the axis at the end of the solenoid would be,

$$H_e' = \frac{2\pi NI}{10l} \text{ gilberts per centimeter} \tag{38}$$

By comparing equations (36) and (38), it is seen that the field intensity at either end of the long solenoid is just half as great as at the center. It is assumed that in the surrounding space there are no magnetic materials; that is, the permeability μ is unity. Therefore, the flux density B_o, at the center, is twice

as great as B_e at the ends of the solenoids. Hence half of the lines of force passing through the center of the coil must pass through the sides of the solenoid, as illustrated by Fig. 5.

The variation of field intensity along the axis of a solenoid whose length is thirty times its diameter is shown in Fig. 6. The ordinates represent field intensity, H, and the abscissæ the length of the solenoid.

It should be observed that the field intensity is nearly constant over the greater part of the solenoid with the decrease quite abrupt near the two ends. That is, the field strength along the axis is fairly uniform for two-thirds the length of the solenoid.

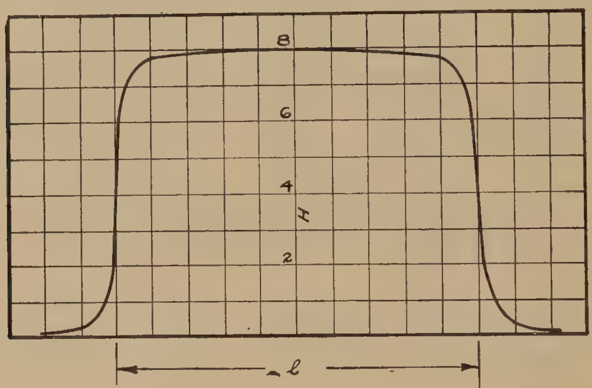


FIG. 6.—Field intensity along axis of solenoid.

For solenoids *in which the length l is large in comparison to the radius r* the field intensity over the central cross-section is uniform, the same as on the axis at the center. Since the permeability is unity, the flux density B equals the field intensity H. Hence the flux density B_o at all points of the central cross-section of a solenoid when in a non-magnetic medium is expressed by equation (39).

$$B = \frac{0.4\pi NI}{l} \text{ gausses} \tag{39}$$

Therefore, the total flux ϕ_o passing through the central cross-section of the solenoid is expressed by equation (40).

$$\phi_o = B_o(\text{gausses})\, A(\text{cm.}^2) = \frac{\dfrac{0.4\pi N(\text{turns})\, I(\text{amps})}{l(\text{cm.})}}{A(\text{cm.}^2)}$$

$$= \frac{0.4\pi NIA}{l} \text{ maxwells} \tag{40}$$

Since $0.4\pi NI$ is the magnetomotive force and l/A the reluctance of the space inside the solenoid, equation (40) expresses Ohm's law for the magnetic circuit and states that the flux passing through the center section of the solenoid is the same as would be produced by the given magnetomotive force in a *magnetic circuit having a reluctance equal to that of the space inside the solenoid.*

Let n = the number of turns per centimeter length of the uniformly wound solenoid and substitute nl for N, the total number of turns, in equations (39) and (40).

$$B_o = 0.4\pi nI \tag{41}$$

$$\phi_o = 0.4\pi nIA \tag{42}$$

From equations (41) and (42) it is evident that for uniformly wound solenoids, in which the length is large in comparison to the diameter, the flux density and the total flux at the center section are independent of the length of the solenoid.

(*e*) *Field Intensity Inside a Conductor.*—When a current flows in a conductor a magnetic field is formed, not merely in the surrounding space as already discussed, but inside the conductor as well. The cylindrical wire, which is the conductor form generally used, may be considered as consisting of an infinite number of concentric cylindrical tubes. The magnetic field produced by the current flowing in each elemental tube appears in the space outside the given tube. The magnetic flux for the current in the surface layer will be in the space outside the conductor, but for each of the inner tubes the flux will appear in all the concentric tubes of larger diameter as well as in the space outside the wire. In Fig. 7 let r be the radius of the conductor (wire) carrying a current of I amp. and x the distance from the center of the wire at which it is desired to find the field intensity. A steady direct current flowing in a straight wire of uniform cross-section is uniformly distributed over the conductor cross-section. The current flowing through the cylinder of radius x, (Fig. 7) is expressed by equation (43).

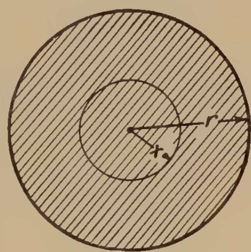


Fig. 7.

$$I_x = \frac{x^2}{r^2} I \text{ amp.} \tag{43}$$

The magnetomotive force produced by the current I_x in the cylinder of radius x, is given by equation (44).

$$F_x = 0.4\pi\frac{x^2}{r^2}I \text{ gilberts} \tag{44}$$

Since the length of path for the flux at radius x is $2\pi x$ the field intensity inside the conductor at a distance x from the center is expressed by equation (45).

$$H_x = \frac{0.4\pi\frac{x^2}{r^2}I}{2\pi x} = \frac{0.2xI}{r^2} \text{ gilberts per centimeter} \tag{45}$$

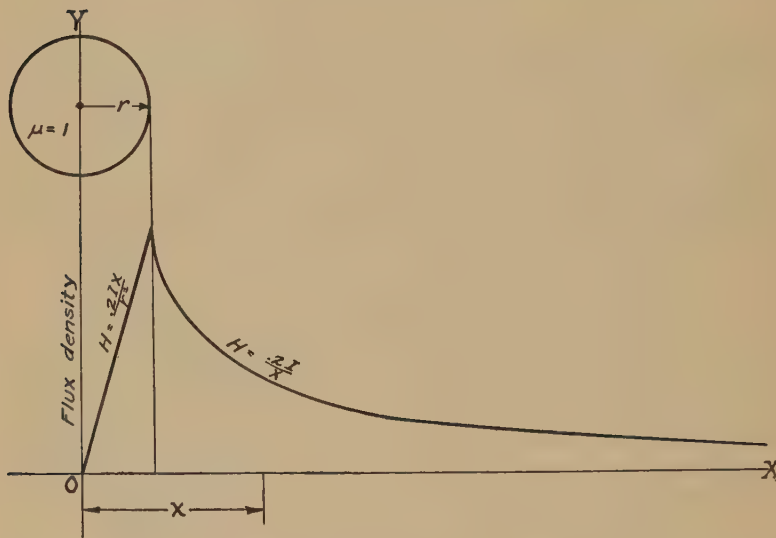


Fig. 8.—Magnetic flux distribution for single conductor.

For cylindrical conductors (wire) of copper or other non-magnetic material ($\mu = 1$) the flux density B_x inside the wire, at a distance x from the center of the wire is given by equation (46).

$$B_x = \frac{0.2xI}{r^2} \text{ gausses} \tag{46}$$

From equation (44) it is evident that under the stated conditions the flux density B_x inside the wire, is directly proportional to the distance from the center of the wire. In the space outside the wire the flux density, equation (18), is inversely proportional to the distance from the center of the conductor. A graphical

representation of the distribution of the magnetic flux both inside and outside the conductor is shown in Fig. 8.

It should be noted that an electric current flowing in a conductor, even if of uniform circular cross-section, may not be uniformly distributed in all cases. If the length of path is not the same for all elemental strands, or, for some other reason, the resistance for all paths be not the same, then the density of even a steady direct current will not be uniform as Ohm's law will apply to each elemental path of strand of the conductor. For alternating currents, or during the transient period when a direct current is rapidly changing in magnitude the inductive effect will cause a greater density of current flow near the surface of the conductor than at the center. This phenomena of unequal current density in the conductor produced by the interaction of the rapidly changing magnetic lines of force is called *skin effect*.

In high-frequency alternating currents the skin effect may be very great, so that by far the greater part of the current is concentrated in the surface layer of the conductor. Necessarily the skin effect produces an increase in the effective resistance of the conductor as the current is forced to flow through a smaller cross-section than would be the case if it were uniformly distributed over the cross-sectional area of the conductor.

(*f*) *Field Intensity and Flux Density between Two Parallel Conductors.*—In Fig. 9 let A and B represent a cross-section of two parallel wires in a transmission line. Let $2r$ be the diameter of each wire in centimeters and D the distance between the centers of the two conductors.

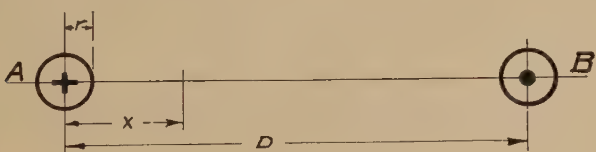


Fig. 9.

Let a steady direct current of I amp. flow in the wires in direction as indicated by the plus and minus signs. Since the current flows in opposite directions in the two conductors the flux between the wires is the sum of the lines of force produced by the current in each wire. Hence the field intensity at any point x in the plane determined by the center lines of the two parallel wires is the sum of the field intensities at the given point due to the current in

each wire. From equation (17) with the current I in amperes and the lengths D and x in centimeters,

$$H_x = \frac{0.2I}{x} + \frac{0.2I}{D - x} \text{ gilberts per centimeter} \tag{47}$$

Similarly from equation (18) and as the permeability of air is unity the flux density,

$$B_x = 0.2I \left[\frac{1}{x} + \frac{1}{D - x} \right] \text{ gausses} \tag{48}$$

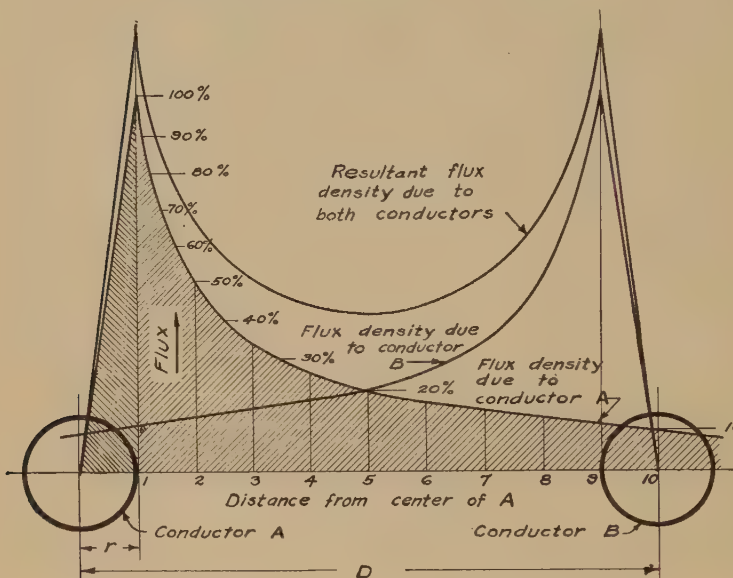


FIG. 10.—Magnetic flux distribution between two parallel wires.

The total flux passing between the wires for 1 cm. length of each conductor is therefore given by equation (49) or equation (51).

$$\phi = 0.2I \int_r^{D-r} \left[\frac{1}{x} + \frac{1}{D - x} \right] dx \text{ lines} \tag{49}$$

$$= 0.2I \Big[\log_e x - \log_e (D - x) \Big]_r^{D-r}$$

$$= 0.4I \log_e \left(\frac{D - r}{r} \right) \text{ lines} \tag{50}$$

$$= 0.1737I \log_{10} \left(\frac{D - r}{r} \right) \tag{51}$$

It is apparent from equation (48) that the distribution of the flux is not uniform and that the greatest density is at the surface of the conductors, and decreasing to a minimum at the midway point. A graphical representation of the magnetic flux distribution for the current in each wire as well as the resultant total value at each point is shown in Fig. 10.

Magnetic Potential.—The magnetic potential of any point represents the work required to bring a unit magnetic pole from a place of zero field intensity to the point specified.

(*a*) *Potential around a Magnet Pole.*—Consider two magnet poles of strengths m and m', a distance x apart, as shown in Fig. 11.

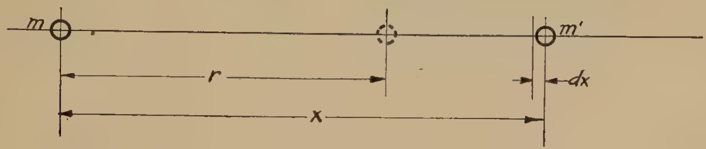


FIG. 11.

The force between the poles is (Chap. III),

$$F = \frac{mm'}{x^2} \text{ dynes} \tag{52}$$

If one of the poles m' is moved a distance dx towards m, the work done would be expressed by equation (53),

$$dU = \frac{mm'}{x^2}\, dx \text{ ergs} \tag{53}$$

The total work expended in bringing pole m from an infinite distance to a distance r from m is:

$$\int_0^{u'} dU = \int_\infty^r \frac{mm'}{x^2} dx \text{ ergs} \tag{54}$$

$$U' = -\frac{mm'}{r} \text{ ergs} \tag{55}$$

This is also equal to the potential energy stored between the two poles at a distance r apart, that would be expended if the poles moved to an infinite distance apart. If m' is a unit pole the energy as expressed by equation (56) is called the magnetic potential of the field at a distance r from the pole m.

$$U_r = \frac{m}{r} \tag{56}$$

The difference in magnetic potential for two points at distances r_1 and r_2 respectively, from the pole m would be given by equation (57),

$$U_{r_1 r_2} = \int_{r_1}^{r_2} \frac{m dx}{x^2} = \frac{m}{r_2} - \frac{m}{r_1} \qquad (57)$$

From equation (57) it becomes evident that the difference in potential between two points is determined entirely by the initial and final positions selected and is not affected by the path followed in passing from one point to the other.

(*b*) *Magnetic Potential on Axis of Circular Coil.*—The expression for the field intensity at any point P on the axis of a circular coil of N turns and radius r *cm.* carrying a current of I amp. is obtained from equation (30).

$$H_p = \frac{2\pi r^2 NI}{10(r^2 + x^2)^{3/2}} \text{ gilberts per centimeter} \qquad (58)$$

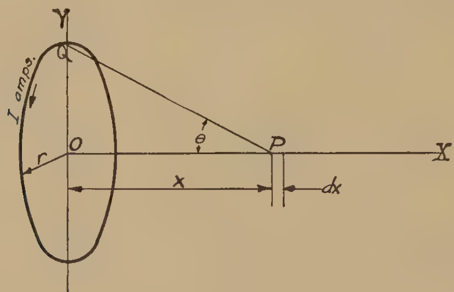


FIG. 12.—Magnetic potential on axis of circular coil.

In Fig. 12 the point P is on the axis of the coil and at a distance x from O, the center of the coil. The work required to move the pole a distance dx away from the coil would be given by equation (59)

$$dU_x = H_x dx \qquad (59)$$

The magnetic potential at a point would be the total work expended to move the unit pole from any point at zero potential to the point P.

$$\int_0^{u_p} dU_p = \int_x^{\infty} H dx = \frac{2\pi NIr^2}{10} \int_x^{\infty} \frac{dx}{(r^2 + x^2)^{3/2}} \qquad (60)$$

$$U_p = \frac{2\pi NI}{10} \left[\frac{x}{\sqrt{r^2 + x^2}} \right]_x^{\infty} \text{ ergs} \qquad (61)$$

$$= \frac{2\pi NI}{10} \left[1 - \frac{x}{\sqrt{r^2 + x^2}} \right] \text{ ergs} \qquad (62)$$

(c) *Magnetic Potential for Any Coil at Any Point.*—Equation (60) which expresses the magnetic potential in terms of the distance x and the radius r of the coil is in most cases a convenient form for numerical computations. To gain a more general concept of the relative value of magnetic potentials under a variety of conditions, it is desirable to express the value of U as given in equation (62) in terms of the solid angle subtended by the coil at the point P.

Let θ be the plane angle (Fig. 12) between the axis OP and a line from P to any point on the circular coil. Let ω represent the solid angle subtended by the circular coil at P. Then equation (62) may be rewritten in terms of either the plane angle θ or the solid angle ω.

$$U_p = \frac{2\pi NI}{10}\left[1 - \frac{x}{\sqrt{r^2 + x^2}}\right] \tag{63}$$

$$= \frac{2\pi NI}{10}[1 - \cos\theta] \tag{64}$$

$$= \frac{NI}{10}\omega \tag{65}$$

That is, the *magnetic potential* at any point on the axis is directly proportional to, or measured by, the *solid angle* subtended by the coil at the given point. That this should be the case becomes evident by considering the distribution of the magnetic lines of force around the unit magnet pole. The solid angle subtended by the coil as expressed by equation (63) represents the flux from the unit pole passing through the coil. As the pole approaches the coil an increasing number of lines of force from the unit pole passes through the coil, that is, in direct porportion to the solid angle subtended by the coil. When the pole reaches the center of the coil 2π lines of force have passed into the coil and ω the solid angle subtended, is equal to 2π.

Fig. 13.—Magnetic potentials and solid angles.

On the basis that the magnetic potential is measured by the number of lines of force from the unit magnet pole subtended by the coil, the relation given in equation (63) expresses the magnetic potential produced by any coil at any point.

Using the solid angle as a measure of the magnetic potential of a coil gives a very ready means for visualizing the relative values of magnetic potentials for any given points. This is illustrated in Fig. 13. The configuration of the coil and the location of the point at which the potential is desired, may make it impossible to derive expressions in other terms than the solid angles subtended by the coil.

It is of interest to observe that if the unit magnet pole is placed at any point in the plane of a circular coil and outside of the coil, none of the flux lines from the unit pole threads the coil; that is, for all points in the plane of the coil and outside the coil, the magnetic potential is zero. For all points in the plane of the coil and inside of the coil, the solid angle subtended is 2π, and hence the magnetic potential is $0.2\pi NI$.

The Line Integral.—In the analysis of problems relating to magnetic potential, electric potential, gravitational potential, magnetomotive force, and similar phenomena, in which the product of force vectors and distances are involved, the equations derived generally have the form of a function known as the line *integral*. If a body or a point, as it moves along a path, is acted upon by either a constant or variable force, the line integral of this force over the given path is equal to the sum of the products of each element of path by the corresponding component of the force in the direction of motion at each element of path. That is, if at every point along the path, the product of the *element of length dl*, the *force F* at the point, and the *cosine of the angle* θ between the force and the direction of the path at each point, be taken, the result is the line integral of the force over the given path. The line integral, therefore, is an expression for the work done in moving the point from an initial to a final position.

To illustrate: (*a*) Let one end of a cord, passed over a cylindrical *axle*, be attached to a weight and the other end to a pencil point as shown in Fig. 14.

Let the pencil point P be moved from A to B along the path shown in the figure. At each point the force on the pencil point due to the weight acts at an angle to the direction of motion. The line integral of this force over the path AB is equal to the work done in moving the pencil point from the initial to the final position. If the force in the direction of the path at each point is represented by the vector F, and each element of path by dl, then the total work done, W_{AB}, in moving the pencil point from

A to B is expressed by equation (66). This is the line integral equation.

$$W_{AB} = \int_l \dot{F} dl \tag{66}$$

If $\dot{F}'$ represents the force along the cord produced by the weight M and θ the angle at each point between the direction of $\dot{F}'$ and the direction of motion for the corresponding dl elements, the line integral of the force $\dot{F}'$ over the line AB is given by equation (67).

$$W_{AB} = \int_l \dot{F}' \cos \theta \, dl \tag{67}$$

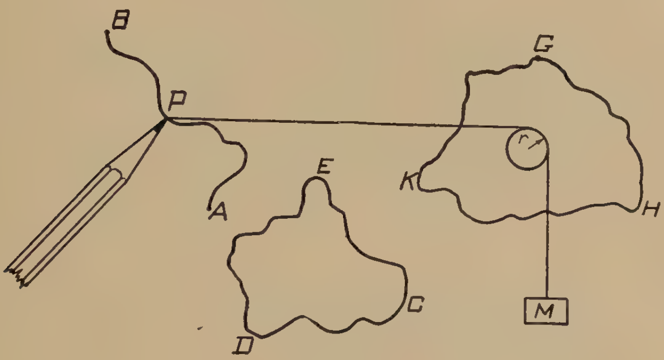


Fig. 14.

From either equation (66) or (67) it is evident that it makes no difference as to the form of the path taken between A and B as the value of W_{AB} depends solely on the initial and final positions.

(*b*) Let the pencil point be moved over the path $CDEC$ in Fig. 14. From the principle of the conservation of energy the total work done against the force from the cord in moving the pencil point completely around the closed circuit, as from C to D to E and back to C, must be zero as the initial and final conditions are identical. Hence the line integral of the force F over the closed path equals zero.

$$\int_l \dot{F} dl = 0 \tag{68}$$

It should be carefully noted that equation (68) is true for any closed circuit only under the assumption that the *initial and final conditions are identical.* This means, not merely that the pencil point must be back in its original position, but that the *final condition in the whole system must be identical with the initial condition.*

(*c*) As a third case let the pencil point be moved over the closed path *GHK* in which it winds the cord once around the cylindrical pulley of radius r. It is evident that although the pencil point moved over a closed path the final condition in the system is not identical with the initial condition. In the first position the cord passes once around the cylindrical pulley and consequently the weight has been raised a distance $2\pi r$ above its initial position. The line integral of the force must be equal to the work done in raising the weight M against the force of gravity $\dot{F}_m$.

$$\int_l \dot{F} dl = 2\pi r \dot{F}_M \tag{69}$$

It should be observed that it makes no difference as to the form of the closed path *GHKG* over which the pencil point moved, only provided that the cord must in the final position encircle by one turn the cylindrical pulley. The work done depends only on the force $\dot{F}_M$ and the circumference of the pulley.

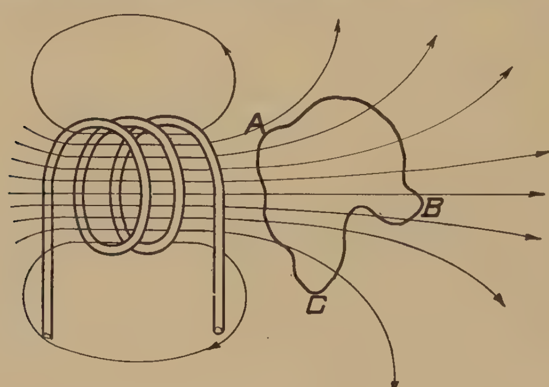


FIG. 15.—Pole moving over closed circuit in magnetic field.

(*d*) Consider a unit magnet pole moving along a closed path *ABC* in a magnetic field, as shown in Fig. 15, without interlinking with an electric circuit; the line integral for the force on the unit pole over any closed path under the above conditions is zero.

$$\int_m H dl = 0 \tag{70}$$

Note that the configuration of the path does not enter into the equation. The requirement is merely that the final condition must be identical with the initial condition.

(*e*) Let the unit pole pass over a closed path which interlinks once with an electric circuit carrying a current of I amp., as

shown in Fig. 16. This is analagous to the conditions in (c) above, in which the cord winds around the cylindrical pulley. For each interlinkage, that is, for each time the unit pole is

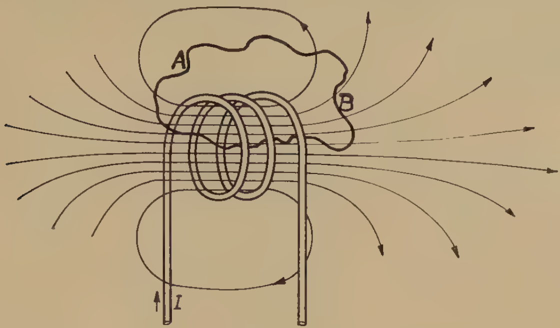


FIG. 16.—Line integral of path interlinking with an electric circuit.

passed over the closed path interlinking with the current I the line integral is expressed by equation (71).

$$\int_l Hdl = 0.4\pi NI \tag{71}$$

The quantity $0.4NI$ is the magnetomotive force of the coil and represents work as explained in the next paragraph.

Magnetomotive Force.—Consider the magnetic potential in the space surrounding a plane coil of any configuration, having

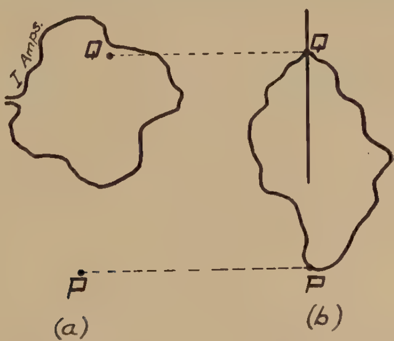


FIG. 17.—Front and side elevation of plane coil on unit pole.

N turns and carrying a current of I amp. Let the front and side or edgewise elevation of the coil be represented by the diagrams, (a) and (b) in Fig. 17.

Let a unit magnet pole be placed at any point P in the plane of the coil but outside the coil. The magnetic potential at P

must be zero as none of the lines of force emanating from the unit pole thread the coil.

Move the unit pole by any path to any point Q in the plane of the coil and inside the coil. The magnetic potential at the point Q is $2\pi NI$ and hence the work done in moving the unit pole from P to Q is,

$$W_{PQ} = 2\pi N\frac{I}{10} \text{ gilberts} \tag{72}$$

Likewise to move the pole from Q in the plane of the coil and passing through the coil back to the point P against the attraction between the pole and the left side of the coil, will require an equal amount of work as the solid angle is again 2π.

$$W_{QP} = 2\pi N\frac{I}{10} \text{ gilberts} \tag{73}$$

Hence the total work required to move the pole from the point P through the coil, making one interlinkage with the current in the coil and back to the starting point P is expressed by equation (74), and this is the *magnetomotive force of the coil.*

$$W_{PQP} = 4\pi N\frac{I}{10} = 0.4\pi NI \text{ gilberts} \tag{74}$$

The expression for the magnetomotive force in equation (74) applies to coils of any shape whatsoever, as in all cases the total solid angle for one complete interlinkage of the coil would be 4π. The magnetomotive force of a solenoid is therefore expressed by equation (74). In respect to magnetomotive force, the solenoid is merely a coil having N turns. Since the field intensity in the space surrounding, as well as inside, the solenoid is not uniform, the total amount of work to move the unit pole over a closed path interlinking all the turns of the solenoid, that is, the *magnetomotive force is the line integral* of the field intensity as expressed by equation (75).

$$\int Hdl = 0.4\pi NI \text{ gilberts} \tag{75}$$

It is evident that if the magnetomotive force of any section of the solenoid is desired, the only change necessary in equation (75) is to let N represent the number of turns in the given section. In all cases the magnetomotive force is proportional to the ampere-turns interlinked with the given path of the unit pole.

Magnetic Induction. Inductance.—Magnetic flux lines encircle or interlink conductors carrying electric currents. For each

centimeter length of a straight conductor, as for example, a copper wire, the number of lines of force is directly proportional to the strength of the electric current. The relation between the magnetomotive force produced by the current, the reluctance of the path for each line of force, and the total flux generated, is expressed by Ohm's law in equation (76),

$$\phi = \frac{\mathfrak{F}(\text{gilberts})}{\mathfrak{R}(\text{oersteds})} = \frac{0.4\pi N(\text{turns})I \text{ amp.}}{\dfrac{l(\text{cm.})}{\mu A(\text{cm.}^2)}} = \frac{4\pi NI\mu A}{10l} \text{ maxwells} \quad (76)$$

In Fig. 18 let a coil of wire be wound around a rectangular shaped iron core as for example the core of an ordinary distribution transformer. Assume that the permeability μ of the

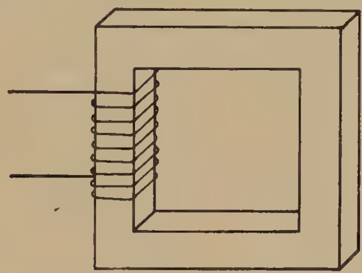


FIG. 18.—Iron-clad circuit.

iron core is constant in value and large enough so that the leakage flux in the surrounding air may be neglected. Also assume that l represents the mean length of the magnetic circuit and that the cross-section of the iron core A is of constant value. From equation (76) it is evident that any change in the current must produce a corresponding change in the magnetic flux. If the current increases by di, the resulting increase in flux $d\phi$ will be expressed by equation (77).

$$d\phi = \frac{4\pi N\mu A di}{10l} \text{ maxwells} \quad (77)$$

The new flux lines will cut the N turns in the coil inducing a counter-electromotive force in the electric circuit.

$$_Le = -N\frac{d\phi}{dt}10^{-8} \text{ volts} \quad (78)$$

$$= -\frac{4\pi N^2\mu A}{l10^9}\frac{di}{dt} \text{ volts} \quad (79)$$

The proportionality factor between the change in the current and the resulting induced counter-electromotive force represents the interlinkage of the magnetic lines of force with the current and is called the *coefficient of self-induction, self-inductance,* or *inductance* of the circuit. The symbol for *inductance* is L.

$$_Le = -L\frac{di}{dt} \text{ volts} \tag{80}$$

From equations (79) and (80),

$$L = \frac{4\pi\mu N^2(\text{turns})A(\overline{\text{cm.}}^2)}{l(\text{cm.})10^9} \text{ henrys} \tag{81}$$

The practical unit for inductance is the *henry.* If the current changes at the *rate of* 1 *amp. per second,* equation (80), and the voltage induced is 1 *volt,* the inductance of the circuit is 1 *henry.* It should be noted from equation (81) that the inductance is directly proportional to the square of the number of turns as well as to the permeability and the cross-sectional area and inversely as the length of the circuit. The relations expressed in equations (76) to (81) are true for all values of the permeability μ and for any configuration of the electric circuit. For non-magnetic materials $\mu = 1$. In most cases the magnetic lines of force spread out around the conductor carrying the current in such forms that it becomes impossible to determine quantitatively either the length l or the cross-sectional area A of the magnetic circuit.

The inductance L, may also be considered as a factor in the amount of energy that can be stored magnetically in the given circuit similar to mass as a factor in the amount of kinetic energy stored in a moving body.

Another expression for the term inductance may readily be derived from equation (81) and equation (76)

$$L = \frac{\phi N}{I\,10^8} \text{ henrys} \tag{82}$$

That is, the inductance in henrys equals the flux interlinkages per ampere divided by the factor 10.[8] A change in the interlinkages is frequently used as equivalent to cutting lines of force for generation of voltage by induction. For the solution of problems it is generally more convenient to use equation (82) than equation (81).

Mutual Inductance, Coupling Coefficient.—The coefficient of self-induction or inductance of a circuit represents the number of

interlinkages of magnetic lines of force with the circuit per ampere current flowing in the circuit as expressed by equation (80). The inductance L of a circuit may also be defined as the proportionately factor between the time rate of change in magnitude of the current and the induced counter-electromotive force. In like manner *mutual inductance* represents the interlinkages of the magnetic lines of force produced by a current flowing in one circuit with a second coil or circuit. The mutual inductance may also be defined as the proportionately factor between the rate of change in the current in one circuit to the electromotive force induced in the second circuit. The symbol for mutual inductance is M.

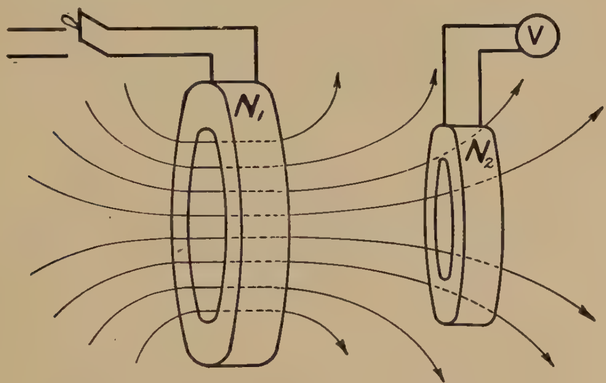


FIG. 19.—Air core coils. Mutual inductance.

In Fig. 19 let the two coils have N_1 and N_2 turns, respectively. Let coil N_1 be connected to a power supply while the circuit in coil N_2 is closed through a voltmeter. If a current is passed through coil N_1 by closing the switch a magnetic field is formed in the surrounding space. As the field increases the lines of force pass outward and part of the flux cuts or interlinks with the turns of coil N_2. This induces a voltage in coil N_2 opposite in direction to the voltage producing the increase in the current in coil N_1. The ratio between the rate of change of the current in coil N_1 to the voltage induced in coil N_2 is the mutual inductance.

Let i_1 = the current in amperes in coil N_1.

ϕ_{1-2} = lines of force produced by i_1 and cutting or interlinking coil N_2.

Hence the voltage induced in coil N_2 is expressed by equation (83).

$$_Le_2 = N_2\frac{d\phi_{1-2}}{dt}10^{-8} \text{ volts} \tag{83}$$

Therefore the mutual inductance of coil N_1 on the circuit N_2 is

$$M_{1-2} = \frac{N_2\frac{d\phi_{1-2}}{dt}10^{-8}}{\frac{di_1}{dt}} = N_2\frac{d\phi_{1-2}}{di_1}10^{-8} \tag{84}$$

It is evident that if a current i_2 were passed through coil N_2 a corresponding voltage would be induced in coil N_1. If under these conditions ϕ_{2-1} represents the lines of force produced by the current i_2 interlinking the turns of coil N_1 the induced voltage $_Le_1$, would be given by equation (85), and the mutual inductance M_{2-1} of coil N_2 on the circuit N_1 by equation (86).

$$_Le_1 = N_1\frac{d\phi_{2-1}}{dt}10^{-8} \text{ volts} \tag{85}$$

$$M_{2-1} = \frac{N_1\frac{d\phi_{2-1}}{dt}}{\frac{di}{dt}}10^{-8} = N_1\frac{d\phi_{2-1}}{di_2} \tag{86}$$

From Fig. 19 it is evident that the number of interlinkages of the flux produced by a current of i_1 amp. in one of the coils with the turns of the other coil will depend on the relative position of the coils. The ratio between the total flux ϕ_1, produced in N_1 to the lines of force ϕ_{1-2} interlinking coil N_2 is called the coefficient of coupling, K.

$$K = \frac{\phi_{1-2}}{\phi_1} \text{ or } \frac{\phi_{2-1}}{\phi_2} \tag{87}$$

In Fig. 20 the coupling coefficient for the two air core coils N_1 and N_2 would be a maximum if placed as near together as possible and with their axes in the same line as in position (*a*). If the coils are placed with their axes at right angles to each other as in (*b*) Fig. 20, $K = 0$. In other positions as in (*c*) K has a value greater than zero but less than for position (*a*). The value of k also depends on the reluctance of the magnetic circuit.

If the coils N_1 and N_2 are provided with an iron core as shown in Fig. 21, a much larger percentage of the lines of force produced by the current in one coil will interlink with the second coil;

that is, the coupling coefficient is greater. By winding the coils closely together on an iron core having low reluctance the leakage flux is very small and the coupling coefficient K becomes nearly

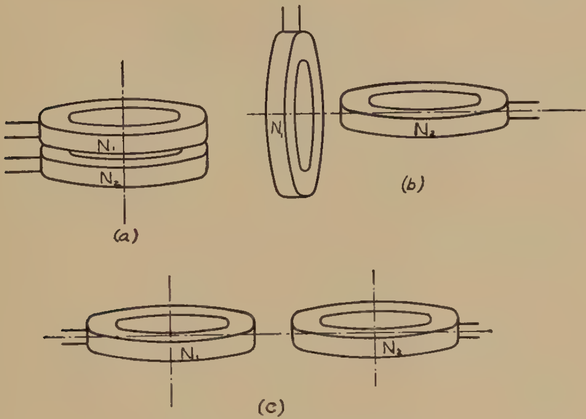


FIG. 20.—Magnetic coupling.

unity. Under the condition that the reluctance $\mathcal{R}$ of the magnetic circuits of coil N_1 as in Figs. (19) and (20) is equal to the reluctance of coil N_2 it is readily proved that $M_{1-2} = M_{2-1}$.

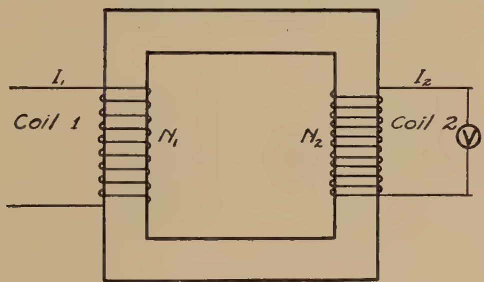


FIG. 21.—Mutual induction. Iron clad circuits.

By applying Ohm's law to the magnetic circuit for coil 1,

$$\phi_1 = \frac{0.4\pi N_1 i_1}{\mathcal{R}} \text{ maxwells} \tag{88}$$

$$\phi_{1-2} = \frac{0.4\pi N_1 i_1 k}{\mathcal{R}} \text{ maxwells} \tag{89}$$

$$\frac{d\phi_{1-2}}{di} = \frac{0.4\pi N_1 K}{\mathcal{R}} \tag{90}$$

Combining equations (87) and (81):

$$M_{1-2} = \frac{4\pi N_1 N_2 K}{\mathcal{R}10^9} \text{ henrys} \tag{91}$$

In like manner:

$$\phi_2 = \frac{0.4\pi N_2 i_2}{\mathcal{R}} \text{ maxwells} \tag{92}$$

$$\phi_{2-1} = \frac{0.4\pi N_2 i_2 K}{\mathcal{R}} \text{ maxwells} \tag{93}$$

$$\frac{d\phi_{2-1}}{di} = \frac{0.4\pi N_2 K}{\mathcal{R}} \tag{94}$$

Combining equations (91) and (83):

$$M_{2-1} = \frac{4\pi N_1 N_2 K}{\mathcal{R}10^9} \text{ henrys} \tag{95}$$

The above equations express the magnetic interdependence or interlinkage of two separate electric circuits. By means of the magnetic circuit interlinking the otherwise independent electric circuits, energy is transmitted through the insulation from one electric circuit to another.

If the two coils N_1 and N_2 do not form two separate electric circuits but are electrically connected in series as shown in Fig. 22, the total inductance in the circuit depends on the self-inductances L_1 and L_2 of the two coils, the mutual inductance M, and whether the coils are connected so that the current in the two

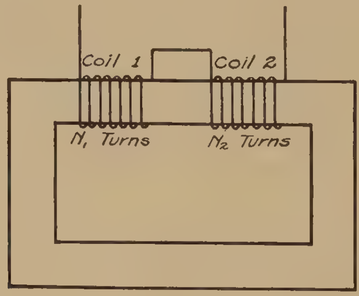


FIG. 22.—Iron-clad circuits. Mutual induction.

coils flows in the same or opposite directions. If the coils are connected as shown in Fig. 22, the magnetic fluxes produced in both coils are in the same direction and the total inductance is the sum of the self-inductance and the mutual inductance of each coil.

$$L = L_1 + L_2 + 2M \tag{96}$$

If the connection is reversed so that the magnetic fields in the two coils oppose or neutralize then the total inductance is expressed by equation (97).

$$L' = L_1 + L_2 - 2M \tag{97}$$

The transformer which is in general use in electric power systems illustrates the application of electromagnetic induction and the properties of the magnetic circuit in transferring electric energy from one electric circuit to another. The ordinary transformer consists of the primary and secondary windings or coils wound on a common iron core. The primary winding is energized by an alternating current which produces a rapidly changing magnetic flux. By electromagnetic induction the flux generated by the current in the primary coil induces a voltage in the secondary winding. The ratio of the voltage impressed on the primary to the voltage induced in the secondary winding is essentially directly proportional to the number of turns in the two windings as the flux leakage is very small. The importance of the transformer can hardly be overestimated as it forms a very simple and economical means for raising or lowering voltages on alternating-current systems.

Other important examples of transmitting energy magnetically and increasing the voltage by electromagnetic induction are induction coils as used in ignition devices on automobile motors or other internal-combustion engines. A few turns of heavy wire wound on an iron core form the primary winding which is energized by a storage battery. The secondary winding consists of many turns of fine wire wound around the iron core on top of the primary winding and with its two ends connected to a spark plug. When the primary circuit is closed and opened in rapid succession the current in the primary and therefore the magnetic flux changes at a very rapid rate and, thereby, induces high voltages in the secondary coil sufficient to cause a spark to pass across the airgap in the spark plug.

Energy in Magnetic Field.—The magnetic field requires energy to produce it and represents energy stored in the space occupied by the lines of force. The relation between the inductance and the current producing the flux is given by equation (80) in which L is the inductance of the electric circuit. Assume the permeability to be constant, that is, unity for non-magnetic materials. A change in the current produces a corresponding

change in the flux and hence absorbs or produces a voltage ${}_Le$ which is proportional to the rate of change in the magnetic flux.

$$
{}_Le = -L\frac{di}{dt} \tag{98}
$$

The power required would be equal to the product of the voltage and the current.

$$
P = {}_Lei \tag{99}
$$

The energy dw for the time dt is given by equations (100) and (101).

$$
dw = pdt = {}_Leidt \tag{100}
$$

$$
dw = Lidi \tag{101}
$$

The total energy required to form the field is therefore expressed by equations (102) and (103),

$$
\int_0^W dw = L\int_0^i idi \tag{102}
$$

$$
W(\text{joules}) = L(\text{henrys})\frac{i^2(\text{amp.})}{2} \tag{103}
$$

The energy stored magnetically in the magnetic field surrounding a conductor and assuming a constant inductance L is *directly proportional to the square of the current.* This statement applies to circuits in which the permeability is constant, as air-cored coils and transmission lines. When the current decreases the stored energy returns to the electric circuit, for, if i, and therefore ϕ, decreases, di/dt and hence ${}_Le$ are negative, which means that energy is returned to the electric circuit.

It is convenient to have an expression for the energy stored magnetically per unit volume as well as for the total energy stored in the magnetic field.

From equations (81) and (103)

$$
W = \frac{2\pi N^2I^2\mu A}{l10^9} \text{ joules} \tag{104}
$$

From equation (76) and since $\phi = BA$

$$
B = \frac{4\pi\mu NI}{10l} \text{ gausses} \tag{105}
$$

Hence,

$$
W = \frac{B^2lA}{8\pi\mu10^7} \text{ joules} \tag{106}
$$

The volume,

$$V = lA \tag{107}$$

Therefore the energy stored magnetically per unit volume is given by equations (108) and (109).

$$W = \frac{B^2}{8\pi\mu 10^7} \text{ joules per cm.}^3 \tag{108}$$

$$= \frac{B^2}{8\pi\mu} \text{ ergs per cm.}^3 \tag{109}$$

In air or material for which the permeability is unity the energy stored in a magnetic field per unit volume is expressed by equation (110).

$$W = \frac{B^2}{8\pi} \text{ ergs per cm.}^3 \tag{110}$$

Magnetic Pull.—In all magnetic fields there is a force or pull along the magnetic lines of force tending to shorten the magnetic circuit and thereby to decrease the volume occupied by the field.

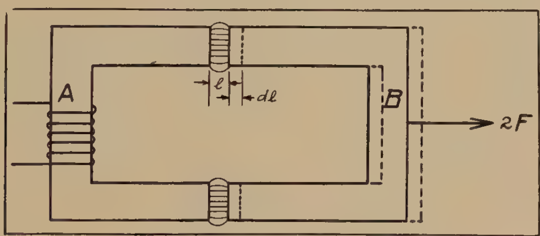


FIG. 23.—Electromagnet.

In lifting magnets, induction coils, horseshoe magnets, etc., the magnetic pull tending to shorten the lines of force forms the basis of action. If there are airgaps in the iron-cored magnetic circuit as in Fig. 23, the force produced by the pull along the lines of force will tend to decrease the length of the airgaps and bring the iron cores together.

In order to find the force or pull exerted by the magnetic field across the airgaps of the electromagnet in Fig. 23, let it be assumed:

(*a*) That the airgaps are short in comparison to the cross-sectional areas of the iron cores, so that the fringing is negligible. That is, the flux passes across the airgap in straight lines. Let the cross-section of each gap $= A$ cm.^{2} and the length $= l$ cm.

(*b*) That the flux is uniformly distributed over the two airgaps.
Let B = the flux density in both air gaps.
From equation (110) the energy stored in each airgap is, therefore, given by equation (111).

$$W = \frac{B^2V}{8\pi} = \frac{B^2Al}{8\pi} \text{ ergs} \tag{111}$$

Let the right-hand part of the electromagnet be moved so as to increase the length of the airgap by dl. Hence the volume will be increased by Adl and the energy stored in the magnetic field when the airgap was increased from l to $l + dl$ is expressed by equation (112).

$$dw = \frac{B^2Adl}{8\pi} \text{ ergs} \tag{112}$$

The increase in the energy stored in the magnetic field must be equal to the mechanical work Fdl expended in moving the iron core the distance dl.

$$Fdl = \frac{B^2Adl}{8\pi} \text{ ergs} \tag{113}$$

Hence, the force in the c.g.s. system of units

$$F = \frac{B^2A}{8\pi} \text{ dynes} \tag{114}$$

In the English system of units the force

$$F = 13.86B1\ A1\ 10^{-9} \text{ lb.} \tag{115}$$

In equation (115),
B_1 = lines of force per square inches.
A_1 = cross-sectional area of airgap in square inches.

It should be noted that *the force is proportional to the square of the flux density*. Hence, for any given total number of lines the force is greater the smaller the area to which the flux is confined.

Kinetic Energy.—The storing of energy in a moving body follows relations exactly analagous to the energy changes discussed in the previous paragraph in forming a magnetic field. Using customary notations for mass (M), force (F), velocity (v), acceleration (a), power (p), and energy (w), the kinetic energy relations are given in equations (116) and (123).

$$F = Ma \tag{116}$$

$$a = \frac{dv}{dt} \tag{117}$$

$$p = Fv \tag{118}$$

$$dw = Pdt = Mvdv \tag{119}$$

$$\int_0^W dw = M \int_0^v vdv \tag{120}$$

$$W = \frac{Mv^2}{2} \tag{121}$$

In the metric system,

$$W(\text{joules}) = \frac{M(kg)v^2\,(\text{meters per second})}{2} \tag{122}$$

In the English system,

$$W(\text{ft. lbs.}) = \frac{M(\text{lbs.})v^2(\text{ft. per second})}{2 \cdot 32.2} \tag{123}$$

The kinetic energy stored in a *moving mass* is *directly proportional to the square of the velocity.* If the moving body is connected to, or is part of, a machine, as the reciprocating parts of a steam engine, kinetic energy is returned to the system when the velocity is reduced. In such reciprocating parts energy is stored kinetically when the velocity of the mass increases, and returned to the system when the velocity decreases. In comparing equations (102) and (120) it is evident that inductance L and mass M are similar coefficients representing respectively the capacity of the electric field for storing energy magnetically by any given current, and the capacity of the moving body for storing kinetic energy at any given velocity.

Discharging a Magnetic Field through Resistance.—In the circuit diagram of Fig. 24 a constant direct-current voltage E causes a current I_0 to flow through the circuit having a resistance R and an inductance L in series. The vibrator element V_2 of an oscillograph is connected in the circuit at the point indicated in the diagram. Closing the switch S forms a short circuit which separates the portion on the right side of the switch from the voltage supply E. Hence the energy stored in the inductance L at the instant the switch S is closed becomes the source of voltage which causes a current i to flow in the circuit. This current will continue to flow until the energy, stored in the magnetic field of inductance L, has been dissipated into heat in the resistance R; that is, by the Ri^2 losses. At any instant after the switch S is

closed the voltage in the circuit SLR is given by equation (124) in accord with Kirchhoff's first law.

$$L\frac{di}{dt} + Ri = 0 \tag{124}$$

Separating the variables,

$$\frac{di}{i} = -\frac{R}{L}dt \tag{125}$$

At the instant the switch S was closed the current

$$I_0 = \frac{E}{R} \tag{126}$$

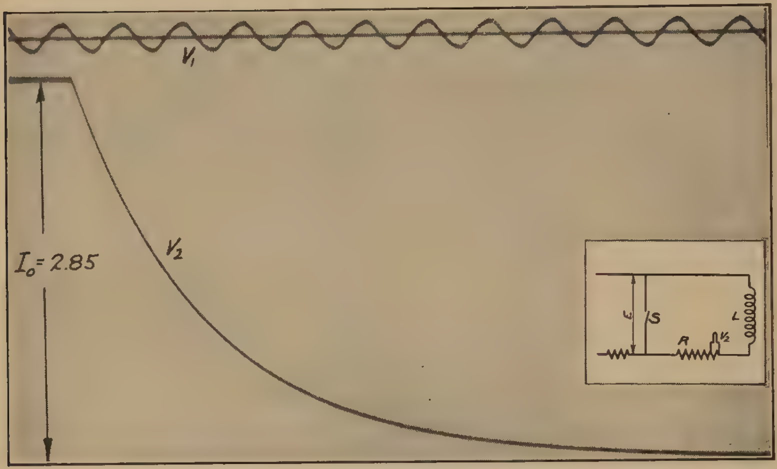


FIG. 24.—Discharging a magnetic field through a constant resistance. Oscillogram. $L = 0.209$ henry; $E = 36$ volts; $R = 12.7$ ohms; $I_0 = 2.85$ amp.; $V_1 = 100 \sim$.

Hence the limits for the variables in equation (124) which expresses the current time relations during the discharge period; that is, the time during which the energy initially stored in the magnetic field would be dissipated into heat, are as stated in equation (127).

$$\int_{I_0}^{i}\frac{di}{i} = -\frac{R}{L}\int_0^t dt \tag{127}$$

$$\log_\epsilon \frac{i}{I_0} = -\frac{R}{L}t \tag{128}$$

$$i = I_0\epsilon^{-\frac{R}{L}t} \tag{129}$$

Substituting numerical values in equation (129) for I_0R and L from the data for the circuit in Fig. 24, gives the values of the current in amperes with the time in seconds.

$$i = 2.85\epsilon^{-\frac{12.7t}{0.209}} = 2.85\epsilon^{-60.77t} \text{ amp.} \qquad (130)$$

The oscillogram in Fig. 24 is a photographic record of the current-time relations expressed by equation (130). The scale for the current in amperes is based on the measured value of I_0. The time scale on the axis of the abscissæ is given by the alternating-current wave V_1, recorded at the top of the oscillogram. The timing wave was produced by passing an alternating current of known frequency, 100 cycles per second, in a separate circuit through vibrator V_1. Hence the distance on the film measured by one complete cycle on the curve represents $1/100$ sec.

Since R is constant and by Ohm's law e is at any instant equal to Ri, therefore, the current-time curve in the oscillogram may also be used to represent the voltage-time relation across either the resistance or the inductance.

$$e = E_0\epsilon^{-\frac{R}{L}t} \qquad (131)$$

The current-time curve V_2 in the oscillogram may be used as a voltage-time curve by drawing a voltage scale for the ordinates. By inserting numerical values for E_0, R, and L in equation (130) the instantaneous values for the voltage across either the resistance or the inductance in the circuit shown in Fig. 24 will be given in volts for the time in seconds.

$$e = 36\epsilon^{-60.77t} \text{ volts} \qquad (132)$$

In order to gain clear physical concepts of the current-time and voltage-time relations during the discharge of the energy in a magnetic field it should be observed that the *rate of change in the current is at any instant proportional to the magnitude of the current at that instant*, and likewise the *rate of change in the voltage is at any instant proportional to the magnitude of the voltage at the given instant.*

Forming a Magnetic Field.—When a magnetic field is built up or formed, energy is stored in the space occupied by the magnetic lines of force. In the circuit shown in Fig. 25, let R, L, and E be constant in value. When the switch S is closed the voltage E is impressed in the circuit having resistance R and inductance

L in series. Since energy must be stored in the magnetic field a counter-electromotive force is produced in the inductance. Hence the current does not reach its full value until all the energy required to form the magnetic field corresponding to the permanent value of the current stored in the field. The value of the current at any instant during the period in which the magnetic field is formed is given by the current-time curve V_2 of the oscillogram in Fig. 25. The switch S was closed at the instant marked O on the oscillogram. The current rapidly increased and reached

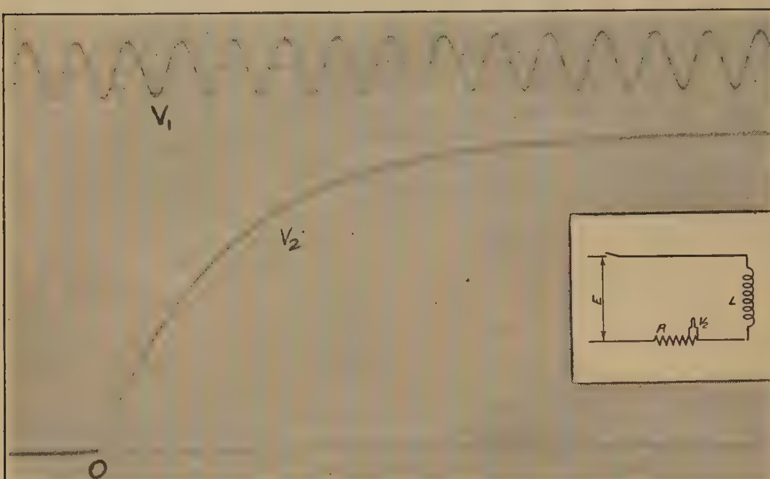


Fig. 25.—Forming a magnetic field. Oscillogram. $L = 0.4$ henry; $E = 6.55$ volts; $R = 15.2$ ohms; $I = 0.43$ amp.; $V_1 = 100 \sim$.

the constant, permanent value I in approximately $\frac{1}{10}$ sec. as measured by the timing wave V_1 on the oscillogram.

In accord with the circuit data in Fig. 25, the voltage relations, directly after the switch is closed, are by Kirchhoff's law as stated in equation (133)

$$L\frac{di}{dt} + Ri = RI = E \tag{133}$$

Separating the variables and applying the limits, under conditions as specified in Fig. 25, gives equation (134).

$$\int_o^i \frac{d(i - I)}{(i - I)} = -\frac{R}{L}\int_o^t dt \tag{134}$$

$$i = I - I\epsilon^{-\frac{R}{L}t} \tag{135}$$

Substituting the numerical values from the circuit in Fig. 25, for I, R and L in equation (135) gives the instantaneous values of the current in amperes with time in seconds.

$$i = 0.43 - 0.43\epsilon^{-\frac{15.2}{.4}t} = 0.43(1 - e^{-38t})\text{amp.} \qquad (136)$$

In direct-current engineering problems the inductance of a circuit enters into the equations only when the *current increases or decreases* in magnitude. When the circuit carries constant direct currents only the resistance need be considered, but any change in load or circuit conditions causing a change in the current necessarily must produce a corresponding increase or decrease in the amount of energy stored in the magnetic field of the system. In general, the mechanical and voltage stresses in the direct-current system are greater, by far, during transient period of rapidly changing values of current and magnetic flux than during the steady full-load operating conditions. If in a direct-current shunt motor the field circuit is opened the voltage produced by self-inductance is several times as large as the normal operating voltage from the supply line.

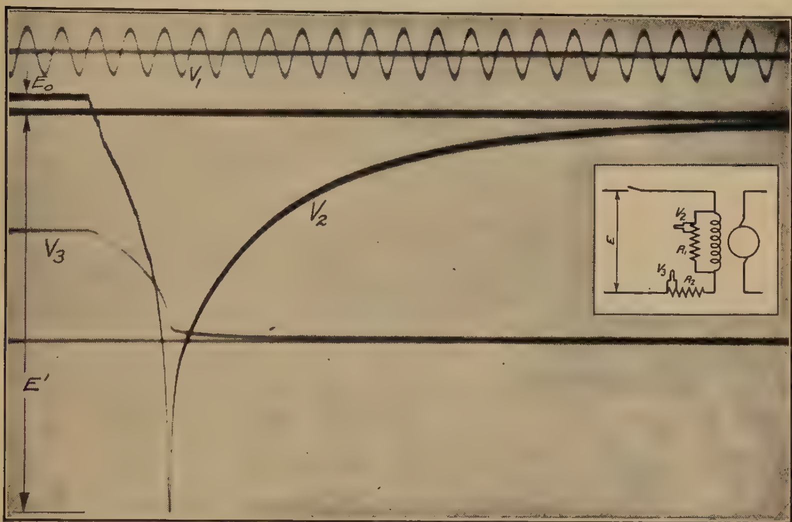


FIG. 26.—Breaking motor field circuit. Oscillogram. Field of a 5 hp. 115-volt motor. V_3 = field current; V_2 = voltage across field terminals; V_1 = 100 ~ timing wave.

The oscillogram in Fig. 26 illustrates the importance of having quantitative data on the sudden changes in current and voltage that occur in direct-current machinery.

The circuit diagram in Fig. 26 shows the location of vibrators V_2 and V_3. Vibrator V_1 was in an independent circuit and provides a convenient means for recording the speed of the moving film when the oscillogram was taken. Each complete cycle of curve V_1 represent $\frac{1}{100}$ sec. Vibrator V_2 records the voltage at the field terminals while v_3 gives the field current-time curve. The line voltage impressed on the motor for normal operation is marked E_0 on the oscillogram. When the field circuit is opened the energy in the magnetic field must escape as the current decreases. The sudden decrease in the current causes a very rapid contraction of the field flux which induces a high voltage in the field winding as indicated on the oscillogram by the maximum value E' on curve V_2. Note that the voltage E' which is generated when the field circuit is broken is over fifteen times the value of E_0, the operating voltage normally impressed on the motor field. It is therefore evident that in the design of the direct-current motor, insulation must be provided in the motor field circuit on the basis of the transient voltage induced when the field circuit is broken and not merely sufficient to withstand the normal operating voltage.

The energy stored in a magnetic field has essentially the same characteristics as the kinetic energy of a moving mass, as for example a bullet. When the circuit is broken the magnetic energy reverts into the electric form and then usually into heat in much the same way as the kinetic energy in the moving body is changed into heat or some other form of energy. In fact, it is as dangerous to be in contact with the terminals of a large magnetic field when the current is suddenly reduced to zero by breaking the circuit, as to be in the path of a bullet moving at high speed. In taking experimental data on magnetic field circuits care must be exercised to prevent injury to the operator or damage to the instruments. If a voltmeter is connected across the terminals of a dynamo field circuit it should be disconnected before the field circuit is opened. Unless this precaution is taken the quick break in the field current when the field switch is opened will cause a very high voltage to be produced in the field circuit which may puncture the insulation of the voltmeter. In large dynamos the voltage induced when the field circuit is opened is so great that protective devices must be employed to prevent puncture and breakdown of the insulation in the field winding itself. Thus the field break-up switch, which opens the field circuit simultaneously at more than one point

separates the field winding into sections and thereby reduces the maximum voltage between the field winding and the frame of the machine. Another method is to permanently connect a non-inductive resistance equal in magnitude to the resistance in the field circuit as a shunt across the terminal of the field circuit. When the switch to the mains is opened the energy stored in the magnetic field is dissipated into heat in the shunt resistance and the terminal voltage does not rise above normal operating values. In order to prevent the loss of power in the discharge shunt circuit during normal operation of the machine, the discharge resistance is connected in series with a small airgap, or a special switch is used which automatically connects the discharge resistance shunt just before the main field circuit is opened. In either case the sudden rise in voltage that otherwise would endanger the field circuit insulation is prevented.

PROBLEMS

1. A wire in the form of a square, whose sides are 4 in. in length, carries a current of 15 amp. Find the flux density at the center of the square. Find the total flux passing through the coil if the size of wire is No. 0 copper.

2. A No. 00 copper wire, in the form of a rectangle, 3 by 5 in., carries a current of 25 amp. Find the flux density at the point where the diagonals cross. Find the total flux passing through the coil.

3. Find the flux density at a point in the plane of the coil in Problem 2 located 2 in. outside of the coil on the perpendicular bisector of one of the 5-in. sides.

4. A power transmission line consisting of two wires, spaced 36 in. apart and 26 ft. above the ground, carries a maximum current of 88 amp. Running parallel with the transmission line and at the same height above the ground is a telephone line having a spacing of 24 in. The two lines are located 38 ft. apart, measured between the wires closest together. Find the flux interlinking the telephone line per mile due to the current in the transmission line.

5. Repeat Problem 4 if the transmission line is 51 ft. above ground; the telephone line remaining at a height of 26 ft.

6. A transmission line of two No. 4/0 copper wires has a spacing of 10 ft. between centers of conductors and carries a current of 250 amp. Find the total flux passing between the parallel wires for 1 mi. of line. What is the inductance of the line per mile under the given conditions?

7. A copper conductor has a diameter of 0.762 in. Find the total flux per mile length inside the surface of the conductor when 800 amp. are flowing in the circuit. Find the total number of flux linkages inside the conductor per mile of length.

8. An air core, standard solenoid, consisting of 764 turns uniformly spaced, has a length of 117 6 cm. and a cross-sectional area of 31.7 cm. If a

current of 10 amp. flows in the solenoid, find the flux passing through the middle of the solenoid by both the approximate and exact methods. What is the error of the approximate method in per cent?

9. An air core solenoid 24 in. long, 12 in. in diameter and having 290 turns, uniformly spaced, carries a current of 20 amp. Find the flux density on the axis of the solenoid, midway between the two ends. Find the flux density at a point on the axis 24 in. outside from one end of the solenoid.

10. Two coils having self-inductances of 0.2 and 0.26 henrys respectively are connected in series and found to have a total inductance of 0.3 henrys. Find the coefficient of mutual inductance of the coils. If the leads of one of the coils were reversed without disturbing their relative positions, what would be the total inductance of the coils?

11. A concentrated coil having 250 turns of wire carries a current of 6 amp. and produces a total flux of 250,000 lines. Find the energy stored in the magnetic field in joules, in foot pounds, in B.t.u. and in gram-calories.

12. A four-pole motor is used to drive a pump by means of a horizontal belt. Due to faulty lubrication and consequent wear of bearings the airgap of one of the poles is lengthened and the directly opposite one correspondingly shortened. The flux density under one pole is 55,000 per square inch in the airgap and 50,000 per square inch under the other. The pole face area is 48 sq. in. The airgaps of the other two poles remain essentially unaffected and may be neglected. Find the extra load imposed on the bearings due to the unbalanced pull of the flux in the airgaps.

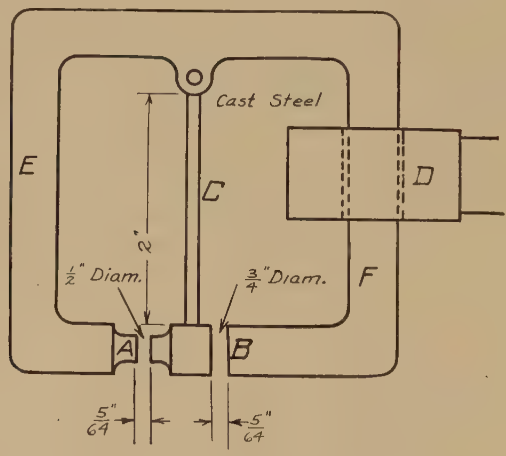


Fig. 27.

13. In Fig. 27 is shown the magnetic circuit of a magnetic relay. For low values of current in the coil *D*, the lever *C* swings to the right toward *B*. For large values of current in coil *D*, the lever *C* becomes saturated and the extra flux produced will pass across the gap at *A*. Since the magnetic pull, equation (114), depends on the square of the flux density and directly as the cross-section, the pull at *A* may be greater than that at *B*, in spite of the

fact that the flux at B is always greater than at A. Consider the air gap dimensions as given in Fig. 27 and neglect fringing. The swing of the lever C is restricted by stops so that it occupies essentially the position shown in the figure at the critical operating point. The lever C is of sheet-steel material. Find the cross-sectional area of the lever C in order to cause it to swing to the left when 5.5 amp. pass through the 100 turns of coil D. Assume that all other parts of the magnetic circuit are of sufficient cross-section that their ampere-turns may be neglected.

14. An inductance coil having a resistance of 56 ohms and an inductance of 0.48 henrys is suddenly connected to the terminals of a 110-volt battery having an internal resistance of 4 ohms.

(*a*) At what rate will the current be increasing after 0.006 sec.?

(*b*) At what rate at the instant of closing the switch?

(*c*) If the current increased continually at a rate equal to the initial rate, how long a time would be required to charge the inductance?

(*d*) Compare this time in seconds with the ratio of L in henrys to R in ohms.

(*e*) What is the value of the current when $t = L/R$?

15. The inductance coil of Problem 14 after being charged is discharged as indicated in the circuit diagram of Fig. 24, except that no additional resistance is placed in series with the coil.

(*a*) Find the initial rate of decrease of current.

(*b*) If current decreases uniformly at the initial rate, how long a time will elapse before the inductance is discharged?

(*c*) Compare the time in (*b*) with the ratio L/R.

(*d*) At what rate is the current decreasing 0.005 sec. after closing the switch?

CHAPTER XI

MAGNETIC PROPERTIES OF IRON AND STEEL

Magnetic Materials.—On the basis of their relative magnetic conductivity or permeability, materials may be classified as para magnetic or *magnetic, non-magnetic,* and *diamagnetic;* the determining factor being whether the permeability is greater than one (magnetic), unity (non-magnetic), or less than one (diamagnetic).

The *non-magnetic group* is by far the largest, including all non-metallic substances and most of the metals and metallic compounds. The diamagnetic group is the smallest and of little practical importance. Bismuth has the lowest permeability and pieces of this metal are generally used in lecture demonstrations on diamagnetism. But even diamagnetic substances conduct magnetic flux to a marked degree. No materials have been found that provide good insulation against magnetic lines of force, and hence it is not possible to confine magnetic flux in as definite paths as may be done with electric currents. Only seven chemical elements, atomic numbers 22 to 28 inclusive, are in the *magnetic group.* Iron in its various forms and combinations is by far the most important element of the magnetic materials. The permeability of iron is affected by many complex factors and varies over a wide range. The permeabilities of iron and steel in electrical machines and appliances vary from 50 to 5,000 under ordinary conditions. Cobalt, nickel, chromium, and manganese are also magnetic but their permeabilities are low as compared with iron, usually less than 50. The alloys of iron, as various kinds of steel, permalloy, etc., have magnetic properties widely different from pure iron. Commercial magnetic materials may be classified as: cast iron, wrought iron, soft or low-carbon steel, high-carbon steel, permalloy and other alloy steels. The magnetic material may also be grouped on the basis of high or low retentivity into two main divisions; one, suitable for apparatus such as dynamos, transformers, electromagnets, etc., in which the retentivity, coercive force, and hysteresis loss must be as small as possible; the other, necessary for permanent

magnets, measuring instruments, magnetos, relays, etc., in which the greatest possible retentivity is the dominating characteristic.

Permeability, Reluctivity.—Permeability is the reciprocal of reluctivity and represents the relative ability of materials to conduct magnetic flux. The permeability of empty space is one and for all non-magnetic materials so nearly one that it is assumed to be unity in all engineering computations. Permeability is the specific permeance of a substance; that is, the permeance per $\overline{\text{cm.}}^3$ Likewise reluctivity is the specific reluctance of a substance; that is, the reluctance in oersteds per $\overline{\text{cm.}}^3$ Permeability and reluctivity are therefore ratios giving the relative values of the magnetic properties possessed by materials with respect to air or empty space. If a piece of iron conducts magnetic flux 600 times better than air it has a permeability of 600 and a reluctivity of $1/600$. The symbol for permeability is the Greek letter μ, and for reluctivity the Greek letter ν.

Although the permeability in each instance is a simple numerical ratio, the magnetic properties of iron and steel are affected by several complex factors. In many cases it is far from a simple matter to determine with the desired degree of accuracy the permeabilities involved in the design and operation of electric machinery and appliances.

The factors affecting the magnetic properties of iron and steel may be grouped under four heads, as follows:

(*a*) Chemical composition.
(*b*) Physical condition.
(*c*) Magnetic flux density.
(*d*) Sequence of values of the flux density.

That the chemical composition of magnetic materials should be of basic importance as regards the permeability of the material is well nigh self-evident. The elements that possess magnetic properties are few in number, in fact, limited to atomic numbers 22 to 28 inclusive, and it seems natural that the proportionate parts of magnetic elements in a compound material should greatly affect its magnetic properties. Thus *permalloy*, composed of approximately 80 parts of nickel and 20 parts of iron, has very high permeability at low magnetic flux densities. Likewise the *Heussler alloys* composed of manganese, copper, and aluminum have permeabilities much higher than manganese, in fact, comparable to those of iron. Soft wrought iron at moderate

magnetization may have permeabilities as high as 7,800. For electrolytically deposited pure iron, melted in vacuo, much higher permeabilities have been obtained. For low magnetization values permalloy has very high permeabilities, even as compared to soft iron or silicon steel. In Fig. 8 are shown permeabilities of permalloy up to 87,000 and in Fig. 10 with the specimens under tensile stress, values in excess of 250,000. The very high permeabilities are for low magnetizations only. Even for moderate values of B and still more at higher values the permeability of soft iron and silicon steel is much larger than for permalloy. Hence it is only under low flux density conditions, as in telephone and telegraph cables that the properties of permalloy are of great practical value.

Cast iron, wrought iron and various forms of steel; that is, alloys of iron with carbon, nickel, silicon and tungsten, form the bulk of magnetic materials used in the manufacture of electrical machinery and appliances.

The physical condition likewise affects the permeability. The magnetic properties may be greatly modified in any given sample by heat treatment, as in the hardening or tempering of steel, by hammering or rolling and by changes in temperature. For temperature above 750°C., at a dull red glow, iron and steel are non-magnetic. Annealing is frequently used to restore desirable magnetic properties in iron or steel subjected to mechanical vibrations or strains or undergoing changes in the crystalline structure of the magnetic material.

The effects of both chemical composition and physical condition on the permeability of magnetic materials are comparable to the effects of changes in composition on the conductivity of electric conductors. There is, however, no counterpart in the electric circuit to the very important effect of magnetic flux density on the permeability of the various irons and steels. The relation is best shown by curves as in Fig. 1, which gives the average values for the materials and flux densities used. Moreover, in general for ferro-magnetic materials, *the permeability factor has more than one value at any given flux density depending on the sequence of changes in the flux densities preceding the conditions under which the measurement is made.* With decreasing flux densities the permeability at any given flux density is greater than the average value while it is smaller if the flux density is increasing in magnitude.

The complex relations between *magnetic flux density* and *permeability* are shown to best advantage by the so-called *B-H curves* which give the quantitative ratios between the flux density B and the field intensity H, for both increasing and decreasing values of the magnetizing force. The *B-H* curve starting at zero values for both B and H and for continuously increasing values of the magnetizing force is called the "*magnetization curve.*"

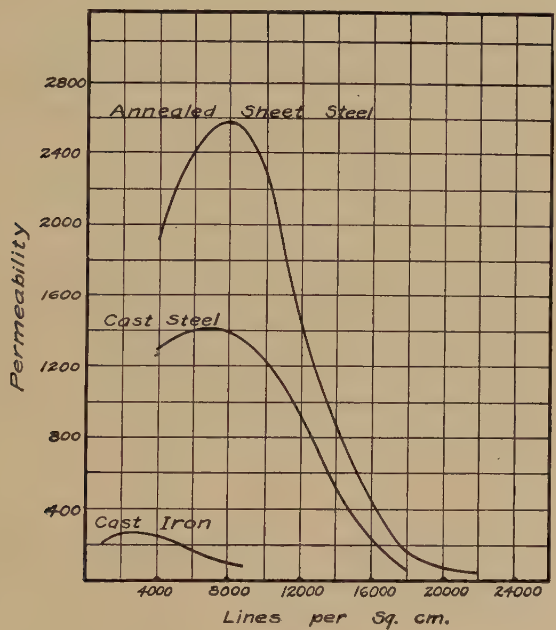


FIG. 1.—Permeability curves.

The B-H Curves. (*a*) *The Magnetization Curve.*—As stated in the preceding paragraph the permeability of iron and steel is greatly affected by the magnetic flux density. Thus the permeability of a given sample of ordinary iron, which may be less than a hundred for very low flux densities increases to 3,000 or 4,000 at moderate degrees of magnetization and then rapidly decreases and approaches unity at very high values of the magnetizing force. The relation between the magnetizing force H and the corresponding flux density B is best shown graphically as in Figs. 2 and 3.

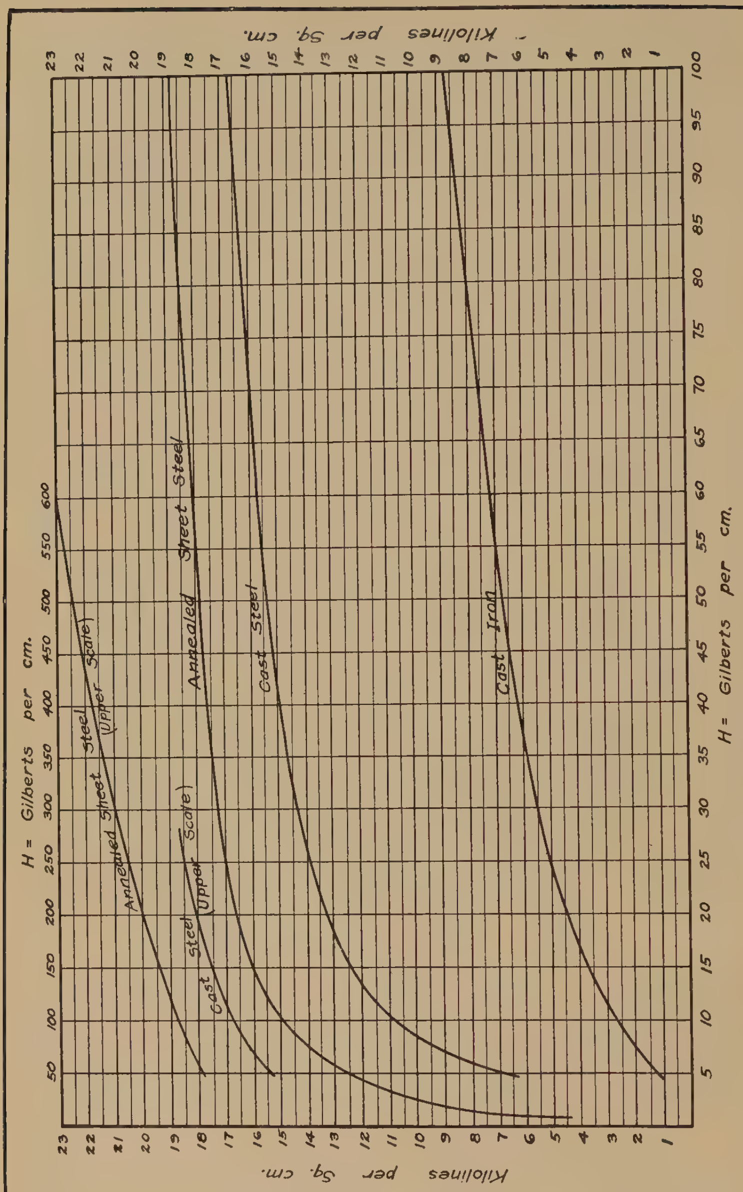


FIG. 2.—Magnetization curves. International units.

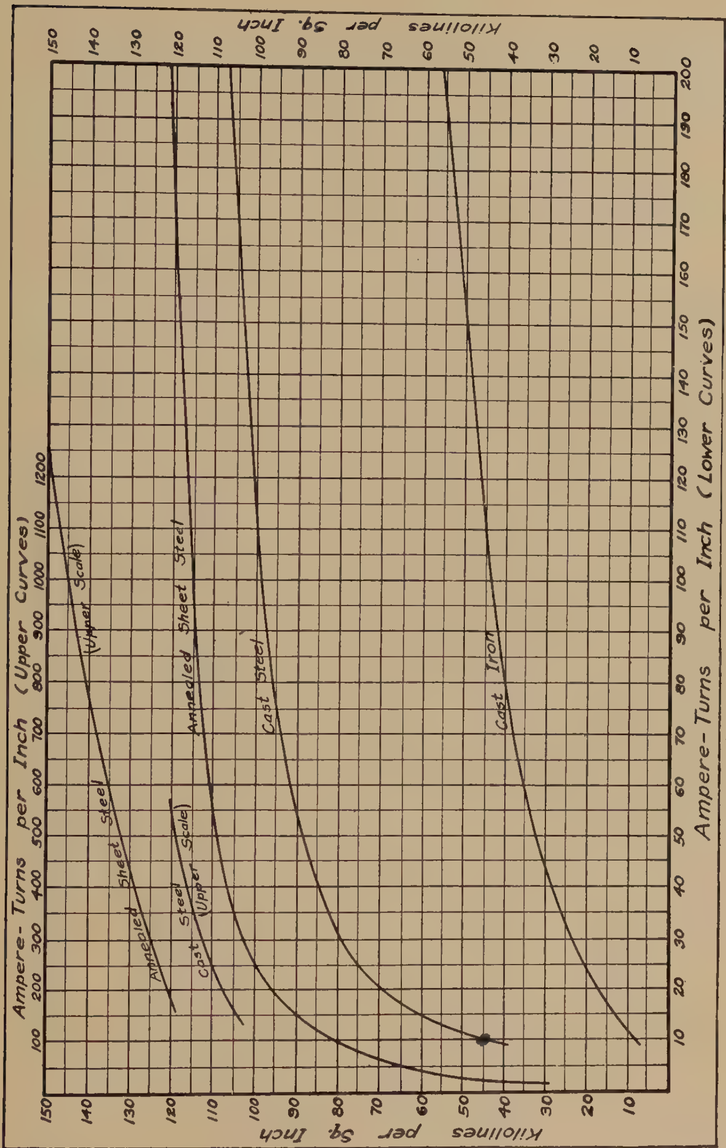


FIG. 3.—Magnetization curves. English units.

For convenience in making computations in ampere-turns and inches, a second graph is provided in Fig. 3 showing the relation H in ampere-turns per inch and B in lines per square inch.

$$\begin{aligned} 1 \text{ gilbert} &= \frac{10}{4\pi} \text{ ampere-turns.} \\ &= 0.796\text{NI} \\ 1 \text{ gilbert per cm.} &= 2.02\text{NI per inch} \\ 1 \text{ gauss} &= 1 \text{ maxwell per } \overline{\text{cm.}}^2 \\ &= 1 \text{ line of magnetic flux per } \overline{\text{cm.}}^2 \\ &= 6.45 \text{ lines of flux per sq. in.} \end{aligned}$$

From the graphs in Figs. 2 and 3 it is evident that in order to secure the benefit of a high permeability the flux density must come below the bend or knee in the magnetization curve. For flux densities above the bend in the curve the ampere-turns increase very rapidly in comparison to the corresponding increase in flux density. Thus for annealed sheet steel, as shown by the magnetization curve in Fig. 2, a magnetizing force of 2.5 gilberts per centimeter produces a flux density of 10,000 gauses while 200 gilberts per centimeter is required to give, in the same sample, a flux density of 20,000. Hence, to double the flux density under the given conditions the magnetic motive force had to be increased eighty-fold.

(*b*) *The Hysteresis Loop.*—Several theories have been advanced to explain the properties of magnets and magnetic fields but none have proved satisfactory. In studying the properties of magnets it is, however, of considerable assistance to have a mental picture or concept, however crude and inadequate, of the mechanism involved. Weber's molecular theory of magnetism is based on the assumption that in magnetic materials as iron or steel, each molecule is a small magnet having definite north and south poles. It is also assumed that under normal non-magnetic conditions the magnetic axes of the many molecular magnets are not arranged in any definite order but grouped in a haphazard manner. When a magnetizing force is applied the molecular magnets align themselves, more or less, in a definite north and south direction and that as a consequence the whole bar or rod acquires the properties of a single large magnet. Necessarily, if the direction of the magnetizing force be reversed all the component molecular magnets would also be reversed in position. It is natural to suppose that to produce the motion of the mole-

cules, when reversing in position, would require work or expenditure of energy in order to overcome friction or resistance to motion, analogous to friction. This molecular friction loss, appearing as heat in the magnetic material, is known as the *hysteresis loss.*

If the magnetizing force H applied to a magnetic circuit of iron or steel passes through a complete cycle, first increasing to some given value in the positive direction, then decreasing to the same magnitude in the opposite direction or negative value and then increasing to the given positive value, the B-H relation will form a closed curve known as the *hysteresis loop.* In Fig. 4 is shown a hysteresis loop for annealed sheet steel. If the cycle is repeated letting the flux density increase to the same maximum positive value as in the first cycle and then decrease to the corresponding negative value, the original hysteresis loop will be repeated. If the magnetizing force is increased so as to give a higher maximum flux density than for the above hysteresis loop, then decreased to the corresponding maximum in the opposite direction and then increased to the same maximum value in the positive direction, a new hysteresis loop will be formed.

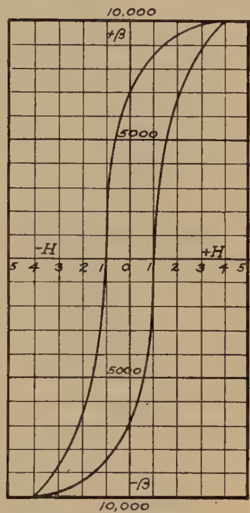


FIG. 4.—Hysteresis loop for annealed sheet steel.

A set of any number of hysteresis loops can, therefore, be formed for any given sample of iron or steel by using a different maximum flux density for each loop, as illustrated by Figs. 5 and 6.

In each loop the flux density B has two values for each value of the magnetizing force H. The higher values are obtained when the flux density is decreasing and the lower values appear for increasing flux densities. Hence, the magnetic properties of iron and steel are affected both by the flux density and the preceding magnetic condition of the material. In alternating-current machinery, as for example, ordinary transformers, the magnetizing force is applied alternately in the positive and negative directions. The same condition prevails in the armatures having iron or steel and carrying alternating currents; the

B-H curve forms a hysteresis loop for each complete cycle of the current producing the magnetomotive force. By joining the tips of successive hysteresis loops as illustrated by the dotted line in Figs. 5 and 6 the mean magnetization curve is obtained.

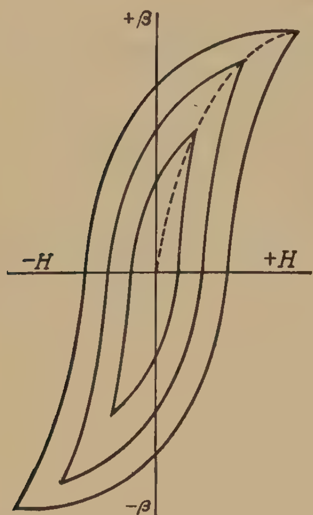


FIG. 5.—Hysteresis loops for carbon steel.

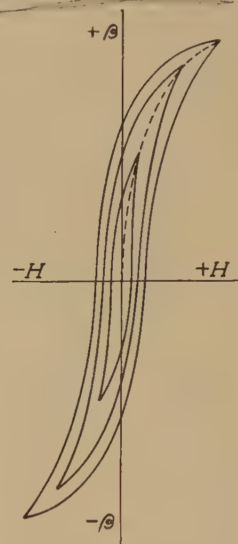


FIG. 6.—Hysteresis loops for soft annealed iron.

Permalloy.—The name permalloy has been given to a series of nickel-iron alloys, developed by the Bell Telephone Laboratories, that have permeabilities at low magnetization values very much greater than those possessed by pure iron or silicon steel. To secure the highest permeabilities the best composition is about 78.5 per cent nickel and 21.5 per cent iron.

In Fig. 7 are shown the magnetization curves for permalloy and armco iron, a standard high-grade soft iron. It should be noted that for low values of H, the flux density B is greater for permalloy than for armco iron, but for higher values of H the reverse is true. In Fig. 8 are shown the corresponding permeability curves. For low magnetization the permeability of permalloy is more than ten times as large as the maximum value for armco iron.

The relative size of the hysteresis loop for permalloy as compared to armco iron may be seen in Fig. 9. In order to show the hysteresis loop to actually be a loop and not merely a single line,

the values of H are plotted to a much larger scale than is usually used for drawing iron and steel hysteresis curves. Since the

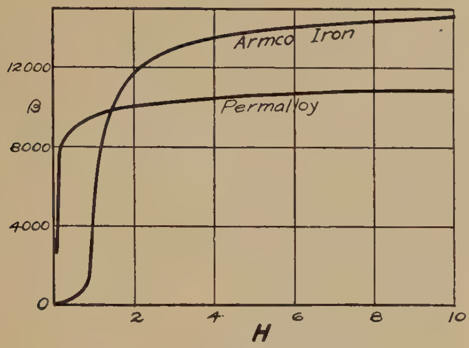


FIG. 7.—Magnetization curves for permalloy and armco iron. (*Bell Telephone Laboratories.*)

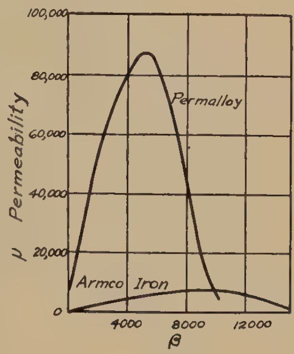


FIG. 8.—Permeability curves of permalloy and armco iron. (*Bell Telephone Laboratories.*)

hysteresis losses are proportional to the area of the hysteresis loop it is evident from Fig. 8 that for permalloy these losses are very small, even as compared to those for armco iron.

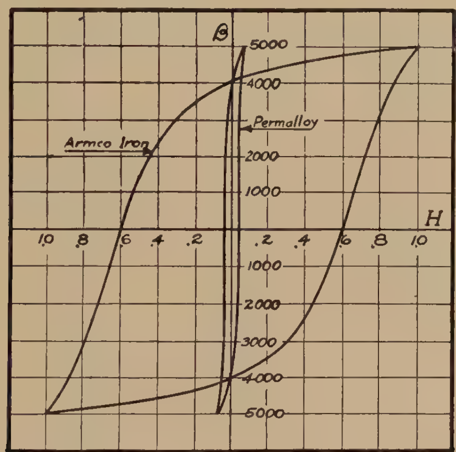


FIG. 9.—Hysteresis loops of permalloy and armco iron. (*Bell Telephone Laboratories.*)

The magnetic properties of permalloy are extremely sensitive to mechanical stresses. Thus the permeability is greatly changed when the permalloy test material is under tension, as shown in Fig. 10.

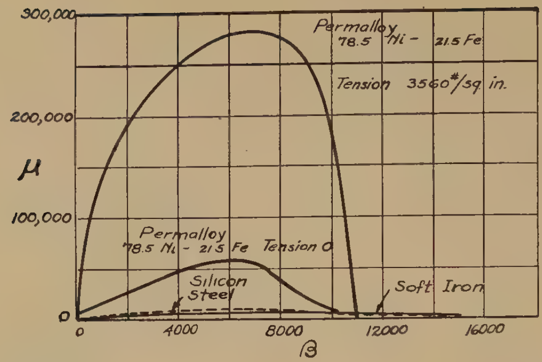


FIG. 10.—Permeability curves of permalloy, silicon steel, and soft iron. (*Bell Telephone Laboratories.*)

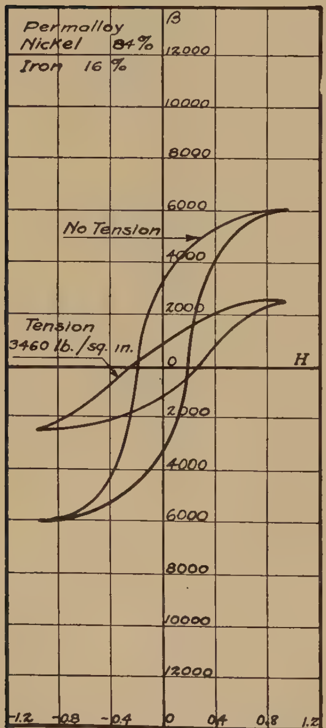


FIG. 11.—Permalloy hysteresis loops. (*Bell Telephone Laboratories.*)

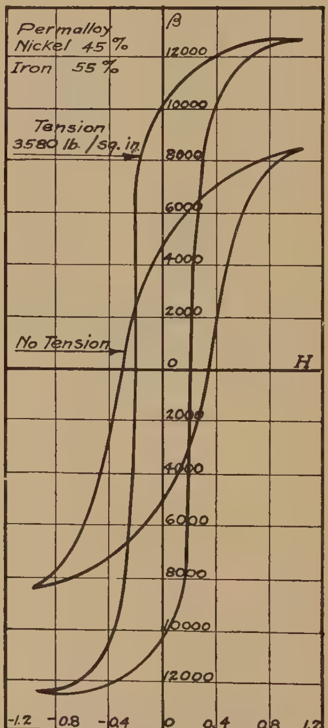


FIG. 12.—Permalloy hysteresis loops. (*Bell Telephone Laboratories.*)

Changes in permalloy hysteresis loops produced by mechanical stress are shown in Figs. 11 and 12. No satisfactory explanation has been found for the remarkable magnetic properties possessed by permalloy, either in the annealed state or when under tension. In fact, very little is known of the intramolecular or intraatomic mechanisms that make it possible to store energy magnetically in some materials, as iron and nickel, while this power is lacking in other metals as copper or silver. The permalloy magnetization and permeability curves and hysteresis loops are presented in this chapter largely as an indication of latent possibilities that may be made available when the nature of magnetism is understood and methods have been devloped to use the properties of magnetic materials to best advantage. A large number of nickel alloys have been investigated and several forms, notably *hypernik* and *perminvar* posess magnetic properties that make these alloys commercially valuable and of special scientific interest.

Froelich's Formula, Magnetic Saturation.—Magnetization curves and hysteresis loops are difficult to express in the form of mathematical equations. Several empirical formulæ have been suggested that approximately represent the B-H relation over a limited range and for restricted groups of materials. The simplest formula applicable to magnetization curves for irons and steels was developed by Froelich about 1881 and is known as "Froelich's formula."

$$B = \frac{aH}{b + H} \tag{1}$$

In the international system of units, the values of a and b would be of such magnitude that if H is expressed in gilberts B would be in gausses. For any given quality of iron or steel the constants must be adjusted so that the curve represented will conform as nearly as possible with the experimental data. The formula gives a close approximation for that part of the magnetization curve which is of greatest practical importance but does not apply for low values of flux density. If it is desired to express Froelich's formula using ampere-turns per inch and the flux density per square inch the constants a' and b' must be given the corresponding numerical values.

$$B = \frac{a'NI}{b' + NI} \tag{2}$$

Froelich's formula is based on the assumption that the permeability of iron is proportional to its remaining magnetizability and states, as shown in equation (4) that the reluctivity of an iron-clad circuit is a linear function of the field intensity. For flux densities above the bend in the magnetization curve the permeability rapidly decreases and approaches unity as the iron or steel approaches a state of *magnetic saturation.* From equation (1).

$$\frac{H}{B} = \frac{1}{a}(b + H) \tag{3}$$

By definition,

$$\frac{H}{B} = \frac{1}{\mu} = \nu, \text{ the reluctivity} \tag{4}$$

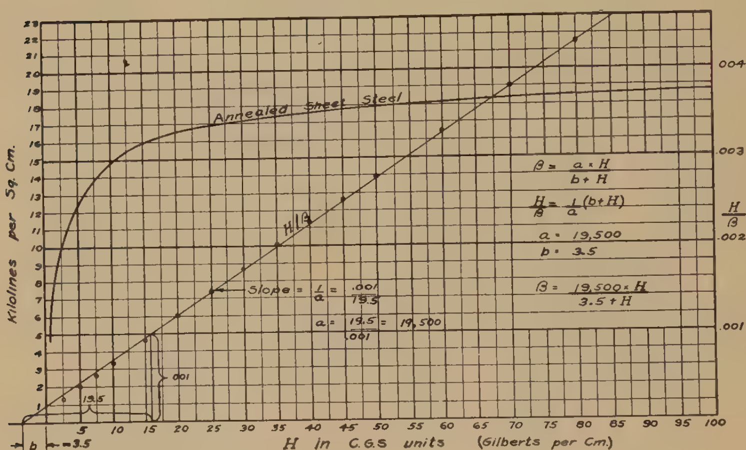


FIG. 13.—ν-H relation, annealed sheet silicon steel. International units.

From equations (3) and (4) the same relation may be expressed in equation (5).

$$\nu = \frac{1}{a}(b + H) \tag{5}$$

If the reluctivity ν and H are taken as the variables this is the equation of a straight line in which $1/a$ is the slope and b the intercept along the H axis. This is illustrated in Fig. 13 using international units and in Fig. 14 for the English system.

From equations (4) and (5) it is evident that the ν-H curve, as illustrated in Figs. 13 and 14 for annealed sheet silicon steel, may be derived from the corresponding B-H curve. The intercept on the X axis gives the value for the constant, while the

slope of the line with respect to the H axis gives the reciprocal of the value for the constant a. While Froelich's formula is helpful for gaining insight into the magnetic properties of iron and steel, it is of little practical value in the design of electrical machines as the magnetic characteristic of the material can be

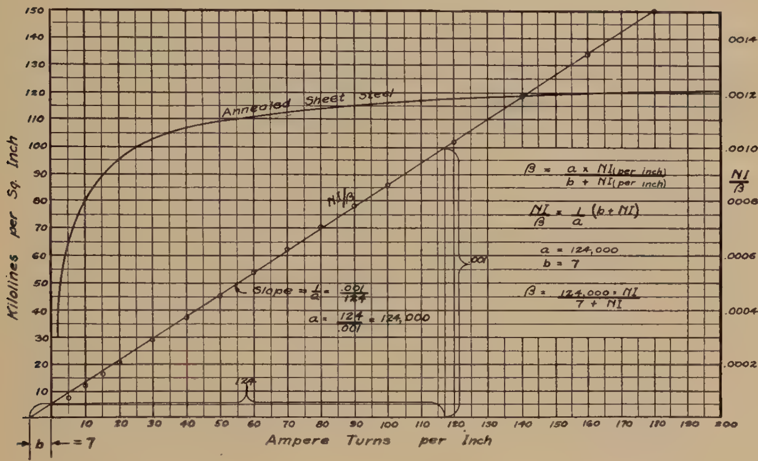


FIG. 14.—ν-H curve for annealed sheet, silicon steel. English units.

determined more readily from the experimental data required to evaluate the constants in the formula before it can be applied to any particular case.

Residual Magnetism, Retentivity, Coercive Power.—Given a piece of iron or steel, preferably of such shape as to form an iron-

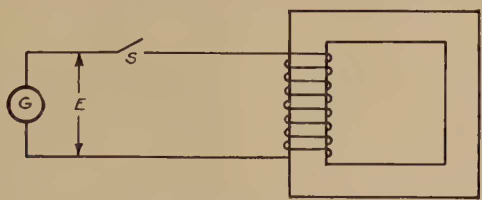


FIG. 15.—Iron-clad circuit.

clad electric circuit; that is, a coil of wire carrying a direct current interlinked with a closed iron circuit for the magnetic flux as illustrated in Fig. 15. The magnetic flux produced in the iron depends on the magnitude of the magnetizing force and the reluctance of the magnetic circuit. If the electric circuit is opened the current will cease to flow and thereby the magnetizing force

drops to zero. The magnetic flux will decrease in magnitude but not to zero value, as a considerable portion will remain in the iron or steel core after the electric current has ceased to flow. The magnetic flux held by the magnetic material after the magnetizing force has been removed is called the *residual magnetism,* while the ability of the iron or steel to retain or hold the residual magnetism is called the *retentivity* or *remanence* of the material. The *coercive force* is measured by the counter-magnetizing force in gilberts per centimeter required to completely remove the residual magnetism. This relation is shown graphically by the hysteresis loop in Fig. 16.

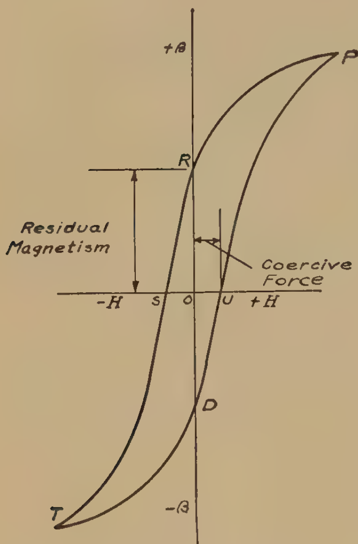


FIG. 16.—Coercive force.

The percentage of the total flux that will be retained as residual magnetism will depend on the chemical composition, the physical condition, the maximum flux density and the geometric dimensions of the magnetic circuit. In an iron-clad circuit, as illustrated in Fig. 15, even soft annealed iron will retain a large part of the flux while a still larger percentage will be retained if steel is the magnetic material used. In a magnetic circuit having a large airgap, as in Fig. 17, the

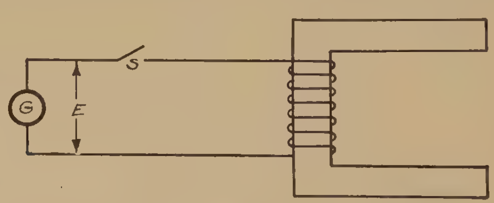


FIG. 17.

retentivity would be small for soft iron, but quite large for tungsten steel.

Steels having from 2 to 8 per cent tungsten with ½ to 1 per cent carbon or chromium and properly hardened by heat treatment possess to a high degree the property of retaining per-

manently the residual magnetism; that is, the permeance or retentivity factor is comparatively large.

Hysteresis Losses. (a) *Area of Hysteresis Loop.*—In Chap. X it is shown that the energy per cm.³ in a magnetic field is expressed by equation (108).

$$W_v = \frac{B^2}{8\pi\mu 10^7} \text{ joules per } \overline{\text{cm.}}^3 \tag{6}$$

If the permeability μ is constant the change in the energy stored with respect to the flux density is obtained by differentiating equation (6).

$$dW = \frac{BdB}{4\pi\mu 10^7} \tag{7}$$

As the increment in dB is very small there will be no appreciable change in the permeability and hence equation (7) applies to all magnetic circuits.

Since $B = \mu H$,

$$dW = \frac{HdB}{4\pi 10^7} \tag{8}$$

Hence the energy per $\overline{\text{cm.}}^3$ is the integral of equation (8).

$$W = \frac{1}{4\pi 10^7}\int HdB \text{ joules per } \overline{\text{cm.}}^3 \tag{9}$$

In Fig. 18, letting OP be the magnetization curve, it is evident that HdB is represented by the elemental area $abcd$ and that $\int_0^Q HdB$ is an expression for the area OPQ between the B-H curve, and the B axis.

$$\int_0^Q HdB = \text{area } OPQ \tag{10}$$

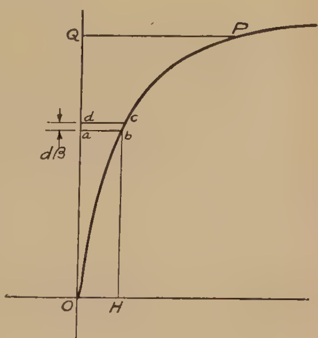


Fig. 18.

By applying this process to the complete hysteresis loop it becomes evident that the area of the loop is Fig. 19 is drawn a typical hysteresis loss per cycle, per $\overline{\text{cm.}}^3$ of iron. In directly proportional to the hysteresis loop in which the cycle of operation follows in sequence the curve $UPRSTDU$. At the instant in the cycle indicated by the intersection of the curve with the Y axis, at the point D, the magnetizing force is zero and

the line OD represents the residual magnetism in the iron. As the magnetizing force increases until $H = QP$ and the corresponding flux, $B = OB$, the energy expended in the magnetic field is proportional to the area $DUPQD$.

$$\int_D^Q HdB = \text{area } DUPQD \tag{11}$$

In the given cycle the magnetizing force then decreases to zero on reading the point R on the Y axis, and the line OR

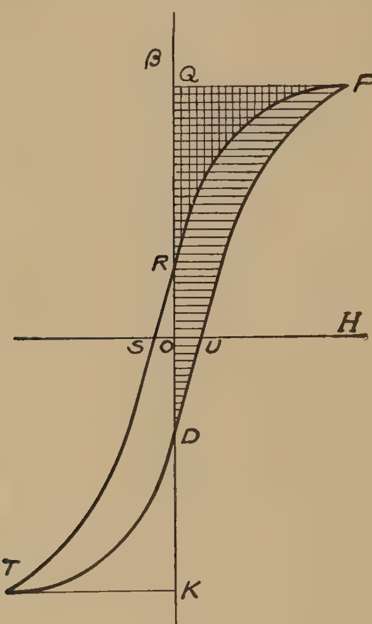


Fig. 19.

represents the residual magnetism. During this stage magnetic energy is returned to the electric circuit as represented by the area PQR.

$$\int_Q^R HdB = \text{area } PQR \tag{12}$$

Necessarily, the loss in energy is the difference in the amount expended while the magnetizing force was increasing from zero to its maximum value and the quantity returned to the electric circuit while the magnetizing force decreased to zero value.

The hysteresis loss during the half cycle is therefore represented by, or proportional to the area $DUPRD$ inside the loop.

$$\int_D^Q HdB + \int_Q^R HdB = \text{area } DUPRD \tag{13}$$

By applying the same process to the other half of the cycle, or from the symmetry of the figure, it is evident that the area of the hysteresis loop respresents the hysteresis energy loss.

$$\int HdB(\text{one cycle}) = \text{area of loop } DUPRSTD \tag{14}$$

To obtain numerical values in practical problems the following relations should be observed:

Let A = area of loop in $\overline{\text{cm.}}^2$ as obtained by planimeter.

B_1 = gausses (lines per $\overline{\text{cm.}}^2$) represented by 1 cm. on Y axis.

H_1 = gilberts represented by 1 cm. on X axis.

V = volume of iron in $\overline{\text{cm.}}^3$

f = frequency in cycles per second.

${}_hW$ = hysteresis loss in joules per cycle, per $\overline{\text{cm.}}^3$ of iron.

${}_hP$ = hysteresis loss in watts for V $\overline{\text{cm.}}^3$

$${}_hW = \frac{B_1H_1}{4\pi 10^7}A, \text{ joules per cycle, per } \overline{\text{cm.}}^3 \tag{15}$$

$${}_hP = \frac{B_1H_1}{4\pi 10^7}AfV, \text{ watts for given volume} \tag{16}$$

If the magnetizing force is expressed in ampere-turns the energy loss is given by the equations (17) and (18).

Let H_1' = ampere-turns represented by 1 cm. on X axis.

$${}_hW = \frac{B_1H_1'}{10^8}A, \text{ joules per cycle, per } \overline{\text{cm.}}^3 \tag{17}$$

$${}_hP = \frac{B_1H_1'}{10^8}AfV, \text{ watts for given volume} \tag{18}$$

As stated in a preceding paragraph Froelich's empirical equation gives an approximate relation between H and B for a part of the magnetization curve but is of little value for application to the complete hysteresis loop. If the hysteresis loop for any given sample is obtained by an oscillograph the area of the loop may be obtained by a planimeter. The hysteresis loss may then be found by comparing the area with a similar loop of known

numerical value or by calibrating the oscillograph. If plotted to the same scale the areas of hysteresis loops are directly proportional to the respective hysteresis losses. Thus the areas of the hysteresis loops in Fig. 5 and Fig. 6 represent, if plotted to the same scale, the relative hysteresis losses in the two samples of iron tested. Similarly, in Fig. 9 the areas of the two loops represent the relative hysteresis losses of permalloy and armco iron under the stated conditions.

(*b*) **Steinmetz' Equation.**—The relation expressed by equation (18) is of great importance in the design and operation of electrical machinery, but in order to apply this equation so as to obtain numerical values in practical problems it would be necessary to obtain the area of the hysteresis loop in each case. This would be a very laborious process and the actual selection of the most desirable material and flux density in the design of electric machines would become well-nigh impossible, as the area of the hysteresis loop varies not only for different materials but also with any change in the maximum flux density. The practical application of equation (18) for computing hysteresis losses was greatly simplified by Steinmetz' discovery of a simple relation that governs the areas of hysteresis loops. From extensive experimental data Steinmetz deduced an empirical equation which can be very readily applied to the solution of practical problems. Steinmetz showed,

(1) That for any given material the areas of the hysteresis loops are directly proportional to the 1.6th power of the maximum flux density,

(2) That for any given maximum flux density the areas of hysteresis loops for different materials are proportional to numerical constants, the coefficients of hysteresis loss.

Let ${}^{M}B$ = maximum flux density in gausses.
1.6 = Steinmetz' exponent of hysteresis loss.
η = coefficient of hysteresis loss.
f = frequency in cycles per second.
V = volume in $\overline{\text{cm.}}^3$ of iron or steel in the magnetic circuit.

Steinmetz showed that,

$$ {}_{h}W = \eta\,{}^{M}B^{1.6} \text{ ergs per } \overline{\text{cm.}}^3 \text{ per cycle} \tag{19}$$
$$ {}_{h}P = \eta f\,{}^{M}B^{1.6}10^{-7} \text{ watts} \tag{20}$$

The hysteresis coefficient η has different values depending on the material. Steinmetz found it to vary from 0.001 to 0.0055

in iron; from 0.0032 to 0.028 in cast steel; up to 0.08 for tungsten and manganese steels; while for silicon steels values even lower than 0.001 were obtained.

For certain grades of silicon steel (probably because of their non-homogeneous nature due to heavy surface scale on the steel sheets) the hysteresis loss is more closely approximated by using 1.7 instead of 1.6 as the exponent of the flux maximum density $_M B$.

Annealing, Aging, Recalescence, Demagnetization.—The hysteresis coefficient and the permeability depend not only on the chemical composition of the iron or steel but on other factors, especially that of heat treatment. By carefully *annealing* iron or steel the hysteresis loss may be greatly reduced. Hardened or tempered steels have greater hysteresis losses as well as greater retentivity or permeance.

The hysteresis coefficient for ordinary carbon steels gradually increases if continuously subjected to a temperature of about 100°C. This process is known as *aging*. Silicon steels having about 4 per cent of silicon, less than 0.1 per cent carbon and, practically free from sulphur and phosphorous, are only slightly affected by aging.

At about 750°C. both iron and steel lose their magnetic properties. For higher temperatures the permeability of iron and steel is unity, the same as for air. This effect is called *recalescence* and the temperature at which the iron or steel loses its magnetic properties is called the *recalescence point*. The magnetic properties are, however, regained by both iron and steel when the temperature is lowered below the recalescence point.

The residual magnetism in iron or steel that has been subjected to a magnetizing force is frequently the source of trouble, as, for example, in watch springs. From the hysteresis loop in Fig. 16, it is evident that the residual magnetism may be removed or the material *demagnetized* by impressing a magnetomotive force in the opposite direction and of just sufficient magnitude to bring the flux density B to zero value. A simpler and more practical method is to apply an alternating magnetic field which can be gradually decreased to zero. To demagnetize a watch place it in or near a solenoid carrying an alternating current and then slowly remove the watch from the magnetic field. As the watch is moved away the successive alternations of the magnetic flux passing through it become weaker and the residual magnetism is finally reduced to zero value. Likewise, if the watch

is brought near a permanent magnet and then rapidly rotated while it is removed from the field of the magnet, demagnetization will be effected.

Eddy Currents.—The lines of force produced by an alternating current move outward radially from the conductor while the current is increasing, and toward the conductor when the current decreases. While thus traversing the space surrounding the conductor carrying the current, the lines of force set up or induce electromotive forces in the conductor. If this voltage is induced in a conducting material like copper or iron, a local current will flow. The magnitude of this current will depend on the difference of potential induced in the conductor and the resistance in the path through which the current must flow. This induced current has also a magnetic field which, in turn, reacts upon the primary field. In solid masses of good conductors, such as iron or copper, the voltage induced would be considerable and the resistance small. As a result large currents would flow in the mass taking curved paths much the same as eddies in a stream. In most electrical apparatus eddy currents are undesirable, and the design must be so made as to reduce the losses from eddy currents to a minimum. This is accomplished by laminating the iron or using iron wire as conductor of the magnetic flux. This method reduces the possible difference in potential between any two points and also increases the resistance of the path through which the eddy currents must flow. Eddy currents are true electric currents but flow in comparatively very short circuits. The induced voltage causing the eddy currents is necessarily proportional to the frequency and flux density.

$$ {}_eE \;\alpha\; fB \tag{21}$$

By Ohm's law the resulting current is equal to the product of the voltage and conductivity λ of the circuit.

$$ {}_eI = \lambda\,{}_eE \;\alpha\; \lambda f B \tag{22}$$

The power is proportional to the product of the current and the voltage,

$$ {}_eP \;\alpha\; {}_eE\,{}_eI \;\alpha\; \lambda f^2 B^2 \tag{23}$$

Since fB is proportional to ${}_eE$, hence,

$$ {}_eP \;\alpha\; \lambda\,{}_eE^2 \tag{24}$$

The loss of power by eddy currents, therefore, is proportional to the square of the voltage and to the electric conductivity of the iron.

To find the power loss due to eddy currents, let:

ϵ = eddy-current coefficient,
V = the volume of the iron in $\overline{\text{cm.}}^3$,
f = cycles per second,
B = flux density in gausses or lines per $\overline{\text{cm.}}^2$

Dividing the expression for power in equation (23) by the frequency gives the energy per cycle per $\overline{\text{cm.}}^3$

$$_{\epsilon}W = \epsilon\lambda f B^2 \text{ ergs per cycle per centimeter} \tag{25}$$
$$_{\epsilon}P = \epsilon\lambda V f^2 B^2 10^7 \text{ watts} \tag{26}$$

As the quality of the iron is included in the conductivity factor, λ, the coefficient of eddy currents ϵ depends only upon the shape of the iron parts in the magnetic circuit. In commercial apparatus thin plates or sheets and wires are used, and the coefficient depends upon the thickness of the sheet, or upon the diameter of the wire.

For thin plates of thickness, d the eddy current coefficient $\epsilon = 1.645d^2\ 10^{-9}$ in ohms per $\overline{\text{cm.}}^3$ The conductivity λ, varies within the limits of 10^4 to 10^5.

In direct-current machinery eddy currents occur whenever there are alternations, fluctuations, or any movement of the magnetic flux lines with respect to the metal parts, as, for example, in the armature and pole tips of direct-current generators and motors.

Magnetic Flux Measurements.—It is much more difficult to measure magnetic flux than to obtain quantitative values of electric currents. Magnetic flux represents stored energy while the current relates to the dynamic flow of energy in electric circuits. Magnetic flux is measured either by means of permeameters as illustrated by the Fahy type in Chap. VIII, or using a ballistic galvanometer and a standard solenoid. If measured by a permeameter the sample to be tested must have the dimensions required for the type of instrument used. If measurements are made by means of a ballistic galvanometer the sample to be tested is in the form of a ring or toroid. A circuit diagram for measuring flux by means of a ballistic galvanometer and standard solenoid is shown in Fig. 20.

In Fig. 20, G represents the ballistic galvanometer; AB the standard solenoid having N_1 turns in the primary winding and N_2 turns in the secondary; EF the primary and HJ the secondary

windings on the ring sample to be tested; L battery and S_2 reversing switch for primary circuit of the toroid to be tested.

In Chap. X it is shown that the flux in the center section of the standard solenoid is given by equation (27), in which N_1 is the number of turns, I_1 the current in amperes, l the length of the solenoid in centimeters and A the cross-sectional area in $\overline{\text{cm.}}^2$

$$\phi_o = \frac{0.4\pi N_1 I_1 A}{l} \text{ maxwells} \tag{27}$$

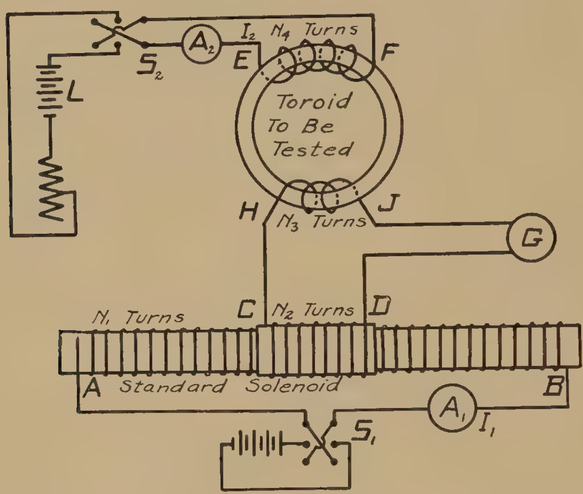


FIG. 20.—Circuit diagram for measuring magnetic flux.

Hence the flux linkages with coil N_2 in series with the galvanometer G and the coil HJ on the ring is given by equation (28).

$$\phi_o N_2 = \frac{0.4\pi N_1 N_2 I_1 A}{l} \tag{28}$$

The induced voltage produced by the changes in interlinkage or cutting of lines of force when the current I_1 is passed through the standard solenoid, by closing the switch S_1 causes a current to flow through the galvanometer. Let the deflection of the galvanometer be d degrees. Similarly, by closing the switch S_2 the current flowing in the primary EF of the ring will produce lines of force cutting or interlinking HJ, the secondary winding of the ring, which causes a current to flow through the galvanometer G. Let, in this case, the deflection of the galvanometer be d^1 degrees.

Since the deflection of the galvanometer is proportional to the current passing through it, and this in turn to the induced voltage, and therefore to the respective change in flux interlinkages as expressed by equation (29),

$$\frac{\phi^1 N_3}{\phi_o N_2} = \frac{d^1}{d} \tag{29}$$

$$\phi^1 = \frac{d^1 N_2}{d N_3}\phi_o = \frac{0.4\pi d^1 N_1 N_2 I_1 A}{d N_3 l} \tag{30}$$

Hence, the total flux in the ring tested is obtained from the ratio of the two deflections d and d^1, the current I_1, the number of turns in N_1, N_2, and N_3, the length l and the cross-section A of the standard solenoid.

It should be noted that the galvanometer circuit is the same whether the induced voltage is produced by changing the current in the standard solenoid or in the ring to be tested. Hence, the deflections of the galvanometer are directly proportional to the two flux-interlinkages $\phi_o N_2$ and $\phi^1 N_3$. Of these quantities N_2 and $\phi' N_3$ are known and ϕ_o can be computed from the dimensions of the standard solenoid.

The m.m.f. impressed on the ring magnetic circuit is the product of the current I_2 and number of turns N_4 of the primary current. Hence the reluctance of the ring is obtained by Ohm's law, as expressed by equation (30).

$$\mathcal{R} = \frac{0.4\pi N_4 I_2}{\phi^1}\text{oersteds} \tag{31}$$

The reluctance of the magnetic circuit is also given by the length l, cross-section A, and the permeability μ for the given flux density.

$$\mathcal{R} = \frac{l}{\mu A} \tag{32}$$

Letting the length l be the average or mean length of the lines of force in the ring, and A the uniform cross-section, the permeability of the iron for the given flux density may be obtained from equation (32).

It is evident that by means of a standard solenoid the ballistic galvanometer can be calibrated, since the deflections are directly proportional to the induced voltages. The calibration of the galvanometer consists in determining the constant ratio between a unit deflection on the scale to the change of one flux line inter-

linking the galvanometer circuit. By taking readings for several values of current and the corresponding galvanometer deflections, the galvanometer constant can be determined to a high degree of accuracy.

To determine the magnetization curve and hysteresis loop for any given sample of steel the step-by-step method is generally used. The circuit diagram is shown in Fig. 21. The range in the magnitude of the current I_2 is controlled by the resistance

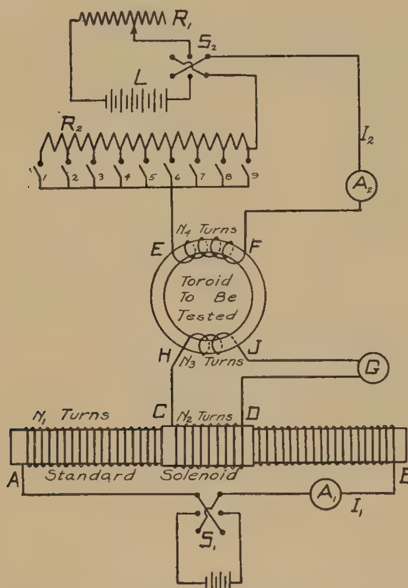


FIG. 21.—Circuit diagram for obtaining hysteresis loops.

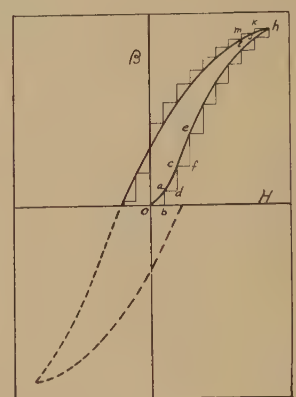


FIG. 22.—Hysteresis loop, step-by-step method.

R_1, while the size of the steps by which the current is successively increased or decreased is determined by the several sections of the R_2 rheostat.

The process is readily understood by reference to the hysteresis loop in Fig. 22 in connection with changes in current, as indicated by the circuit diagram in Fig. 21.

By closing switch No. 1 in R_2 and also the reversing switch S_2 a small current flows through the primary of the test ring. Operating the reversing switch three or four times will remove any residual magnetism in the test ring.

In Fig. 22 lay off on the H axis the length o-b representing to scale the magnetization force of N_4I_2 ampere-turns. At b erect

an ordinate ba representing the flux density for the deflection d. The point a is, therefore, on the magnetization curve of the test specimen.

By closing switch No. 2 the current is increased to I_2 and hence the magnetizing force is increased by $N(I_2 - I_1)$ ampere-turns. Let d_2 be the deflection of the galvanometer. Along the H axis lay off bd to represent the increase in field strength, $N(I_2 - I_1)$ ampere-turns and on the ordinate the length dc equivalent to the galvanometric deflection d_2. It is evident that c is a point on the magnetization curve. Repeating the process by closing successively switches Nos. 3, 4, 5, etc. a series of points are obtained through which the continuous magnetization curve may be drawn. After the maximum flux value desired has been reached the current is decreased in successive steps and readings taken of the magnetizing current and the galvanometer deflection for each change in the current. Plotting the values thus obtained as jk, lm, etc. (Fig. 22) a series of points on the hysteresis loop will be located. When the current has been reduced to zero the reversing switch S_2 is thrown. The process of increasing the magnetization current in steps in the opposite or negative direction is then continued until the same maximum value is reached as in the positive direction. For obtaining points in the lower side of the hysteresis loop the magnetization current is decreased in steps from the maximum value in the negative direction until zero value is reached. The switch S_2 is then reversed and the step-by-step process is continued until the previous maximum value of the current, in the positive direction, is obtained. A continuous curve drawn through the series of points located by the above step-by-step process forms the complete hysteresis loop as illustrated in Fig. 22.

Cut-and-try Method for Solving Magnetic Circuit Problems.—In general, magnetic-circuit problems relate to finding the quantitative relation, under given conditions, of the magnetomotive force to the total flux or the flux density in the several sections of the magnetic circuit. For airgaps and other sections of non-magnetic materials the permeability is constant ($\mu = 1$) and the desired relation is obtained by direct application of Ohm's law to the magnetic circuit. In sections composed of iron or steel or other magnetic materials the permeability depends on the flux density and this relation is so complex that it cannot be fully expressed by a mathematical equation. Hence, since the reluct-

ance of the circuit depends on the permeability and therefore on the flux density, it is evident that the quantitative value of the m.m.f. to flux density relation cannot be obtained by the application of Ohm's law, even if the permeability-flux density curve were known, which is seldom the case.

Solutions for magnetic-circuit problems are obtained by the *cut-and-try method* in applying Kirchhoff's laws and using data from the magnetization curves for the materials of which the magnetic circuit is composed. The procedure is illustrated by the following examples:

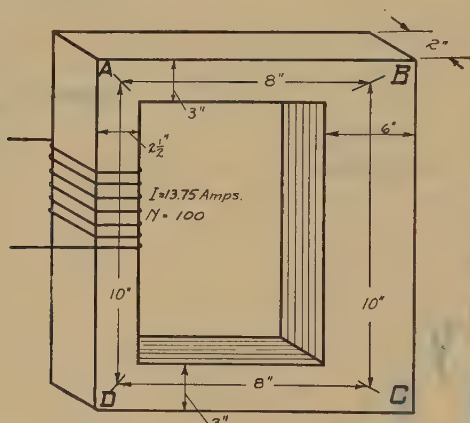


FIG. 23.

Example 1.—Given a magnetic circuit of cast steel with dimensions as shown in Fig. 23. Find the total flux and the flux density in each section of the magnetic circuit when 13.75 amp. flow in the coil of 100 turns wound around the core AD.

First, assume B_{BC} = 45,000 lines per square inch for the section BC, Fig. 23.

NI_{BC} per inch = 10 ampere-turns (from magnetization curve in Fig. 3)

NI_{BC} total = 100 ampere-turns

ϕ = 45,000 × 12 = 540,000 lines. mag. flux ϕ

$$B_{AB} = \frac{540{,}000}{6} = 90{,}000 \text{ lines per square inch}$$

NI_{AB} per inch = 54 ampere-turns (magnetization curve Fig. 3).

NI_{AD} total = 432 ampere-turns

$NI_{DC} = NI_{AB}$ = 432 ampere-turns

$$B_{AD} = \frac{540{,}000}{5} = 108{,}000 \text{ lines per square inch}$$

NI_{AD} per inch = 210 ampere-turns (magnetization curve Fig. 3).

NI_{AD} total = 2,100 ampere-turns

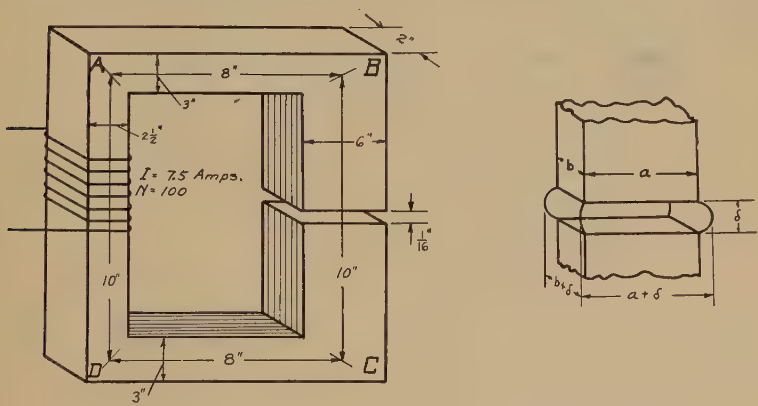
NI_{ABCD} total = 100 + 432 + 432 + 2,100 = 3,064 ampere-turns.

The impressed ampere-turns are, however, only 1,375 and hence a new value for B_{BC} must be assumed and the process repeated.

Second, assume B_{BC} = 40,000 lines per square inch.

$$
\begin{aligned}
NI_{BC} \text{ per inch} &= 9 \text{ ampere-turns (magnetization curve Fig. 31).}\\
NI_{BC} &= 90 \text{ ampere-turns}\\
\phi &= 40{,}000 \times 12 = 480{,}000 \text{ lines}\\
B_{AB} &= 80{,}000 \text{ lines per square inch}\\
NI_{AB} \text{ per inch} &= 31 \text{ ampere-turns (magnetization curve Fig. 3).}\\
NI_{AB} &= 248 \text{ ampere-turns}\\
NI_{DC} &= 248 \text{ ampere-turns}\\
B_{AD} &= 96{,}000 \text{ lines per square inch}\\
NI_{AD} \text{ per inch} &= 80 \text{ ampere-turns (magnetization curve Fig. 3).}\\
NI_{AD} &= 800 \text{ ampere-turns.}\\
NI_{ABCD} &= 90 + 248 + 248 + 800 = 1{,}386 \text{ ampere-turns.}
\end{aligned}
$$

This result is within 1 per cent of the impressed m.m.f. and should be considered as a satisfactory solution as it is well inside the degree of accuracy that can be expected. Variations in the material, as compared to the sample from which the magnetization curve was obtained, may well vary up to 3 or 4 per cent and hence the actual flux in the circuit will be subject to a like variation from the computed values.



(*a*) Magnetic circuit with airgap. (*b*) Magnetic fringing.

FIG. 24.

Example 2.—Assume that an air gap of 1/16 inch is cut across the core BC in Fig. 23 as illustrated by Fig. 24 (*a*). The fringing of the magnetic lines will cause the flux density in the air gap section to be less than in the core BC since the same total flux passes through all sections of the magnetic circuit. The effect of the fringing is assumed to be equivalent to an increase of the air gap cross-section dimension by a length equal to that of the airgap. This is illustrated in Fig. 24 (*b*) in which S represents the length of the airgap. For airgaps in which S is fairly large in proportion to the cross-sectional area the effect of fringing is more nearly equivalent to an increase of $2S$ in the dimensions of the airgap cross-section.

The method of solution of the circuit in Fig. 24 (b) is similar to the process used in Example 1. The cross-section of the airgap, as indicated in Fig. 24 (b) is obtained by adding δ, the length ofthe air gap, to the cross-sectional dimensions. Since the permeability is unity the flux density is readily found. The ampere-turns m.m.f. required for the airgap may be obtained by the direct application of Ohm's law.

$$B\text{ (lines per } \overline{\text{cm.}}^2) = \frac{0.4\pi N(\text{turns})I(\text{amperes})}{l(\text{centimeters})}$$

$$NI = \frac{Bl}{0.4\pi}$$

$$= 0.796B(\overline{\text{cm.}}^2)l(\text{centimeters})$$

$$= 0.313B(\text{square inch})l(\text{inches})$$

The process for the other sections of the circuit is the same as described under Example 1. The ampere-turns for the airgap are then added to the m.m.f. required in the other sections in the magnetic circuit to find the total m.m.f.

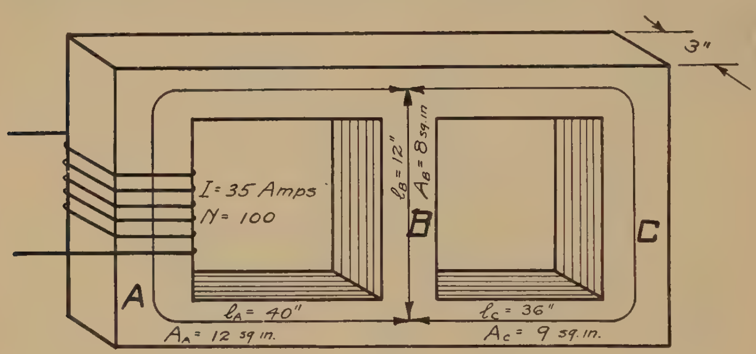


FIG. 25.

Example 3.—Let the problem be to find the flux density in all sections of the magnetic circuit shown in Fig. 25. All parts of the magnetic circuit in Fig. 25 are made of cast iron. Let the cross-sectional areas of the legs A, B, and C (Fig. 25) be 12, 8, and 9 sq. in., respectively. The length of each path is shown on the diagram in Fig. 25. Let there be 100 turns in the magnetizing coil through which a current of 35 amp. is flowing.

First, assume B_C = 20,000 lines per square inch
NI_C per inch = 24 ampere-turns (magnetization curve Fig. 3).
$NI_C = NI_B$ = 864 ampere-turns.
NI_B per inch = 72 ampere turns.
B_B = 38,000 lines per square inch (from Fig. 3).
$\phi_A = \phi_B + \phi_C$ = 484,000 lines.
B_A = 40,333 lines per square inch
NI_A per inch = 81 ampere-turns (from Fig. 3).
NI_A = 3,240 ampere-turns.
NI total $= NI_A + NI_B = NI_A + NI_C$
$= 3{,}240 + 864 = 4{,}104$ ampere-turns.

Since the impressed m.m.f. is only 3,500, the assumed value of 20,000 lines per square inch for B_C is too large. A somewhat lower value for B_C should be assumed and the process repeated.

PROBLEMS

1. Find the constants in Froelich's equation for the magnetization curve of cast steel in Fig. 2. Between what limits of flux density does the equation hold?

2. Find the constants in Froelich's equation for the saturation curve of a generator at 1,000 r.p.m. as given in Problem 1, Chap. XV.

3. The hysteresis loop of a sample of iron is plotted on cross-section paper and found to have an area of 9.64 sq. in. An ordinate of 1 in. is equivalent to 25,000 lines per square inch and an abscissa of 1 in. to 50 ampere-turns per inch. Find the hysteresis loss per $\overline{\text{cm}}.^3$ per cycle.

4. A transformer has an eddy current loss of 246 watts and a hysteresis loss of 116 watts at a frequency of 25 cycles and a flux density of 15,000 per $\overline{\text{cm}}.^2$.

(*a*) What is the eddy current and hysteresis loss at 60 cycles?

(*b*) Find the eddy current and hysteresis loss if the frequency remains at 25 cycles but the flux is changed to 20,000 per $\overline{\text{cm}}.^2$.

(*c*) Find the eddy current and hysteresis losses if both frequency and flux density are changed simultaneously.

5. The core loss of a transformer is 480 watts at 60 cycles. When the flux and frequency are both reduced to one-half the core loss is 60 watts. Find eddy current and hysteresis losses at each frequency.

6. If the flux density in the core *AB* of Fig. 23 is 75,000 lines per square inch, find the current in the exciting coil of 100 turns. Core material is cast steel.

7. An airgap ⅛ in. long is cut in the core *BC* of the magnetic circuit shown in Fig. 23. Find the flux in all parts of the circuit if 15 amp. flow in the winding of 100 turns. Take fringing into account.

8. A magnetic circuit having the same dimensions as Fig. 25 is made up of annealed sheet steel of which the stacking factor (ratio of actual metal area to total area in cross-section) is 0.9. Find the flux density in all three parts. *A*, *B*, and *C*, if the coil of 100 turns on *A* carries a current of 8 amp.

9. Repeat Problem 8 when *C* has an airgap of 0.2 in. and the current in the coil on *A* is increased to 30 amp. Effect of fringing included.

10. A magnetic circuit of cast steel has the same dimensions as in Fig. 25. A ⅛ in. airgap is cut in leg *A*. A coil of 2,000 ampere-turns is wound around leg *B*. Find the flux density in all parts of the magnetic circuit. Take account of fringing.

CHAPTER XII

THE DYNAMO

The *dynamo* is a machine for converting mechanical energy into electrical energy, or conversely, for transforming electrical energy into mechanical work. If used for the first purpose dynamos are called *generators*, but if employed in the latter process they are called *motors*. The design of the dynamo, whether used as generator or motor, is based on two fundamental principles, or well-established physical relations:

(*a*) That a mechanical reaction or force exists between a magnetic field and a conductor carrying an electric current;

(*b*) That voltage is generated in a conductor cutting lines of force in a magnetic field.

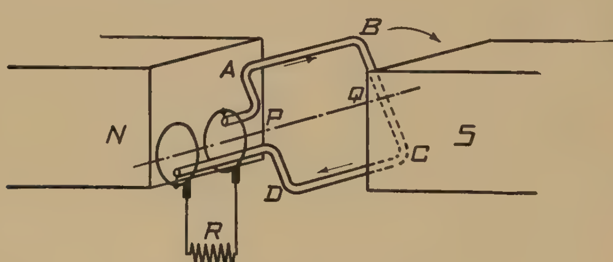


FIG. 1.—Elementary dynamo.

The dynamo was invented in 1831 by Faraday, who constructed a simple disc machine which was used in connection with his epoch-making investigations on the laws of electromagnetic induction. The dynamo, however, did not become a commercially practical machine until about 40 years later.

In Fig. 1, let a conductor $ABCD$ in the form of a loop be placed in a magnetic field NS. Let the loop be attached to a shaft PQ so that it may rotate in the magnetic field. During each complete revolution of the loop each of the conductors AB and CD cut all the lines of force twice that pass through the loop. At any position of the loop the voltages generated in the two sides of the loop, by cutting of the lines of force, are additive in the

electric loop circuit; that is, the voltage between A and D is the sum of the voltages induced in AB and CD. Let the conductors be connected through a resistance R (Fig. 1) to complete the electric circuit. Assume a rotation at constant speed in the clockwise direction. In order to more readily study the reaction in the machine a cross-section of the magnetic field and the loop in a vertical position is shown in Fig. 2; similarly, for the loop in a horizontal position in Fig. 3.

In Chap. V it was shown that the voltage generated in a conductor moving in a magnetic field is directly proportional to the rate of cutting lines of force. If at any instant the conductor moves parallel with the lines of force, as in Fig. 2, no voltage is generated; while the maximum voltage will be produced when the conductor moves at right angles to the lines of force, as in Fig. 3. With the direction of the magnetic lines of force and of rotation

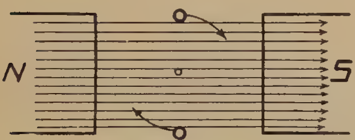


Fig. 2.

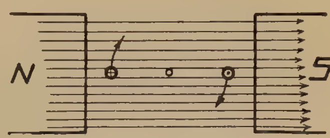


Fig. 3.

of the loop as shown by arrows in Fig. 3, the flow of the current will be in the direction indicated by the dot and cross-mark. The reaction between the magnetic lines of force produced by the current in conductor A or conductor B against the stationary magnetic field produces a mechanical force opposite in direction to the applied torque that causes the loop to rotate. The mechanical work applied in making the loop rotate is therefore transformed into electrical energy in the electric circuit. The moving element is the *armature* of the dynamo while AB and CD are *armature conductors*.

It is evident that the diagrams in Figs. 1 to 3 of the elements of the dynamo as a generator, may be used to illustrate the basic features of the electric motor. In all dynamos, whether operating as generators or motors there is a mechanical force between the magnetic field and the electric current in the armature conductors.

If a dynamo is used as a motor and the direction of both the armature and field currents remain unchanged, then the direction of rotation is reversed as compared to the rotation if used as a generator. This necessarily follows from the reversal of the flow

of energy. When used as a generator mechanical work is applied to the armature forcing it to revolve against the direction of the reaction of the magnetic field, while if used as a motor the armature necessarily rotates in the direction of the force exerted by the magnetic field.

In the transformation of energy in the dynamo, whether used as a generator or a motor, part of the energy will be dissipated as heat due to friction, to the resistance loss, etc. Since energy cannot be destroyed or created, the transformations produced in the dynamo may be expressed by the general energy equations (1) and (2).

The dynamo as motor:

Input (mechanical energy) − energy losses in the machine (friction, windage, RI^2 losses, etc.) = output (electrical energy) (1)

The dynamo as motor:

Input (electrical energy) − energy losses in the machine (friction, windage, RI^2 losses, etc.) = output (mechanical energy) (2)

In either case the dynamo operates on the basic principle discovered by Oersted in 1820. That is, if a conductor carrying a current be placed in a magnetic field a mechanical force is exerted on the conductor by the interaction of the magnetic field and the electric current.

The magnitude of the mechanical force depends on the flux density of the magnetic field, the length of the conductor, the current flowing in the conductor, and the angular inclination of the conductor to the direction of the lines of force in the field, as expressed by equation (3).

$$F = \frac{BlI \sin \theta}{10}. \tag{3}$$

F = force exerted on conductor in dynes.
B = magnetic field flux density in gausses.
l = length of conductor in centimeters.
I = electric current in amperes.
θ = angular inclination of conductor to magnetic lines of force, in degrees.

Mechanically the dynamo machine consists of two distinct sections: the *armature* and the *field*. The more important parts

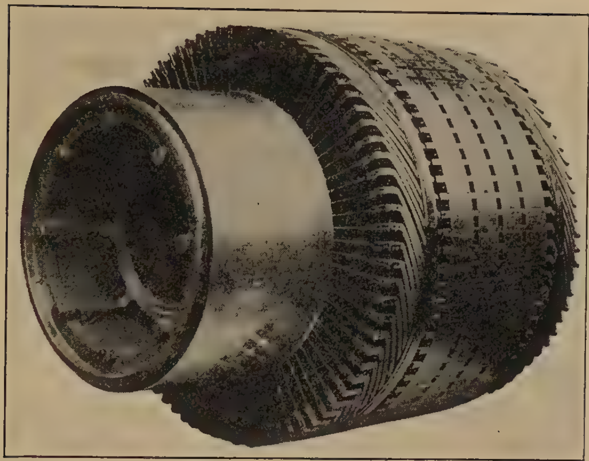


FIG. 4.—Dynamo armature. (*General Electric Company.*)

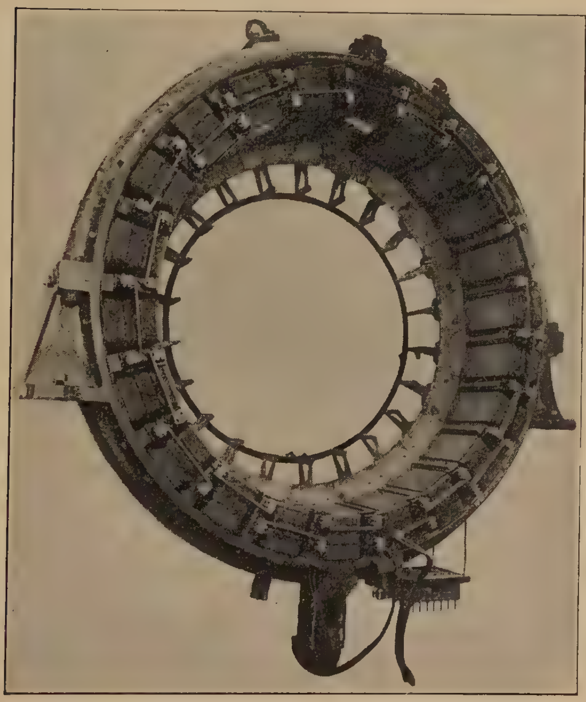


FIG. 5.—Dynamo field. (*General Electric Company.*)

of the armature (Fig. 4) are *armature core, armature conductors,* and *commutator*. The principal parts of the field (Fig. 5) are the *yoke*, the *pole cores*, the *pole shoes*, and the *field windings*. Other integral parts of the dynamo are the brushes, brush holders, rocker arm, spider, shaft, bearings, frame, bedplate, etc.

Electrically the dynamo consists of an electric circuit or circuits interlinked with a magnetic circuit or circuits, and so constructed that the armature conductors can move with respect to the magnetic field.

FIG. 6.—Electric circuit of a dynamo. Ring type.

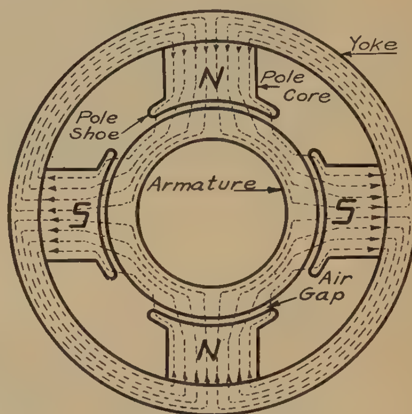


FIG. 7.—Magnetic circuit of a dynamo. Ring type.

The *electric circuit* (Fig. 6) consists of the armature conductors, the commutator, and the field winding (the latter not shown). The *magnetic circuit* (Fig. 7) may be traced by starting at some point as in the yoke, then passing through the north pole core, north pole shoe, airgap, armature core, airgap, south pole shoe, south pole core and back to the starting point in the yoke. The space between a pole shoe and the armature core is called the *airgap*.

The essence of the dynamo action is the voltage generated and force produced by the relative motion of the armature conductors with respect to the magnetic field. The basic action is precisely the same whether the field is stationary and the armature rotates or the field revolves while the armature remains stationary. In alternating-current systems, small generators generally have stationary fields and rotating armatures, while in larger units the armatures are stationary and the fields rotate. In direct-current systems the problem of producing satisfactory com-

mutation limits the design to the first form, so that practically all direct-current generators, as well as motors have stationary fields and rotating armatures.

The Yoke.—In dynamos of the *rotating armature-stationary field* type the yoke has two distinct functions: (*a*) to provide a path of low reluctance for the magnetic flux from pole to pole, and (*b*) to serve as mechanical support to the poles by forming the frame of the machine.

The yokes are made of cast iron, cast steel, or rolled steel. In small machines the yoke and bedplate are cast as a unit. In moderate sizes the yoke is split on the horizontal diameter for convenience in assembling and repairing. In larger sizes the dynamo frame, of which the yoke is the essential part, is fabricated from rolled-steel sections. In general, the reluctance requirements of the magnetic circuit determines the cross-section area of the yoke as in most designs this will provide ample mechanical strength. The yoke flux density in modern machines is in the neighborhood of 80,000 lines per square inch.

The Poles.—The pole is generally considered as consisting of two parts: the *pole core* and the *pole face* or *pole shoe*. The field winding is placed around the pole core, which is attached by bolts to the yoke. The pole face is on the end nearest the armature. The poles are generally made of laminated sheet steel; the laminations being about 0.025 in. in thickness. In modern designs both the core and the face are formed as a single unit by the same sheet-steel punchings. If solid cores are used a laminated shoe should be provided and attached to the core in order to prevent excessive heating in the pole tips. This increases the reluctance in the magnetic circuit so that the advantage of requiring less copper in the field windings is overbalanced, and therefore practically all modern direct-current dynamos have laminated poles.

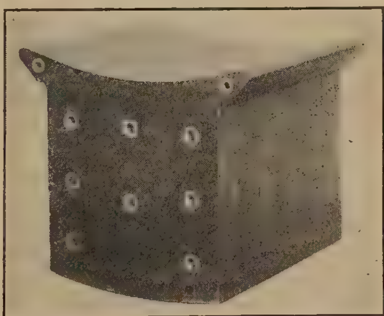


Fig. 8.—Pole core. (*General Electric Company.*)

In order to reduce the reluctance of the magnetic circuit and to provide a more nearly uniform distribution of the magnetic

flux across the airgap, the end of the pole, nearest to the armature is spread out, so that the pole face has a considerably larger cross-section area than the pole core. Thus in direct-current dynamos the flux density in the airgap is from 45,000 to 60,000 lines per square inch while that of the core may be even more than 100,000 lines per square inch.

The Armature Core.—The armature core forms part of the magnetic circuit and provides mechanical support for the armature conductors. Since the armature conductors and core are in motion relative to the field flux the armature core must be

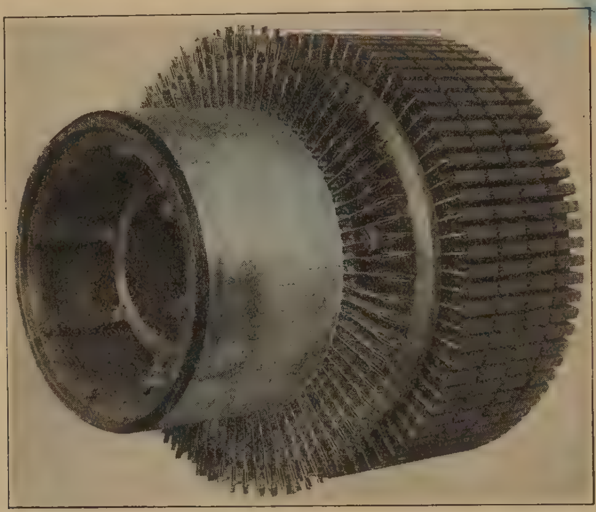


FIG. 9.—Armature core and commutator. (*General Electric Company.*)

laminated so as to reduce the otherwise excessive eddy currents. It is evident that if the core were made of a single solid piece of steel considerable voltage would be generated when rotated in the field of the dynamo.

The voltage generated in the core would be in direction at right angles to the lines of force as well as to the direction of rotation; that is, parallel with the armature conductors. Necessarily the laminations of the armature core must be at right angles to the direction of the generated voltage. Therefore, in the usual radial-pole type of machine the armature core (Fig. 9) is built up of circular sheet steel punchings insulated from each other so as to prevent the eddy currents from flowing between the laminations. The loss due to eddy currents is directly propor-

tional to the square of the thickness of the laminations. To reduce the heat loss and still keep the required cross-sectional area of the iron in the magnetic circuit the thickness generally used for armature core laminations is about 0.014 in. Core punchings are generally keyed directly to the shaft, or if composed of segments, attached to the spider by means of a dovetail joint. The heat formed by eddy currents is removed through ventilating ducts formed by spacing pieces placed at intervals along the axis of the armature.

On the surface of the armature core, grooves or *slots* are cut in which the armature conductors are imbedded. This construction serves two important purposes:

(*a*) It decreases the thickness of the airgap, which directly reduces the reluctance of the magnetic circuit;

(*b*) It provides the necessary mechanical support for the armature conductors.

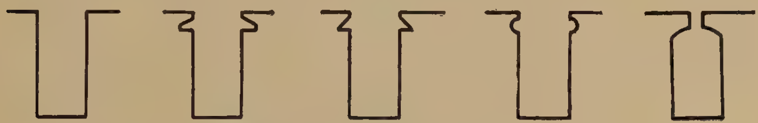


FIG. 10.—Typical teeth and slots.

The projections between the slots are called the *teeth* of the armature core. Several typical shapes of teeth and slots are shown in Fig. 10.

The Commutator.—In any given armature conductor the generated voltage reverses in direction as the conductor passes successively magnetic poles of opposite polarity. Hence the current in the armature conductors is alternating; making a complete cycle for each pair of poles passed by the given conductor. By connecting several properly spaced conductors in series and providing the armature with a commutator and brushes the alternating current generated in the armature is converted into a direct current in the load circuit.

A two-pole dynamo having a ring-wound armature with commutator and brushes is shown in Fig. 11. In the ring-wound armature the armature core is in the form of a torus. The lines of force coming from the north pole pass through the airgap and the outside layer of conductors on the left side of the armature, then follow the armature core and pass out of the armature through the conductor layer and airgap on the right side to the

south pole. Hence the conductors on the outside of the ring cut the field flux while the lines of force do not cross the conductors on the inside of the ring armature. All the conductors on the left side, moving across the face of the north pole, will have current generated in the same direction as indicated by the arrows. Similar action is produced in the conductors passing in front of the south pole. If the brushes are located midway between the poles, *current from both sides of the armature* will flow into the positive brush and pass through the outside load resistance and return to the negative brush, thus completing the circuit. The brushes are stationary and, as the armature revolves, make con-

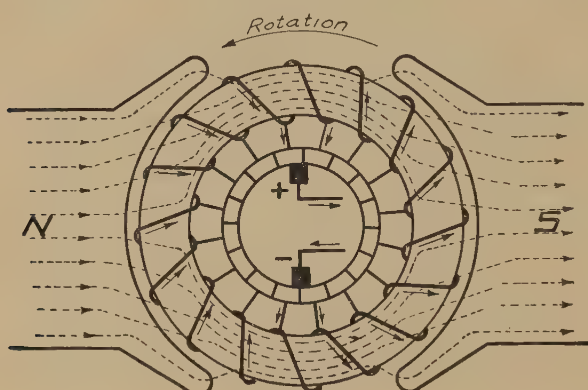


FIG. 11.

tact with successive segments of the commutator; thus connecting the external or load circuit to the armature continuously at the two points having the greatest difference in potential. Hence, by means of the commutator and sliding contact brushes the alternating currents flowing in the armature conductors are converted to a direct current in the external or load circuit.

In alternators collector rings are used in place of the commutator in the direct-current generator. The alternating currents generated in the armature of the alternator are transmitted as such through the collector rings, the brushes, and the load circuit.

The commutator is made up of wedge-shaped segments of hard-drawn or drop-forged copper, insulated from one another by thin sheets of mica. The voltage between the adjacent commutator segments is in most designs only from 10 to 15 volts.

The connections of the commutator segments to the armature conductors will be described under armature windings, Chap. XIII.

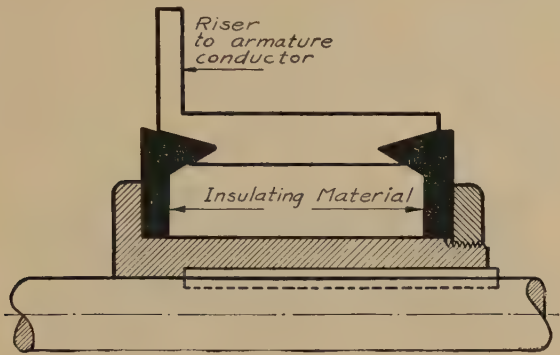


Fig. 12.—Sectional view of commutator.

Brushes, Brush Holders, Rocker Arm.—The brushes, through which the electric circuit in the revolving armature is connected to the external circuit, are made out of a composition consisting chiefly of graphitic carbon. In small low-voltage machines the

Fig. 13.—Brush and brush holder. (*Westinghouse Electric and Manufacturing Company.*)

brushes are sometimes made of copper. Carbon brushes are manufactured in several degrees of hardness to meet the requirements for sparkless commutation of dynamos differing in design. The graphite serves both as a conductor of the current and as lubricant for the sliding contact on the commutator.

The brushes are usually set at an angle, trailing with respect to the direction of rotation. In machines designed to operate in either direction, however, as railway motors, the brushes are set radially with the commutator.

Each of the brushes is held in position by *brush holders* (Fig. 13) which in turn are attached to the *rocker arm* or *rocker ring* (Fig. 14). The brushes are held in contact with the commutator by an adjustable spring so attached to the holder that the spring

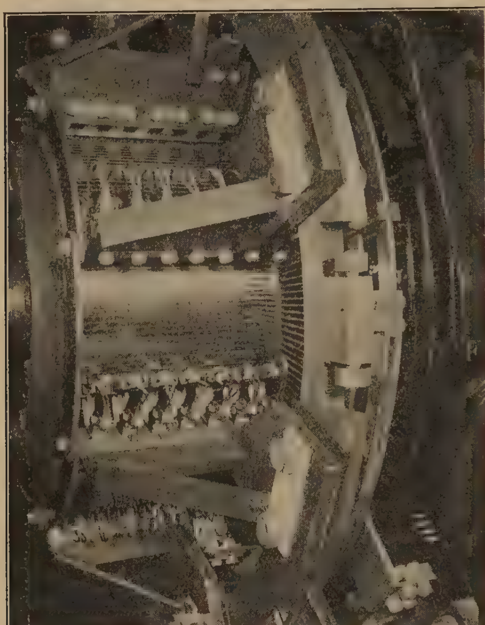


Fig. 14.—Brush rigging. (*Westinghouse Electric and Manufacturing Company.*)

does not carry any current. The carbon brushes are generally copper plated at the outer end to insure good contact with the *pig tail*, a braided copper band, which provides electrical connection with the brush holder.

Field Excitation.—In magnetos and other small dynamos used for special purposes as for ringing call bells in small telephone systems, operating igniters in gas engines, etc., the magnetic field consists of permanent magnets. Aside from these relatively unimportant exceptions the magnetic fields in all dynamos whether used as generators or motors are electromagnetic.

The arrangement of the field windings give rise to the following groupings or types of field excitation.

1. Separate excitation.
2. Self-excitation.
   (*a*) Series excitation.
   (*b*) Shunt excitation.
   (*c*) Compound excitations.

Separate Excitation.—As implied by the title, the field excitation in this type is produced by an electric current coming from a source outside the machine itself, usually either a storage battery or an exciter generator. Alternators and synchronous motors are separately excited. Practically all direct-current dynamos are self-excited. As an exception may be mentioned

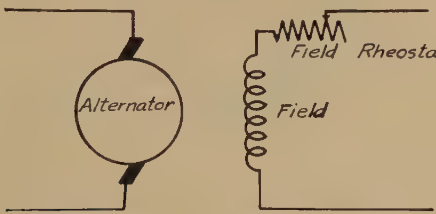


FIG. 15.—Alternator. Illustration of separate excitation.

low-voltage generators used for electroplating; for which a higher voltage than the machine generates is necessary for the field current.

Self-Excitation.—The principle of self-excitation of dynamos was discovered by Werner Siemens in 1867. The armature current, either all or part of it, flows through the field windings, thereby causing the dynamo to produce its own magnetic field. This discovery proved to be of very great practical importance and made possible the rapid expansion in the use of the dynamo for commercial purposes. Self-excited dynamos may be divided into three groups on the basis of the field-winding connection; namely *series*, *shunt*, and *compound* excitation.

Series Excitation.—In a series-excited dynamo all the current in the external circuit passes through the field windings. The circuit connections for a four-pole, ring type, series-excited dynamo are shown in Fig. 16. The corresponding conventional circuit diagram is shown in Fig. 17. The series-field excitation is used for electric railway motors and for motors having load requirements of a similar character. Large series generators are

used in the Thury system (Chap. XV) but aside from this important type of development there are very few series-connected direct-current generators in commercial operation. In the early development of electric lighting, series generators were used to

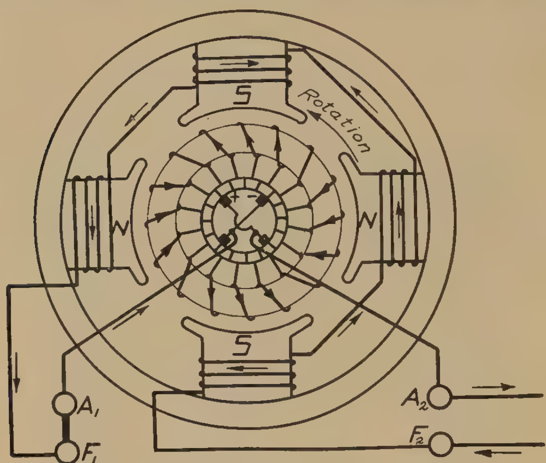


Fig. 16.—Four-pole, series-excited dynamo.

provide current for the now obsolete direct-current arc lamps. The series-connected generator is in principle a constant-current machine while modern power systems are designed for constant-potential operation.

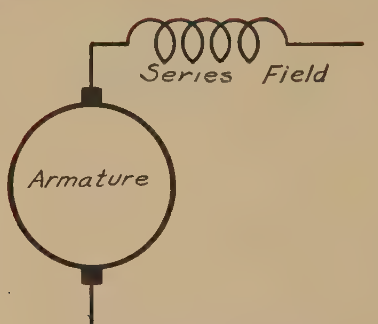


Fig. 17.—Circuit diagram for series-excited dynamo.

The field winding of series machines has relatively few turns of heavy wire. The characteristic features of the design and operation of series dynamos can be studied to best advantage in connection with series railway motors as it is in the electric rail-

way field this type of machine has its most important commercial application (Chap. XV).

Shunt Excitation.—In the shunt-connected dynamo only a part of the available armature current is used for field excitation. The field winding is connected in parallel with the main load circuit and the field excitation is regulated by means of a special field rheostat.

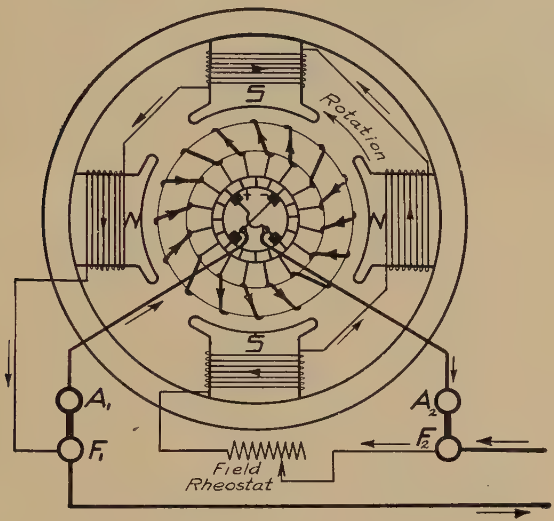


FIG. 18.—Four-pole, shunt-excited dynamo.

The circuit connections for a four-pole, ring-type, shunt-excited dynamo is shown in Fig. 18 and the corresponding conventional circuit diagram in Figure 19. From the circuit diagrams in Figs. 18 and 19 it is evident that the exciting current depends on the difference of potential between the brushes, the resistance of the field winding and the adjustable resistance of the field rheostat. For any given generator the resistance of the field winding is constant except for slight variations due to change in temperature. To regulate the field current a rheostat whose resistance can be varied is inserted in series with the field winding. From the circuit diagram it is evident that the *voltage generated* by the shunt dynamo is independent of the load current, except for the effect produced by the so-called *armature reaction*, discussed in Chap. XIV. Full *terminal voltage* will be developed by the shunt generator when the load circuit is open, that is, when

no current flows in the load or receiver circuit. If no change is made in the field rheostat the terminal voltage of the shunt generator will decrease slightly as the load current increases. This is due to three factors:

(*a*) The armature reaction which decreases the effective field flux, that is, the number of lines of force cut by the armature conductors (Chap. XIV).

(*b*) The voltage lost in the armature by the resistance drop R_aI_a.

(*c*) The decrease of the terminal voltage, due to factors (*a*) and (*b*), reduces the current in the field windings which in turn reduces the field excitation and thereby causes a proportionate decrease in the field flux.

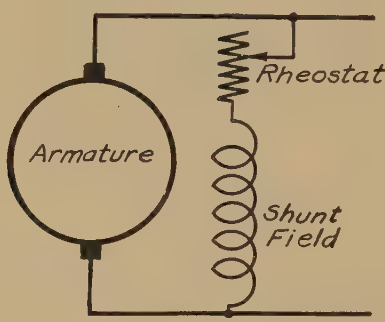


FIG. 19.—Circuit diagram for shunt dynamo.

In order to provide constant voltage at the terminals of the generator or at some point on the load circuit the simple shunt generator could be provided with a regulator that would automatically decrease the resistance in the field rheostat with increase in load current. The requirements of constant-voltage operation are, however, met more economically by the compound generator and hence comparatively few simple shunt generators are used in commercial operation.

The field windings of shunt dynamos consist of many turns of fine wire. This necessarily follows as the full voltage of the machine is consumed in the field winding and necessarily only a small part of the total power developed may be used for field excitation. In the shunt generator the armature current i_a is the sum of the field current i_f and the load or line current i_l while in the shunt motor the armature current is the difference in the line current and the field excitation current.

In a shunt generator:

$$i_a = i_l + i_f \tag{3}$$

In a shunt motor:

$$i_a = i_l - i_f \tag{4}$$

Compound Excitation.—Most power systems operate on a constant-potential basis; that is, the voltage is held practically constant while the current varies with changes in load. For example, to obtain the best results in lighting by incandescent lamps the voltage must be kept very nearly constant at the terminals of the lamp itself. To gain this end the voltage generated in the dynamo must increase with the load sufficiently to provide for the resistance drop in the generator and in the wires

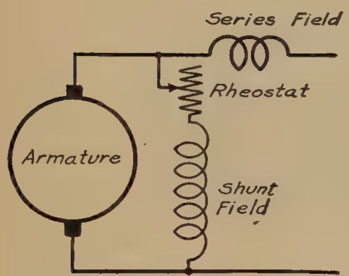


FIG. 20.—Circuit diagram for short-shunt, compound generator.

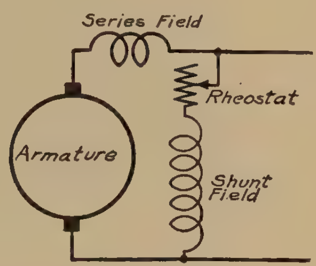


FIG. 21.—Circuit diagram for long-shunt, compound generator.

leading to the incandescent lamp. This requirement is met automatically by the compound-excited generator. In essence compound excitation is a combination of series and shunt excitations, with the two field windings so proportioned as to produce the desired increase in field excitation as the load current increases.

Two forms of compound generators, known as the *short-shunt* and the *long-shunt* connection, are in general use. The conventional circuit diagram for the short-shunt connection is shown in Fig. 20, and a similar diagram for the long-shunt connection in Fig. 21.

The circuit connection of a four-pole compound-wound, short-shunt, dynamo is shown in Fig. 22.

The excitation produced by the shunt winding will give a terminal voltage that decreases as the load increases while the series winding causes an increase in the terminal voltage with

increase in load current. It is, therefore, evident that by proper proportions in shunt and series windings, generators can be constructed that automatically will give at the terminals of the machine *decreasing voltage*, *constant voltage*, or *increasing voltage* with increase in load.

If the series and shunt excitation is so proportioned that a constant voltage is maintained for all loads the dynamo is said to be *flat compounded*. If the series-winding excitation more than balances the voltage losses in the machine so that the terminal

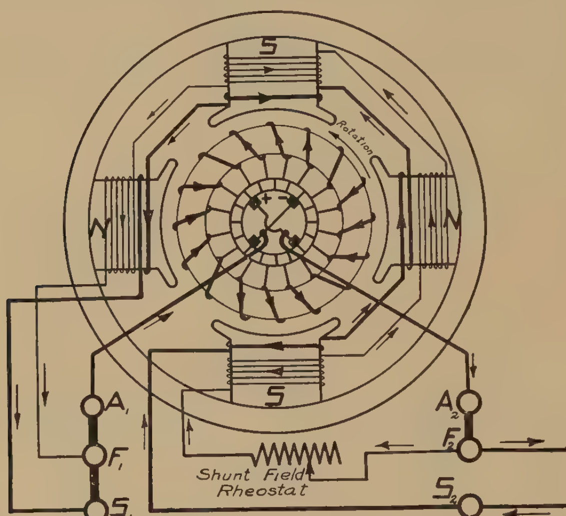


FIG. 22.—Connections of compound generator. Short shunt.

voltage increases as the load is increased the dynamo is said to be *overcompounded*. On the other hand, if the series excitation is less than the turns required to hold the voltage constant so that there is a decrease in terminal voltage for increase in load the dynamo is said to be *undercompounded*.

The percentage of compounding whether over- or undercompounded is defined as the change in voltage between no load and full load divided by *no-load* voltage.

Field Windings.—The field magnetic flux is produced by the current flowing in the field windings. If the reluctance of the field magnetic circuit is known the ampere-turns necessary to produce the desired flux lines Φ may be computed by the application of Ohm's law. Besides the reluctance and permeability of

NI $\qquad R = \frac{\mathcal{E}}{\Phi} = \frac{N}{I}$

the magnetic circuit, other construction details enter into the design of the field windings, such as economy of copper, size and shape of wire, number of turns, insulation, heat radiation, distribution of field flux, and the use of form-wound coils. The cross-section and length of the core in various types of dynamos differ greatly. The cross-section is generally either round or of a rectangular shape with more or less rounded corners.

Shunt coils are generally of many turns, made from cotton-covered copper wires. The coils are wound on forms and then impregnated with insulating compound. The coils are either frame wound or shaped on forms so as to slip on the field cores

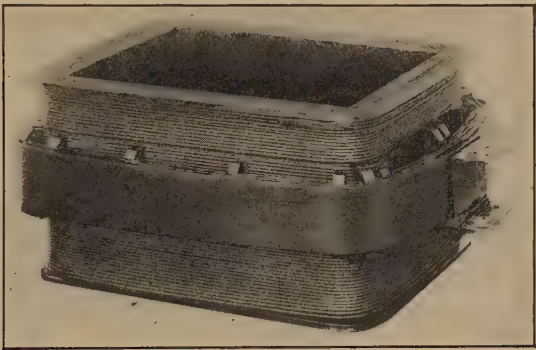


FIG. 23.—Field coils. (*General Electric Company.*)

properly insulated. The series coils of compound and interpole machines (see Chap. XVII) are generally made of copper strap wound on edge.

The several turns of the field windings are insulated from each other and the field coil as a unit is insulated from the core and field frame. Since the voltage induced in the coil when the field circuit is opened becomes many times as large (Chap. X, Fig. 25) as the normal operating voltage, the insulation must be proportionately greater than that required for the operating voltage of the dynamo.

Field Rheostats.—A variable resistance, called the *field rheostat*, is connected in series with the shunt-field winding to provide means for voltage regulation. The resistance in the rheostat is tapped at short intervals, as indicated diagrammatically in Fig. 24. A movable contactor passes over the resistance taps so that the resistance in the field circuit may be varied by turning the rheostat handle shown in Fig. 25.

Rheostats are usually rated at both a maximum and minimum value of current capacity. The reason for the double rating is that the resistance wire is not of the same cross-section throughout the rheostat but varies from a large size to a small size as indicated in Fig. 24. It is obvious from Fig. 24 that the field itself must have sufficient resistance so that when the rheostat is all out the current will not be greater than the carrying capacity of the large-size wire of the rheostat. Furthermore, the voltage of the field circuit must not be so high that the rheostat resistance is not sufficient to throttle the current to the minimum rating. Therefore, besides the current rating the voltage rating of the rheostat is also usually given, indicating that it shall not be used on circuits above that voltage.

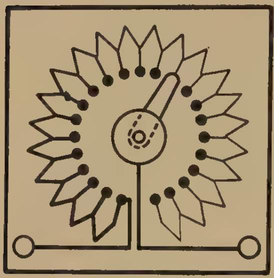


Fig. 24.—Diagram of field-rheostat circuit connections.

The terminals of generator-field rheostats are so connected that by turning the hand in a clockwise direction the resistance in the field circuit is increased, and, consequently, the field current and generated voltage are decreased. Motor-field rheostats are connected just the opposite so that clockwise turning

Fig. 25.—Field rheostat (*Westinghouse Electric and Manufacturing Company.*)

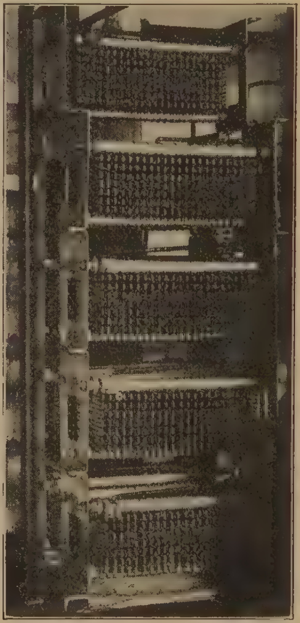


Fig. 26.—Large field rheostat. Iron grid.

of the handle increases the field current, which, as will be shown in Chap. XVI, decreases the speed. Properly selected rheostats will thus always produce a throttling effect when rotated in the clockwise direction.

Field rheostats are generally mounted on the back of the switchboard with the controlling handle on the front of the board. The materials used, resisting surface required, and the form of design are largely determined by the size of the dynamo the field rheostat is designed to regulate.

Field Discharge Resistance.—The energy stored magnetically in the field of a dynamo as explained in Chap. X, is expressed by equation (5):

$$W = \tfrac{1}{2}Li^2 \text{ joules} \tag{5}$$

When the field circuit is broken this energy must be transformed into electric energy and then into heat or mechanical energy in a manner similar to the conversion of the kinetic energy of a moving body into heat or mechanical energy when the motion is checked. Even in small dynamos the energy stored in the magnetic field is considerable. A quantitative comparison of the kinetic energy in a motor field and a rifle bullet, may be of assistance in forming concepts of the magnetic field.

In one of the laboratory, four-pole, 40-hp., 500-volt, 1,200-r.p.m., direct-current motors,

$$\begin{aligned} L &= 1{,}200 \text{ henrys,} \\ i_f &= 2.2 \text{ amperes at normal excitation,} \\ \text{Hence } W &= 2{,}904 \text{ joules.} \end{aligned}$$

The kinetic energy of a bullet from the Springfield rifle, at muzzle velocity of 2,700 ft. per second, is 2,781 ft.-lb. or 3,771 joules.

When the field switch of a motor is opened, an arc is formed which will break when the stored energy has been dissipated. The more abruptly the circuit is opened the higher the induced voltage. Various devices are used to prevent excessive rise in voltage that otherwise might puncture the field insulation. In large machines *field breakup switches* are used that open the field circuit at several points simultaneously. In other designs the field switch is provided with a *discharge resistance* that provides a bypath in which the greater part of the magnetically stored energy will be dissipated as heat when the field winding is dis-

connected from the current supply. In Fig. 27 is shown the diagram of connections for the field switch with discharge resistance.

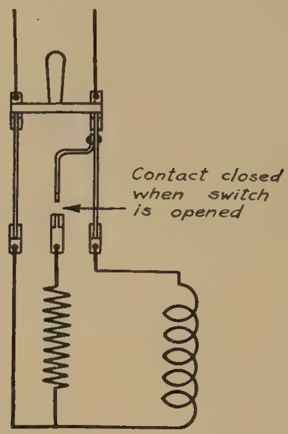


FIG. 27.—Circuit diagram of field discharge resistance.

Unipolar or Homopolar Generators.—Difficulties in securing satisfactory commutation were for many years the ever-present bugbear of the designer, manufacturer, and user of direct-current dynamos. The commutator was the center of endless troubles and poor commutation the greatest handicap in the use of direct-current machines. Not until recently has the complex process of commutation been understood and so thoroughly has it been mastered that it is no longer a serious obstacle to the operation of direct-current machinery. It is, therefore, not surprising that during the years when commutation difficulties appeared well-nigh insurmountable, many attempts were made to escape the dilemma by eliminating the process of commutation from the design of the direct-current generator. To gain this end many types and forms of so-called *unipolar or homopolar* dynamos were designed and constructed. The essential principle, common to all, is that the armature conductors shall cut magnetic lines of force continuously in the same direction; in order that the voltage generated and, as a consequence, the currents flowing in the armature as well as in the outside circuit shall be continuously in the same direction. The basic principles of the design can readily be understood by reference to the diagram in Fig. 29, showing the electric and magnetic circuits. The hatched areas represent the iron or steel-core sections of the magnetic circuit.

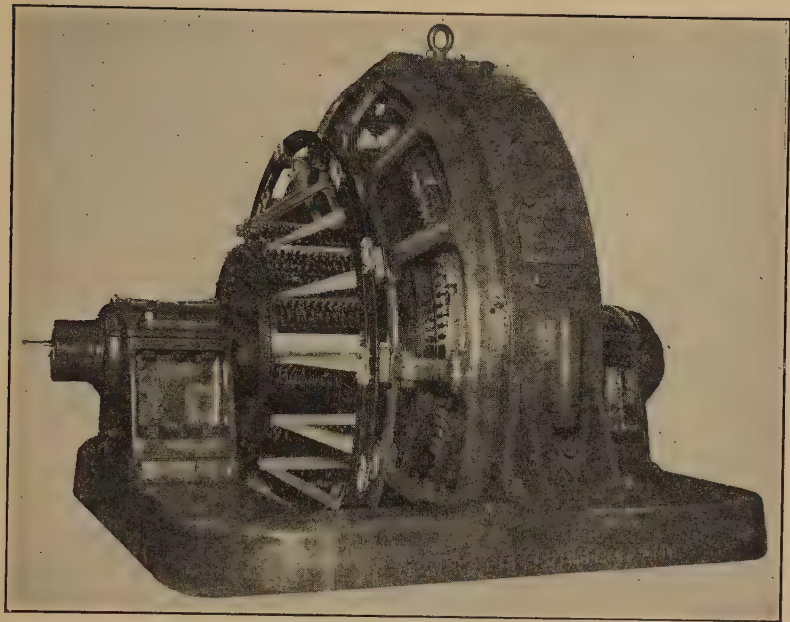


FIG. 28.—Large mill motor. (*General Electric Company.*)

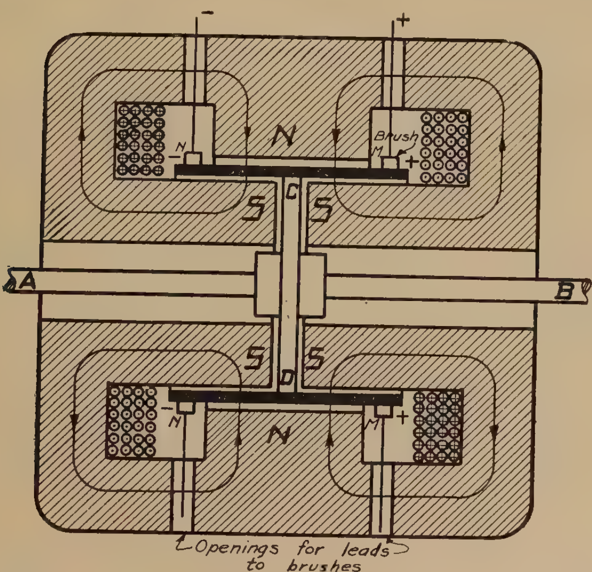


FIG. 29.—Cross-section of a unipolar or homopolar generator.

Two field coils circular in shape are shown in cross-section with the direction of the field current in each case indicated by crosses and dots. On the shaft AB is attached a spider CD which carries the armature, a hollow copper cylinder, shown in cross-section as two heavy bars parallel to the shaft. The brushes MN are shown on the armature in sliding contact with the outer surface of the copper cylinder. The indicated direction of the field currents shows that the magnetic flux will at all points on the armature be in direction from the outside to the center, as marked by the arrows and by N and S in the diagram.

If the rotation of the armature is in the clockwise direction, looking from the A end of the shaft, the voltage generated will be in direction from brush N to brush M inside the armature and consequently from the + to the − terminal in the external circuit. Since the voltage generated and the currents flowing are continuously in the same direction the commutation process is eliminated and hence no commutator required. The advantages gained are more than overbalanced, however, by the limitations in voltage at the machine terminals, by the difficulties incidental to collecting current by brushes; in sliding contact on surfaces moving at high speed; by eddy current losses in the armature, and by other minor adverse factors.

PROBLEMS

1. A shunt generator supplies a load with 106 amp. at 125 volts. The shunt-field resistance is 29.2 ohms and the armature resistance 0.105 ohms. Find the shunt-field and armature-copper losses.

2. A shunt motor having the same constants as the generator in Problem 1 takes 98 amp. from 125 volt mains. Find the shunt-field and armature-copper losses.

3. A short-shunt, compound generator delivers 216 amp. to a load at 250 volts. The shunt-field resistance is 26.8 ohms, the shunt-field rheostat 6.2 ohms, the series-field resistance 0.042 ohms, and the armature resistance 0.096 ohms. Find the copper loss in the shunt-field winding, in the shunt-field rheostat, in the series-field and in the armature winding.

4. A long shunt motor having the same constants as the generator in Problem 3 takes 254 amp. from 246-volt mains. Find the copper losses in the shunt field, in the shunt-field rheostat, in the series field and in the armature winding.

CHAPTER XIII

ARMATURE WINDINGS

The primary division in the customary classification of armature windings is based on the form of the armature core. *Ring, drum,* and *disc windings* are merely groups adapted for use on ring,- drum,- or disc-armature cores. The *disc type* is of little, if any, practical importance and need not be considered. The *ring type,* frequently called the Gramme ring, was used in early designs but the advantages of the *drum type* are so pronounced that, with few exceptions, all dynamos in practical operation have drum-armature cores and some form of drum-type armature winding. The drum type of winding, however, is more difficult to picture and visualize in its operation than the ring winding. Hence, although the ring type of winding is seldom used in practical designs it is more desirable for illustrations in preliminary studies of armature windings. A clear insight into the principles involved in the design of armature windings can be gained more readily by studying the ring type as a preliminary step than by a direct analysis of the generally used drum type of armature windings.

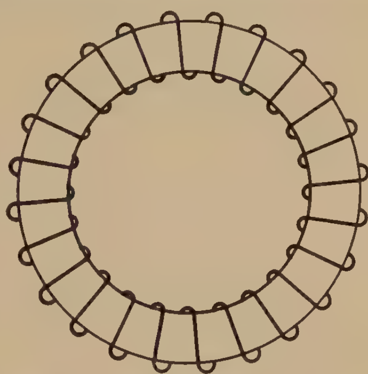


FIG. 1.—Ring-armature core and winding.

Ring Windings.—In Fig. 1 is shown a simple form of ring winding, consisting of an insulated conductor wound around an iron ring or armature core, each turn connecting to the next one. After completing the winding for a full revolution of the ring core, the terminal and starting ends may or may not connect, depending on the space between the successive turns or the number of grooves in which the winding is placed. In the latter case the winding does not close on itself but will have both start and finish

terminals. This form is known as an *open-circuit* winding. As this type of winding is not used in direct-current machines it will not be considered here. If the two ends of the conductors connect at the starting point a *closed* winding is formed. The winding shown in Fig. 1 is classified as a closed simplex ring winding. In Fig. 2 is shown a ring-type armature rotating in a two-pole magnetic field. Each turn of the winding is connected to a segment of the commutator. The brushes are shown on the inner surface of the commutator, merely to avoid overlapping of the lines thereby simplifying the diagram. The lines of force

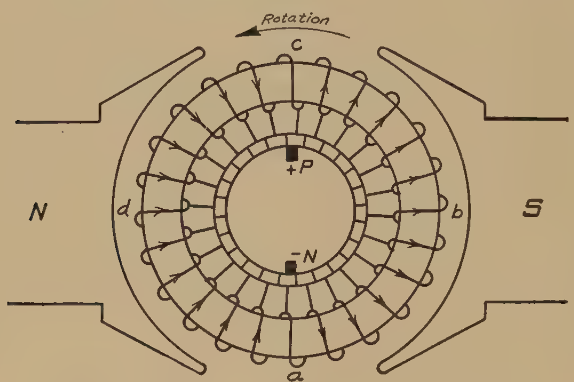


FIG. 2.—Simplex, two-pole, ring armature.

in the magnetic field in which the armature rotates (Fig. 2) pass from the north pole to the south pole and for as great a part of the path as possible in the iron core of the armature in preference to the air space inside the core or between the pole tips. It is evident that armature conductors when at either *a* or *c* do not cut field flux and, consequently, do not generate voltage. The space in which no lines are cut is known as the "neutral zone." With the rotation as indicated by the arrow in Fig. 2, all conductors when not in the neutral zone and on the outside of the iron core cut lines of force and generate voltage. Since the motion of the conductors on the opposite halves of the armature or at *b* and *d*, is reversed in direction with respect to the direction of the magnetic field the voltages generated will be in opposite directions, and as indicated by the arrow heads. The total voltage generated on each side *abc* or *adc* is cumulative and is equal to the sum of the voltages generated in the several conductors. The

voltage generated on the right side *abc* is obviously equal in magnitude to that generated on the left side *adc* since the same flux passes through both sides and the same number of conductors in series is found in the two paths. The two halves of the armature produce voltages equal in magnitude but opposite in direction and, therefore, unless an outside circuit between the brushes is provided, no current flows in the armature although the circuit is closed. It is evident that from the point *a*, the position of the negative brush, to point *c*, the position of the positive brush are two paths in parallel both generating voltage of the same magnitude. As the armature rotates the voltage generated in any given conductor reverses in direction when passing the neutral zone at *a* or *c*, that is, when passing under the brushes. The brushes are stationary and located at the neutral zone, the voltages generated in the several armature conductors are all in the same direction and will cause currents to flow continuously in the armature from *a* to *c* or from the negative brush to the positive brush.

In Fig. 3 is shown a simple ring armature rotating in a four-pole field. Since the poles are alternately north and south it is evident that, in this case, neutral zones exist at four points, that is, at *a*, *b*, *c*, and *d* midway between adjacent pole tips. Under the assumption that the four poles are of the same size and carry the same number of lines of force it is evident that the voltages generated in the four sections, *a-b*, *b-c*, *c-d*, and *d-a*, are equal in magnitude, but for each pair opposite in direction and, therefore, no current will flow in the closed armature winding unless an outside path is provided through the brushes. The voltage generated in the *a-b* sector is balanced by that generated in the adjacent *b-c* sector. Similarly the voltages generated in *a-d* and *d-c* although equal in magnitude are opposite in direction and hence balance. The points *a* and *c* are therefore at the same electric potential and may be connected together as indicated in the diagram. Similarly, the points *b* and *d* must be at the same potential and hence can be connected together. The brushes are placed at the neutral zones and in four-pole machines the two positive brushes as well as the two negative brushes are connected together. The voltage between the positive and negative brushes is therefore the highest generated in the machine and unless an outside load circuit is connected to the brushes no currents flow in the armature. The importance of placing the

brushes in the neutral zone and the causes for sparking at the brushes is discussed under Commutation (Chap. XVII).

It may be well to note that the number of paths in the ring armature of a four-pole generator (Fig. 3) is four, and two for the corresponding two-pole machine (Fig. 2). In general, the number of paths in a simplex ring winding is numerically equal to the number of poles.

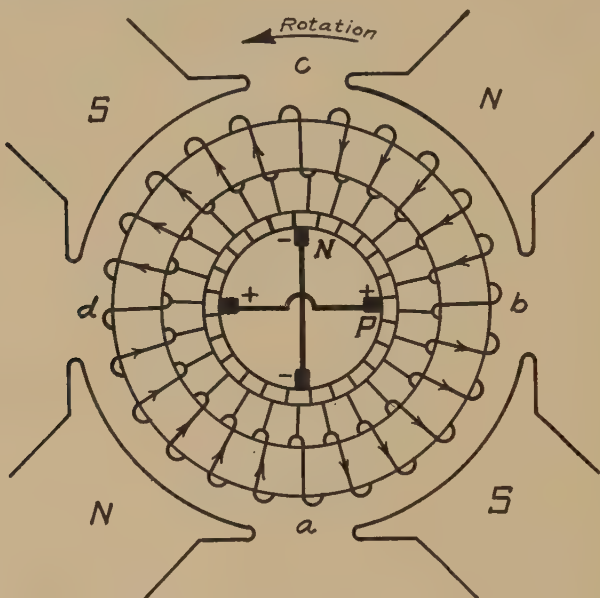


FIG. 3.—Simplex, four-pole, ring-armature winding.

Multiplicity and Reentrancy.—In Fig. 4 is shown a ring winding differing somewhat from those already discussed. In explanation of Fig. 1 it was noted that if, on going around the armature once the winding closed, it would be a simplex winding. Such a winding is shown by the full lines in Fig. 4. If only half of the armature surface is thus used, a second winding, a duplicate of the first may be wound on the in-between spaces on the core, as shown by the broken lines in Fig. 4. It will be noted that the two windings have separate sets of commutator segments placed alternately together to form one commutator. The brush width has a larger span than for the simplex winding in order to collect current from both windings at the same time. The only connection between the two windings is by way of the brushes since the

segments themselves are completely insulated. There will, however, be no interchange of current from one winding to the other through the brushes because the voltages of the two windings are equal both in magnitude and direction. The two wind-

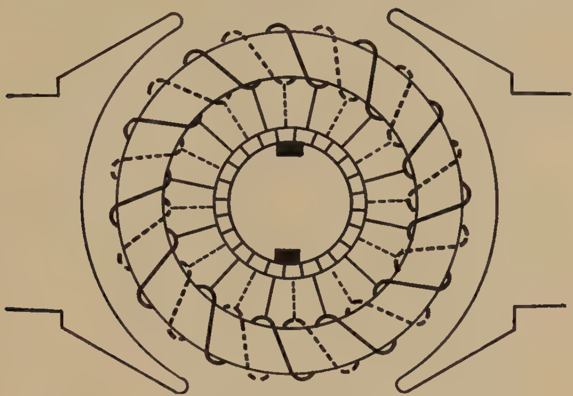


FIG. 4.—Doubly reentrant duplex winding.

ings are merely operating in parallel to supply current to the external circuit. The effect of the addition of the second winding is to increase the current capacity of the machine but has practically no effect on the voltage rating. Actually the complete

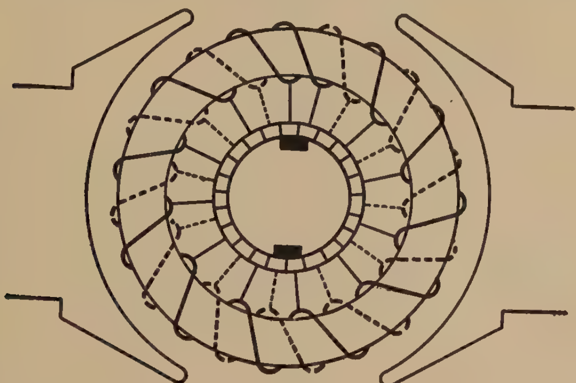


FIG. 5.—Singly reentrant duplex winding.

winding is identical to two simplex windings operating in parallel. The winding illustrated in Fig. 4 is called a *duplex multiple winding*. In large machines more than two windings are placed on the armature core giving rise to *triplex*, *quadruplex*, or windings of

any desired multiplicity. In the preceding paragraph it was stated that the number of paths through a *simplex* ring winding is numerically equal to the number of poles. It follows from the foregoing that the number of paths through a *multiplex* winding would be increased by the multiplicity and would be numerically equal to the product of the multiplicity and number of poles.

In Fig. 5 is shown a ring winding very similar to the one in Fig. 4 but having one less turn. In passing around the armature surface once it will be noted that the winding does not close as in Fig. 4 but proceeds a second time around the surface and then connects to the starting terminal. This winding is called a singly reentrant duplex winding while the winding of Fig. 4 is called a doubly reentrant winding. In similar manner three-fold, four-fold or any degree of reentrancy may be obtained in corresponding multiplex windings. It was previously noted, in connection with the doubly reentrant winding, that no current flows in the armature windings under no-load conditions although connected together by means of the brushes. For the same reason no current interchanges in the singly reentrant winding because of balanced voltages. The number of paths will be the same whether singly or multiply reentrant because every brush spans a sufficient number of segments so as to connect at all times to all windings regardless of the reentrancy. Therefore, between adjacent brushes there will always be a number of paths equal to the multiplicity and the total paths will be equal numerically to the product of the multiplicity and the number of poles.

Drum Windings.—From the description and sketches of the ring winding it is evident that only one side of each coil wound around the armature core is useful in generating voltage between brushes. Moreover, to construct a ring winding will require much more labor than if all the conductors could be placed on the outer surface of the armature. With the drum type of winding both of these disadvantages are avoided and as a consequence some form or other of drum winding is used in practically all commercial dynamos. The drum winding may be divided into three different types known as the *lap winding*, the *wave winding* and the grasshopper or *frog-leg winding*, the latter being a combination of the other two. Lap windings are also known as parallel or *multiple* windings and wave windings as *series* windings.

A lap-winding diagram is shown in Fig. 6. The armature conductors, numbered consecutively from 1 to 36, are indicated

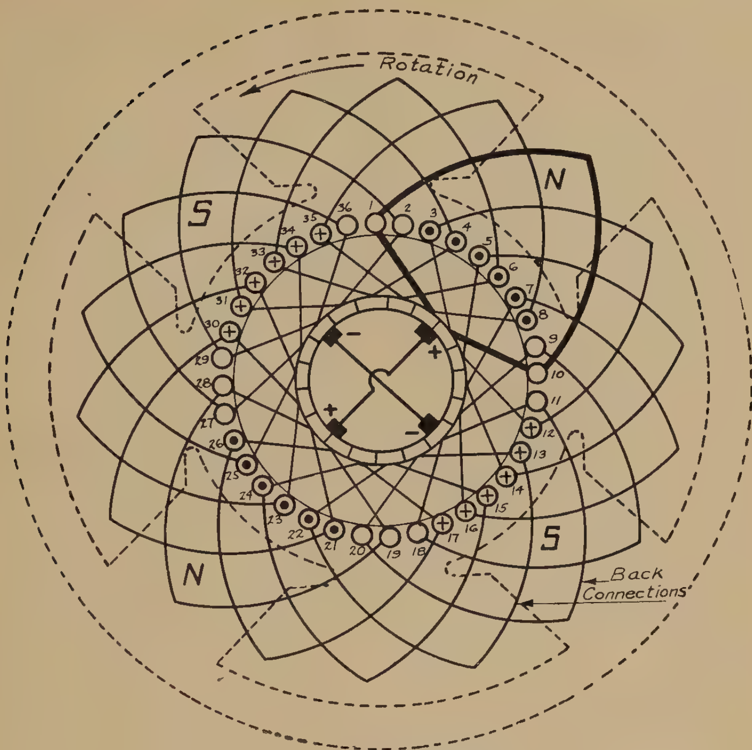


FIG. 6.—Simplex lap winding.

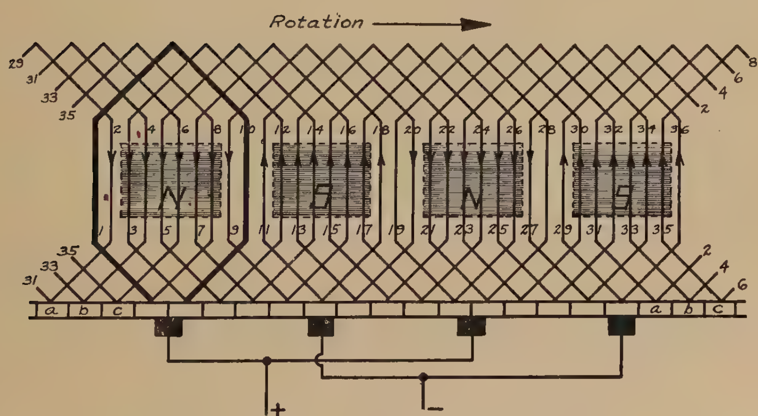


FIG. 7.—Development of lap winding in Fig. 6.

by small circles distributed around the surface of the armature core, in which the dots and crosses denote the directions of the generated voltages.

The back connections on the pulley end, as noted in Fig. 6, serve to connect a conductor under one pole to one similarly located under an adjacent pole. On the commutator end a front connection connects this latter conductor back to one displaced two conductors away from the first one. The whole procedure continues until eventually the winding closes as shown in Fig. 6. It will also be noted that the front connections are tapped to the commutator segments. In order to gain a clearer insight into the winding of Fig. 6, the armature surface has been rolled out on a plane to give a developed view as shown in Fig. 7. The location of the pole pieces is also shown. It is essential to note, as before, that the brushes are in contact with segments that connect to conductors located in the neutral zones. The actual position of the brushes with respect to the poles depends entirely on the shape of the front connection which in Fig. 7 is symmetrical and, consequently, the brushes are opposite the center of the pole pieces. By varying the shape of the front connection as indicated in Fig. 10 (*c*) the brush location may occur at any desired position.

Tracing the circuit from brush to brush it will be found that the voltage of all the conductors is in the same direction and hence add. The voltages between successive pairs of brushes, however, are in opposite directions. In tracing through the total winding it will be found that alternate brushes are at the same potential as in ring windings and therefore may be connected together as shown in Fig. 6, thus giving the equivalent of a single pair of brushes as positive and negative terminals for the machine. The number of paths for a simplex lap winding, as shown in Fig. 6, is numerically equal to the number of poles. This is like the rule given for the simplex ring winding and may be verified by tracing the circuits in the diagram. The principles of multiplicity and reentrancy may be applied to lap windings with the same results as were found for the ring windings. The number of paths between terminals will accordingly be equal to the product of the number of poles and the multiplicity regardless of reentrancy.

For lap windings:

$$\text{Number of paths} = \text{number of poles} \times \text{multiplicity factor} \qquad (1)$$

In Fig. 8 is shown a diagram of a *wave winding* and in Fig. 9 the corresponding development. The essential differences between the wave and lap windings is that the front connection, although still attached to a commutator segment, does not lap back but travels forward a distance practically equal to that spanned by the back connection. Furthermore, in passing around the armature once, the winding does not close but continues in sequence around the armature, each time connecting to a set of conductors immediately adjacent to those followed on the preceding round. With all conductors included, the winding automatically closes at the starting point. An interesting feature of the wave winding is the number of paths between terminals. In Fig. 9 it will be noted that the two positive brushes are connected by a comparatively short path, consisting of two armature conductors and the front and back connections. Since these conductors are in the neutral zone no voltage is generated in them so that the two brushes may be considered as located at essentially the same point in the circuit. Therefore, if one of the brushes be made sufficiently large so as to be capable of carrying the total load current the other brush may be omitted. The same reasoning applies to the negative brushes. The general statement may then be made that only two brushes are necessary for a wave winding, provided they are large enough to carry the current. This characteristic of a wave winding is almost a determining factor in the design of railway motors where restricted clearance and accessibility demand that the brushes be located on the top side of the commutator. For this reason railway motors generally have wave windings. While the preceding argument is based on a machine having only four poles, it will be found to be applicable to machines having any number of poles since all brushes of the same polarity have merely a short connection between them and therefore may be considered as equivalent to one brush. From the above it is evident that every *simplex wave winding* has only two paths regardless of the number of poles.

If in Figs. 8 and 9 only half of the armature surface had been used, a second and similar winding could be placed on the same core thus producing a *duplex wave winding*. With the brushes wide enough to collect current from both windings at all times, two more paths between terminals would be produced. In

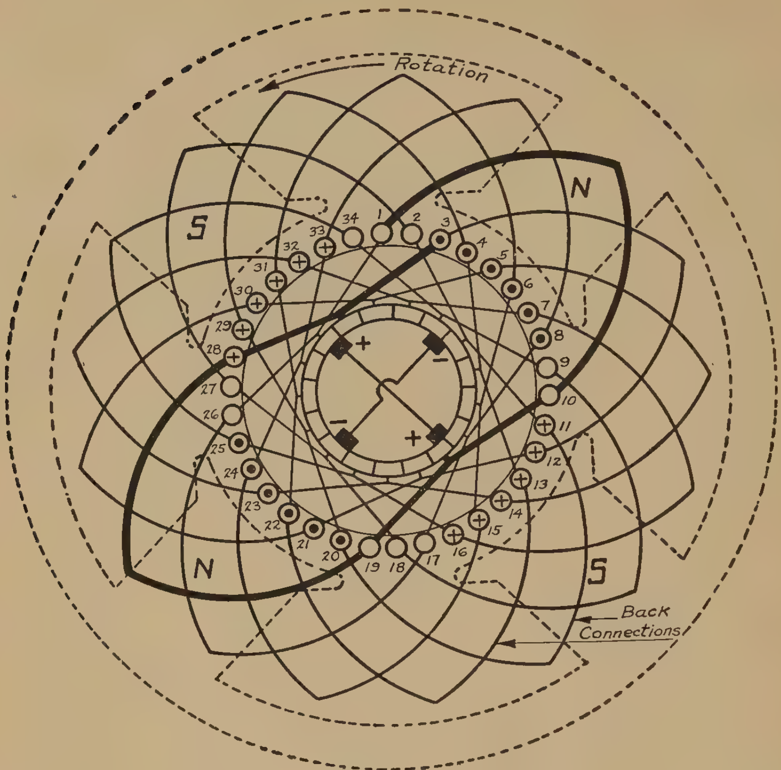


FIG. 8.—Simplex wave winding.

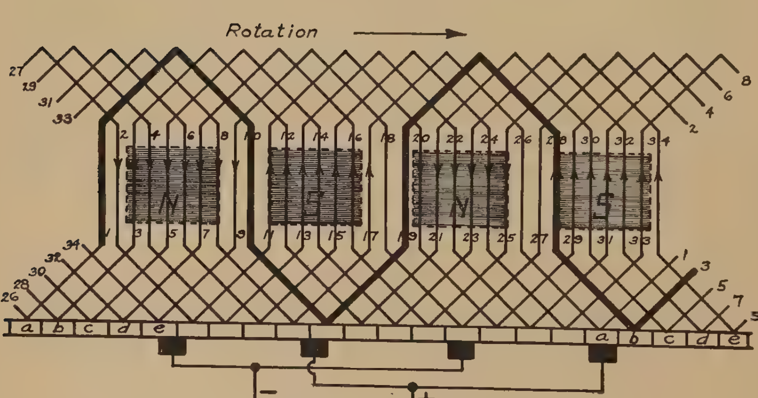


FIG. 9.—Development of wave winding in Fig. 8.

general, the number of paths for a wave winding is twice the multiplicity of the winding.

For wave windings:

$$\text{Number of paths} = 2 \times \text{multiplicity factor} \qquad (2)$$

Reentrancy in a wave winding, similar to the other types of windings, does not effect the number of paths. As already stated reentrancy is simply the number of times the winding must be entered in order to include every conductor when tracing the armature circuits.

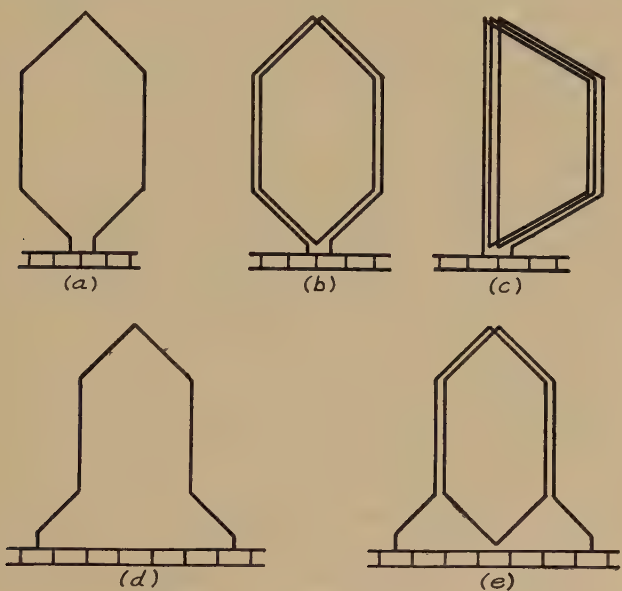


FIG. 10.—Winding elements.

Winding Elements.—It may be well to guard against the possible impression that the back and front connections are separate leads connecting to active armature conductors, whereas in reality, they are sections of the same piece of wire. That part known as the active conductor usually is imbedded in a slot cut into the armature core and held in place by a wedge while the front and back connections are simply extensions which lie on cylindrical surfaces at both ends of the armature and which, because of the centrifugal force, are held in place by banding wire. It is evident that the section of winding between two seg-

ments of the commutator forms a unit in the winding and that these units or *winding elements* are all of the same size and shape. Necessarily, the ends of the winding element always terminate at commutator segments. Because of this fact the number of commutator segments must always be equal to the number of elements in either the lap or wave winding. In Fig. 10 (*a*) is shown a typical lap-winding element having one turn and in Fig. 10 (*b*), one with two turns. In Fig. 10 (*d*) and (*e*) are shown similar types for wave windings. Figure 10 (*c*) shows a lap-winding element of three turns illustrating the possible variation of the shapes of end connections. It is evident that the shape of the end connection will affect the position of the brushes, which in some designs are shifted away from the point opposite the middle of the poles. If a winding element consists of more than one turn the conductors are taped together and inserted into the armature slot as a unit. In large machines there is seldom more than one turn per element, but for small high-voltage machines several turns may be necessary. The maximum number of turns is limited both by commutation difficulties and the insulation between segments. It is important that the self-inductance of the element, which varies as the square of number of the turns, be kept low in order to permit the required rapid reversal of current in the element when passing through a neutral zone while in contact with a brush. The normal voltage between segments is from 8 to 10 volts but increases to a maximum of about 20 volts in railway motors.

Numbering of Element Sides, Winding, Pitches.—In Figs. 6 and 8 the conductors are shown distributed in one layer on a smooth armature surface. This arrangement is for two basic reasons not a practical design. In the first place, it does not provide the necessary support for holding the conductors in place. In the second place the large airgap between the field pole and armature core would include a large reluctance in the field flux circuit, which would require an excessive field excitation. Both of the objectionable features are eliminated by placing the conductor in slots or grooves cut into the armature core. This provides the necessary mechanical support for the conductors and likewise reduces the length of the airgap between the field poles and the armature core. Since a systematic method of design demands the connecting together of specific conductors a definite system of numbering of the conductors should be followed.

Assuming that only two-layer windings will be considered, which includes most windings, the coil or element sides in the slots are numbered as shown in Fig. 11. The upper conductors have the odd numbers while the even numbers are in the bottom layer.

By numbering the element sides rather than the conductors the rules established will be found applicable to all two-layer windings. If the winding has but one turn per element the numbers will apply to each separate conductor since the number of element sides will be identical with the number of conductors. If as indicated in Fig. 11 (*c*), there are several turns per element,

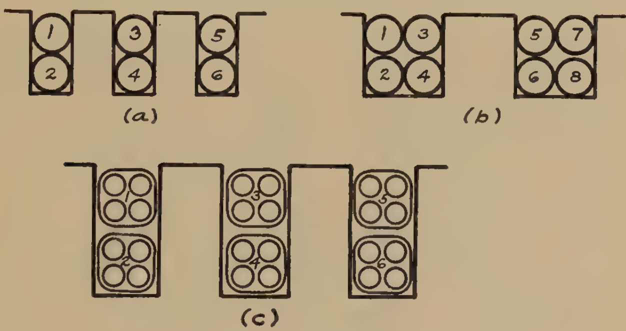


FIG. 11.—Two-layer windings showing method of numbering.

however, then the numbers will apply to the total bundle of wires which taped together forms an element side. Let the total number of conductors be denoted by Z and the number of element sides by Z_1. For windings having more than one turn per element Z will not be equal to Z_1. If n = turns per element, Z_1 will be given by equation (3)

$$Z_1 = \frac{Z}{n} \tag{3}$$

The number of element sides spanned by the back connection is called the *back pitch* y_b of a winding while those spanned by the front connection is termed the *front pitch* y_f. Front and back pitch for both the lap and wave types of elements are illustrated in Fig. 12. It will be noted that in tracing through any element from segment to segment as in Figs. 7 and 9, the outgoing sides bear odd numbers and the return side the even numbers. This results from the sequence necessary to form a systematic winding that will be easy to construct and which will uniformly fill the bottom of the slots at the same rate as at the top.

To gain this end the elements are always placed so that one side of the element occupies the top and the other side occupies the bottom of the slot. For this reason it is evident that both the front and back pitches must be odd numbers since both are computed by finding the difference between an even and an odd number. In Fig. 7 the back pitch is accordingly $12 - 1 = 11$ and the front pitch is $3 - 12 = -9$. In Fig. 9 the back pitch is $12 - 1 = 11$ and the front pitch is $21 - 12 = 9$. The *average pitch* of an element is the average of the front and back pitches.

$$y_{av} = \frac{y_b + y_f}{2} \tag{4}$$

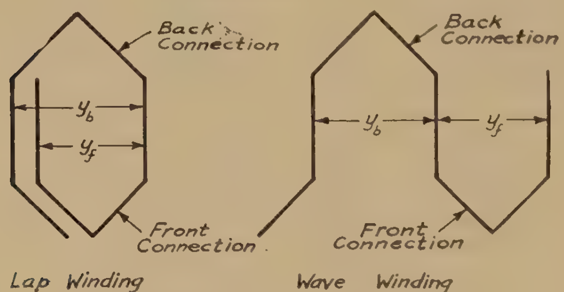


FIG. 12.—Winding pitches and connections.

Lap Windings.—(*a*) *Selection of Pitches, Progressive and Retrogressive Windings.*—In a preceding paragraph it is stated that the back connection served to connect a conductor under one pole to another conductor similarly located under an adjacent pole. The distance spanned by the back connection may or may not be exactly equal to the pole pitch. In order that no voltage will be induced in an element while undergoing commutation, that is, while it is being short circuited by a brush, it seems evident the back pitch should equal the pole pitch. On account of armature reaction (see Chap. XIV), however, it is often necessary to make the back pitch considerably larger or smaller than the pole pitch as shown in Fig. 13. In either case the winding is then known as a *shortpitch* or *chorded* winding. Leaving this point for discussion in Chap. XIV under Armature Reaction, the full-pitch winding will be considered more in detail.

During commutation the brushes short circuit the conductors connected by the back pitch and if both are to be in the neutral

zone at the same instant then the back pitch y_b must be very nearly equal to the number of element or coil sides spanned by a pole pitch. The number of element sides spanned by a pole pitch y_p is $\frac{Z_1}{\text{poles}}$ which may be an even, odd, or fractional number. The value of y_b, however, due to the method of numbering the

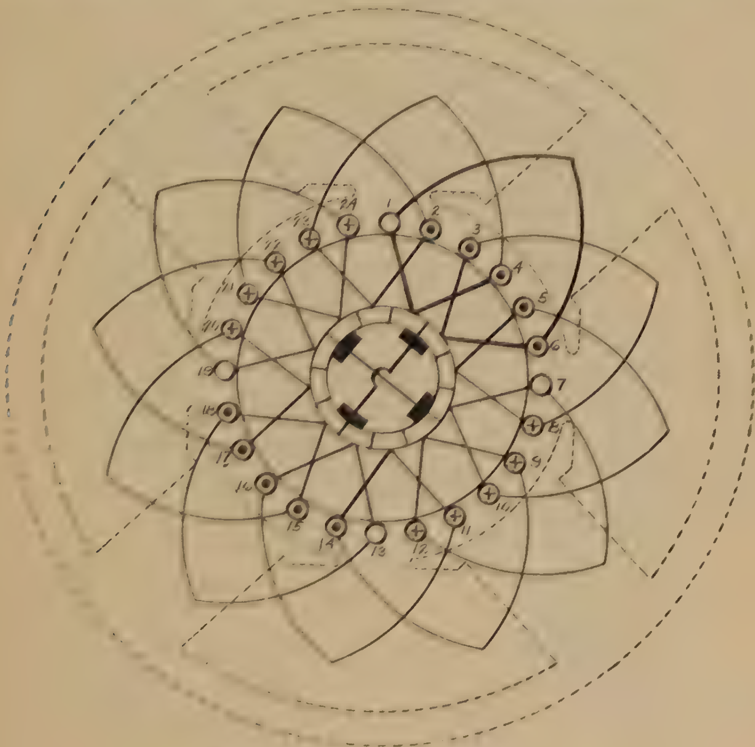


FIG. 13.—Short-pitch, lap winding.

coil sides, can only be an odd number, but the number selected for y_b should be as near as possible to that of the pole pitch.

The front pitch y_f for a simplex lap winding connects to a top coil side immediately adjacent to the first coil side passed through in tracing the circuit. From Fig. 14 it is seen that the number of this element differs by 2 from the first one. In the case of a duplex winding the front connection would have skipped conductor 3 and gone directly to 5, or in general y_b and y_f differ by 2 × multiplicity. It is obvious that the value of y_f is not

restricted to a number smaller than y_b but can also have values which are correspondingly larger. The general rule for lap windings is therefore,

$$y_b = y_f \pm 2m \qquad (5)$$
$$m = \text{multiplicity factor.}$$

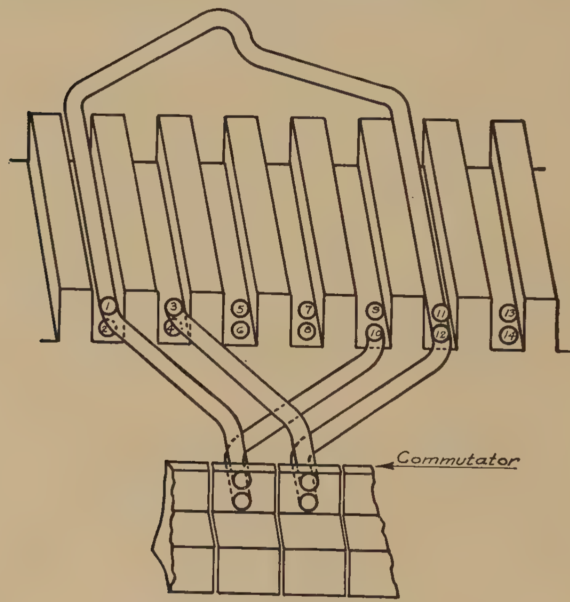


Fig. 14.

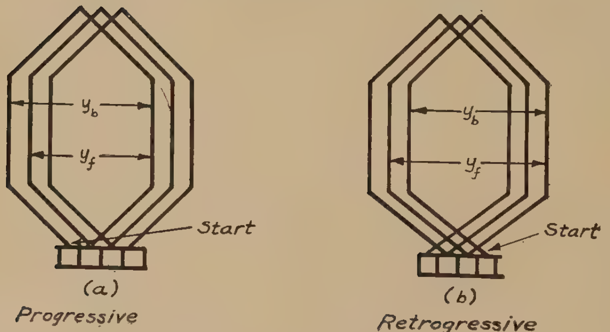


Fig. 15.—Progressive and retrogressive windings.

If y_b is greater than y_f the winding will proceed in a clockwise direction around the armature and is accordingly called a *progressive winding* (Fig. 15*a*). If y_b is less than y_f the winding

travels backwards and is called a "retrogressive winding" (Fig. 15*b*).

(*b*) *Commutator Pitch.*—The *commutator pitch* of a winding is simply the number of commutator segments spanned by the two ends of an element. For lap windings the commutator pitch Y_c is always equal to the multiplicity. That is, for simplex windings the ends of any element connect to adjacent segments or a span

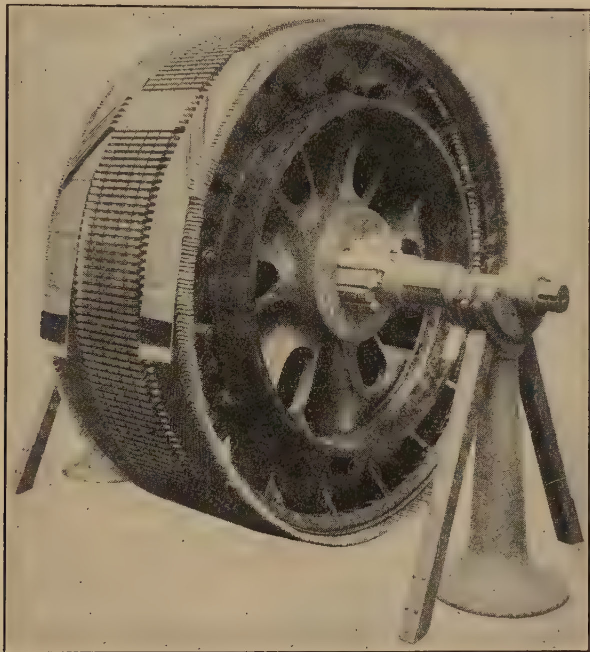


Fig. 16.—Photo of lap-winding element on armature. (*General Electric Company.*)

of one segment; duplex-winding elements connect to segments separated by one segment or a span of two segments and in similar manner for a lap winding of any multiplicity.

$$y_c = m \tag{6}$$

(*c*) *Reentrancy.*—Equation (6) states that on tracing through a lap winding, of *m* multiplicity, connection is made successively to the commutator at segments displaced from each other by *m* segments. After having passed once around the commutator it is obvious that connection may fall on the initial segment connected to and thus close the winding, or it may skip over the first

segment and proceed again round the commutator, connecting to a new set of segments, also displaced m segments from each other. From the foregoing, the test, necessary to determine whether the winding in a given machine is singly or multiply reentrant, is to merely note whether the number of commutator segments is divisible by the commutator pitch. If divisible, the winding will close on passing once around the armature or commutator and the winding will be multiply reentrant. If not divisible, the winding will be singly reentrant in the case of all lap windings up to and including triplex windings. The rule holds for other windings also, but not for all, as for instance a quadruplex lap winding. In this case the number of commutator segments may not be divisible by $y_c = 4$ but at the same time it need not be singly reentrant due to the fact that on passing once around the commutator it may skip the starting point by two segments. On passing around the second time the creepage would be two more segments and the winding would close. Only half of the segments and conductors, however, would have been passed through in closing the circuit. This winding would therefore be a quadruplex doubly reentrant winding. Windings higher than triplex are seldom used in practice, so that the above rule covers most cases.

Lap Windings with Several Coil Sides per Slot.—In a machine having many conductors it is not always possible to restrict the number of coil sides to two per slot as the armature teeth may become too thin and lack the necessary mechanical strength. For this reason several coil sides are often placed in each slot. If this be the case, care must be taken in the selection of the

FIG. 17.—Diagram to illustrate choice of pitch.

winding pitches. For instance, let Fig. 17 represent the slots of a 10-pole machine having 920 coil sides for which the pitches are desired for a simplex winding. For this winding, y_b should be approximately equal to

$$\frac{Z_1}{p} = \frac{920}{10} = 92$$

If Y_b is chosen as 93 and y_f as $y_p - 2 = 91$, conductor 1 will connect to conductor 94 by way of the back connection then to 3

and next to 96 and 5. The next connection, however, will be to conductor 98 in the 13th slot. While the span of this last element is the same as the other two in terms of coil sides, it is not the same by actual dimensions. Such a winding would be very undesirable due to difficulty in winding, difference in size of elements and also because the coil sides forming the upper and lower layers could not be taped together as a unit before inserting in the slot.

The above difficulties may be avoided by selecting back pitches of either 89 or 97 thus making all elements of equal dimensions. In the case of wave windings having several coil sides per slot there is little chance for avoiding the above difficulties because very little choice is possible in determining y_b and y_c, since they must satisfy equation (7).

Wave Windings. (*a*) *Selection of Pitches; Dummy Coils; Progressive and Retrogressive Windings.*—For lap windings the only requisite necessary for obtaining a winding of the desired multiplicity is that equations (5) or (6) be fulfilled. If this equation is satisfied regardless of whether y_b is taken correctly, that is, near a pole pitch span, the winding will always close and pass through every armature conductor. In wave windings the selection of y_b and y_f is more restricted in that the values must satisfy the following equation in which P is the number of poles:

$$\frac{P}{2}(y_f + y_b) = Z_1 \pm 2m \qquad (7)$$

Equation (7) may be derived as follows: The wave winding differs from the lap winding in that the front pitch instead of lapping back travels forward in the same manner as the back pitch. Hence, for every pair of poles the winding travels a distance of $y_f + y_b$ element sides or in going around the complete armature $\frac{P}{2}(y_f + y_b)$ elements must be spanned. The distance $\frac{P}{2}(y_f + y_b)$ must not equal Z_1, the total number of element sides, otherwise the winding will close on the first round (see Fig. 18).

In the case of the simplex wave winding the element terminal must connect to the coil side immediately adjacent to the starting point, the number of which is 2 greater or less. For a duplex wave winding the difference is 4 greater or less. In general,

equation (8) will hold for any degree of multiplicity. In wave windings as well as for lap windings the front and back pitches must both be odd numbers, but unlike the lap winding y_b and y_f, may be equal to each other in wave windings if only equation (7) is satisfied.

In cases where special limitations are imposed, as, for example, to use standard armature stampings it may be found that there are no possible values of y_f and y_b that will satisfy equation (7). If all slots are filled the maximum number of conductors is fixed.

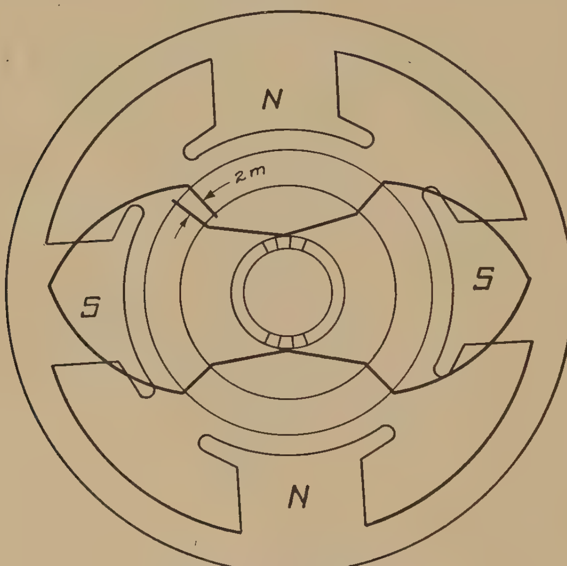


FIG. 18.—Wave winding.

The only alternative is to change the values of Z_1 to some smaller number that will satisfy equation (7). The coils which are thereby discarded are called *dummy coils* and although serving no useful purpose as far as generation of voltage is concerned they are left in the slots with the ends taped, merely for the purpose of keeping the armature dynamically balanced against centrifugal forces. In any case where dummy coils are necessary, that winding should be selected which gives the least number of dummy coils.

If in equation (7) the positive sign is used the winding will travel ahead in a clockwise direction and is then known as a

progressive winding. If the negative sign is used the winding is retrogressive.

(*b*) *Commutator Pitch.*—In the wave winding the number of segments between the ends of any element will be found to be

$$y_c = \frac{y_f + y_b}{2} \tag{8}$$

This readily follows from the fact that the winding travels ahead $y_f + y_b$ element sides per coil which in terms of commutator segments is $\frac{y_f + y_b}{2}$ since there are one-half as many segments as there are element sides.

(*c*) *Reentrancy.*—Figure 19 shows a portion of a four-pole duplex wave winding having $y_f = 19$, $y_b = 21$, therefore $Y_c = 20$ by equation (8).

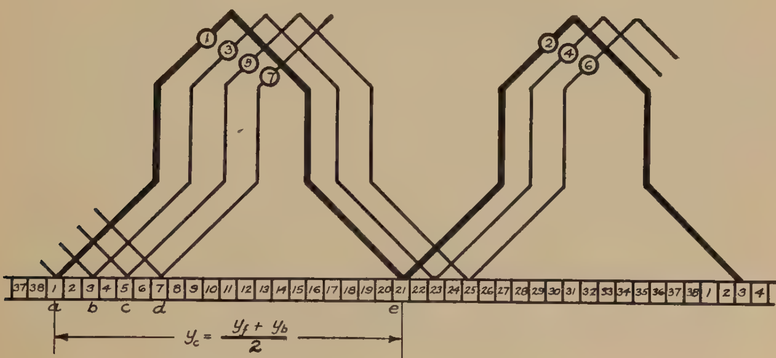


FIG. 19.—Reentrancy of wave winding.

In tracing through the several elements, 1, 2, 3, etc. in Fig. 19 it will be noted that after going around the armature each time the winding creeps ahead two segments from *a* to *b* to *c* and so on, each time lessening the number of segments between *a* and *e* by two. If on arriving at *e* the winding automatically connects to segment 21, the winding will close, but if it skips 21 the winding will not close until every conductor and segment has been used. In the first case the winding is multiply reentrant, in the second, singly reentrant. The criterion of reentrancy in the wave winding is therefore to note whether or not the commutator pitch y_c

is divisible by the multiplicity factor. If it is divisible, then the winding is multiply reentrant, if not, it is singly reentrant.

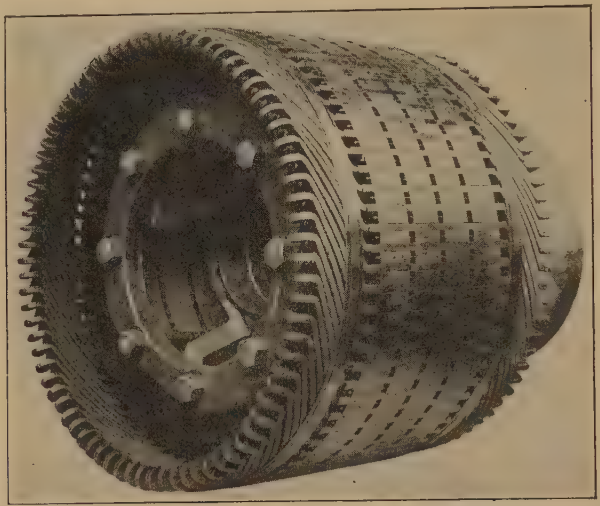


FIG. 20.—Armature with wave winding. (*General Electric Company.*)

Equalizer Connections.—For simplex lap windings it has been demonstrated that there are as many paths through the armature

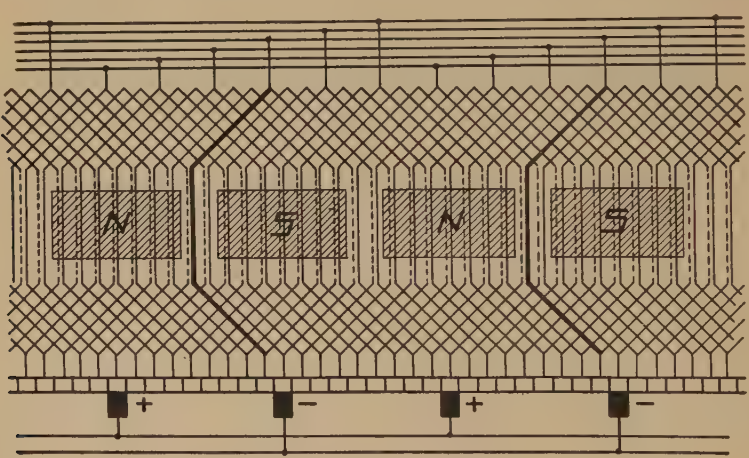


FIG. 21.—Winding diagram showing equalizer connections.

as there are poles, and that for multiple lap windings the number of paths is increased in direct proportion to the multiplicity.

The paths are all connected in parallel by uniting all the positive brushes together to form the positive terminal of the machine and likewise for the negative brushes to form the negative terminal. It is evident that if for any reason the generated voltages in the several paths are not exactly the same, circulating currents will flow through the brushes even when no load is being delivered. This condition may be due to various factors, as the wearing of the bearings, inequalities of the airgap or poor mechanical alignment. It is desirable to keep the circulating currents as small as possible to avoid excessive heating and losses but it is

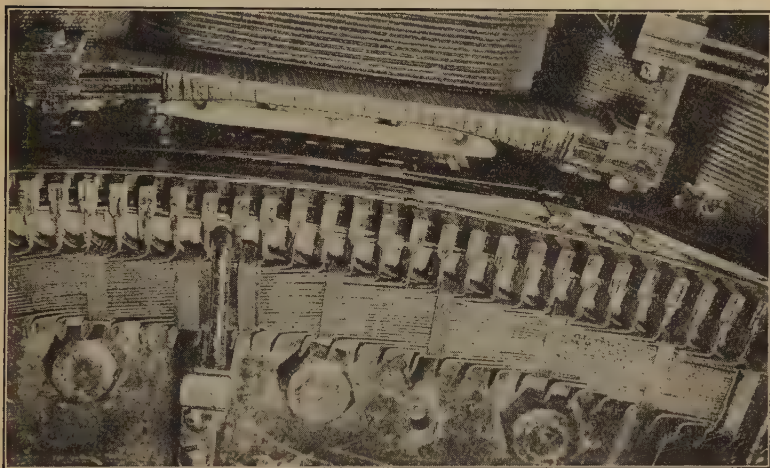


FIG. 22.—Generator showing equalizer ring, vertical involute type. (*General Electric Company.*)

still more important to keep them away from the brushes and the commutator. The brushes are primarily intended for carrying current to and from the outside mains and the superposition of circulating currents would impair the commutation. This is accomplished by connecting together, by means of circuits called *equalizer connections,* several points of the armature which are simultaneously at the same potential.

These connections are usually made at the back end of the machine as shown in Fig. 21. It is evident from Fig. 21 that points of equal potential are separated a distance of exactly two pole pitches, and that as a consequence the number of slots should be a multiple of the pairs of poles when equalizer connec-

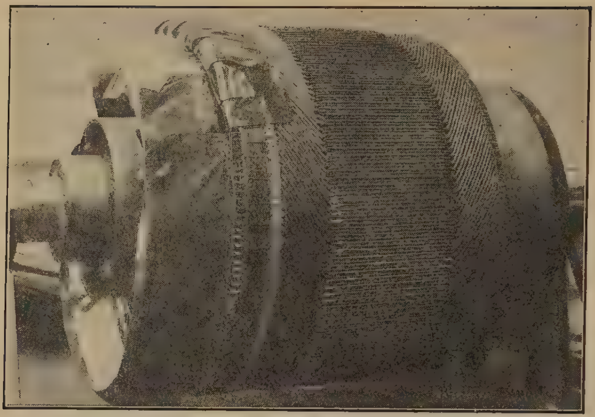


FIG. 23.—Armature of generator showing equalizers. *(General Electric Company.)*

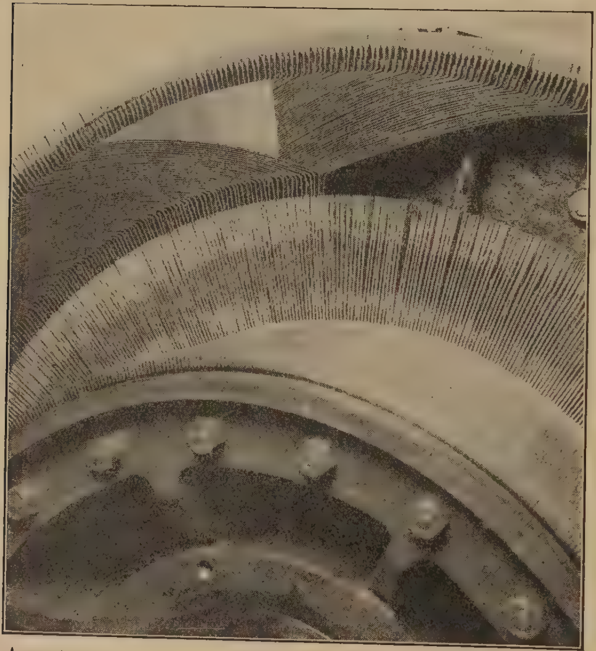


FIG. 24.—Armature of motor showing details of construction of equalizer commutator leads. *(General Electric Company.)*

tions are to be used. It would be possible to connect an equalizer connection to each conductor but this is rarely done as it has been found sufficient to connect every third or fourth conductor as illustrated in Fig. 21. In order that the equalizer connections may be equally distributed along the armature circuit, the number of slots per pole should be some multiple of a small number such as 3 or 4.

Three types of equalizer connections are shown in Figs. 22, 23, and 24. The vertical involute type shown in Fig. 22 is on the back end of the armature, while the corresponding horizontal or surface position of epualizer connections is shown in Fig. 23. In Fig. 24 are shown the details of construction for gaining the same goal by using equalizing commutator leads.

In all cases the equalizer provides a low resistance path for the circulating currents, which otherwise would pass through the brushes and seriously interfere with the proper commutation of the load current.

In wave windings due to the manner in which the voltage per path is generated no equalizer connections are necessary. That is, for lap windings, the voltage in any path is generated by cutting flux from not more than two poles while in wave windings the conductors of each path are equally distributed among all the poles so that if the flux of any pole is weakened the generated voltage of all paths will be equally affected.

Frog-leg or Grasshopper Winding.—This winding, which derives its name from the shape of the winding coils, has been developed in machines manufactured by the Allis Chalmers Mfg. Co. It is essentially a combination of a lap and wave winding; the primary purpose of which is to eliminate the usual type of equalizer connections. Both windings in the actual machine are placed in the same slots but in Fig. 25 the two windings for the sake of clearness are shown separately, one on each end of the commutator segments for a six-pole machine. The outlines of all six poles have been drawn adjacent to both windings in order to more readily note the location of the conductors with respect to the poles. It will be noted that between the segments *A* and *B* there are three lap-wound coils composed of six conductors and also three wave-wound coils of six conductors. Further examination of Fig. 25 reveals the fact that each one of the wave-wound conductors, numbered consecutively from 1 to 6, is identically located with respect to a pole, as are the correspond-

ing lap-wound conductors numbered from 1 to 6. That is, conductors 1 and 2 of both windings are located for the particular instant at a left-side pole edge, conductors 3 and 4 midway between poles, and conductors 5 and 6 at a right-side pole edge. This is obviously accomplished by making the front and back pitches of both windings the same, and also by placing the initial conductors, numbered 1, in the same slot. The total number of conductors in the machine is so chosen that the number of resulting segments causes the wave winding to fall short of closing

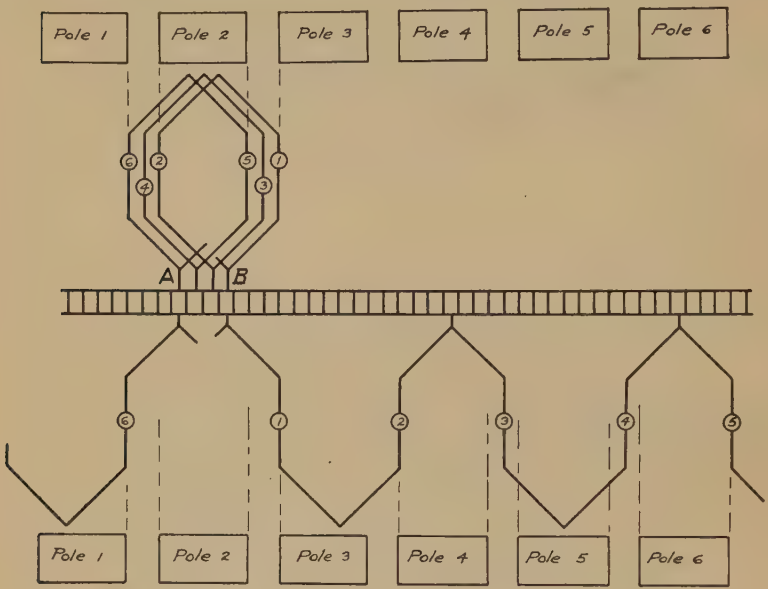


Fig. 25.—Frog-leg or grasshopper winding.

by three commutator bars. The wave winding of the machine is essentially a triplex wave winding. If the machine had eight poles the wave winding would be quadruplex and four lap-wound coils would be used.

The advantage of the above type of winding lies in the fact that the wave-wound conductors, while serving as equalizer connections, are also voltage generating and current carrying conductors in the same sense as the lap-wound conductors. The advantage over the wave winding alone is mainly due to the use of a narrower brush being possible in the frog-leg winding and to the elimination of special equalizer connections.

Recapitulation of Winding Rules.

(*a*) *Lap Winding.*—1.

$$Z_1 = Z/n,$$

where

n = number of turns per element
Z = armature conductors
Z_1 = element or coil sides.

2. The back pitch y_b and the front pitch y_f must both be odd numbers satisfying the equation $y_b = y_f \pm 2m$ and y_b should approximately equal Z_1/p; m = multiplicity; P = number of poles.

3. The winding is progressive if y_b is greater than y_f and retrogressive if y_b is less than y_f.

4. The number of elements,

$$n = Z_1/2$$

5. The number of commutator segments, $s = n$, the number of elements.

6. The commutator pitch, $y_c = m$, the multiplicity.

7. The winding is multiply reentrant if the number of commutator segments S is divisible by the commutator pitch Y_c; otherwise, it is singly reentrant.

8. The number of paths through the machine $= mp$; that is, the product of the multiplicity factor and the number of poles.

9. For lap windings there are no dummy coils.

(*b*) *Wave Windings.*—1.

$$Z_1 = Z/n,$$

the same as for lap windings.

2. y_b and y_f must both be odd numbers and may be equal to each other, but must satisfy the equation

$$\frac{P}{2}(y_b + y_f) = Z_1 \pm 2m.$$

y_b is approximately equal to Z_1/p.

If no possible values of y_b and y_f can be found to fulfill the above equation then the actual number of element sides must be selected less than Z_1.

3. If the positive sign is used in the equation

$$\frac{P}{2}(y_b + y_f) = Z_1 \pm 2m,$$

the winding is progressive; if the negative sign is used, the winding is retrogressive.

4. The number of elements,

$$n = Z_1/2$$

5. The number of commutator segments, $S = n$, the number of elements.

6. The commutator pitch,

$$y_c = \frac{y_b + y_f}{2}$$

7. The winding is multiply reentrant if the commutator pitch y_c is divisible by the multiplicity m, otherwise it is singly reentrant.

8. The number of paths through the machine equals $2m$.

9. The number of dummy coils or dummy elements is found from the number of coils carrying current with respect to those actually on the armature.

Application of the above winding rules is illustrated by the following problem:

Given a four-pole machine with 184 conductors on its armature, having two coil sides per slot and one turn per element. Find all the design data as listed in the following table for simplex, duplex, and triplex lap and wave windings.

From the given data: $Z_1 = \frac{Z}{1} = Z = 184.$

Table XII

Type of winding	y_b	y_f	y_c	S	Active coil sides	Dummy coils	Reentrancy	Paths	Progressive or retrogressive
Simplex lap	47	45	1	92	184	0	1	4	Progressive
Duplex lap	47	43	2	92	184	0	2	8	Progressive
Triplex lap	47	41	3	92	184	0	1	12	Progressive
Simplex wave	47	45	46	91	182	1	1	2	Progressive
Duplex wave	45	45	45	92	184	0	1	4	Retrogressive
Triplex wave	47	47	47	91	182	1	1	6	Progressive

It is evident that the above windings are only a few of the many that would satisfy the given conditions.

In the design of armatures the selection of a specific type of winding usually depends on several factors. In general, *lap windings* (also called *parallel* or *multiple windings*) are used for machines with comparatively low voltage and high current carrying capacity. *Wave* windings or *series* windings will better

meet the controlling factors in the design of machines to operate at comparatively high voltage and correspondingly low-current carrying capacity.

Thus, in the design of a generator for charging automobile batteries, which requires low-voltage and comparatively high-current capacity, a highly multiple lap winding would be selected. This type has many parallel circuits, each having only a few conductors in series. On the same basis a simplex wave winding would prove most desirable for the armature of a 5-hp. 500-volt motor as there would be only two paths between the terminals and, therefore, half of the conductors would be connected in series.

PROBLEMS

1. A simplex, four-pole, lap winding has 48 coil edges. The front and back pitches are +13 and −11, respectively. Draw the development of this winding showing the location of the poles, the brushes, and the direction of current in each conductor. Number the conductors on the drawing.

2. Draw the development of a duplex lap winding using the same data of Problem 1 except that $y_f = -9$.

3. A simplex, four-pole, wave winding has 46 coil edges. Let $y_b = +13$ and $y_f = +11$. Draw the development of this winding similar to Problem 1.

4. Draw the development of a duplex four-pole wave winding having 48 coil edges. Let $y_b = y_f = +11$.

5. A six-pole machine has 180 conductors on the armature, with one turn per element and two coil sides per slot. Fill out a table like Table XII on page 256 for both lap and wave windings up to and including triplex windings.

6. An eight-pole machine has 432 conductors on its armature with two turns per element and two coil sides per slot. Fill out a table similar to that called for in Problem 5.

7. An eight-pole, simplex, lap-wound machine has 792 conductors arranged in a two-layer winding with 8 conductors per slot, one turn per element. Find suitable front and back pitches and give reasons for your selection.

8. If the simplex lap-wound machine of Problem 5 has a voltage rating of 500 volts and a current rating of 1,000 amp., what is the voltage and current rating of the other machines in Problem 5? What is the kilowatt rating of each machine?

9. An eight-pole, simplex, lap-wound machine has generated in its eight different paths the following voltages respectively: 125, 125.1, 125.3, 125.6, 125.5, 125.2, 125, and 124.8 due to the wear in the bearings. If there are no equalizer rings, what is the current in each path when the armature is delivering 800 amp. (including field excitation) to the load? The armature resistance between terminals is found to be 0.0025 ohms. What is the terminal voltage of the machine at this load?

CHAPTER XIV

ARMATURE REACTION

Armature Reaction in Generators.—The change in magnitude and distribution of the magnetic field surrounding the armature of a dynamo due to the lines of force produced by the currents flowing in the armature conductors is called the armature reaction. Consider a simple, separately excited, two-pole generator with the brushes located midway between the poles, as illustrated diagramatically in Fig. 1 (*a*). Let the arrows indicate the direction of the field and of the armature rotation. If the field is excited but no current flowing in the armature the magnetic flux, ϕ_f will be distributed as indicated in Fig. 1 (*a*). The direction and magnitude of the field is represented as the vector ϕ_f in Fig. 1 (*b*); while a development of Fig. 1 (*a*) is shown in Fig. 1 (*c*). The ordinates in the curve ϕ_f in Fig. 1 (*c*) represents the field strength at successive points on the armature. At successive points around the circumference of the armature, as represented by the curve ϕ_f, the flux density will depend both on the field excitation and on the reluctance of the path followed by the flux lines. The shape of the pole pieces, the length of the airgap, the size of the armature teeth, the density of the flux in the pole tips and, especially in the armature teeth, etc. affect the reluctance and hence the flux distribution. At the points *a* and *b* there is no flux; that is, the brushes are in the neutral plane or neutral zone of the armature with respect to the main magnetic field.

Let the machine in Fig. 1 be represented in Fig. 2 (*a*) without field excitation but with currents flowing as indicated in the armature circuits. The magnetomotive force at any point of the armature is proportional to the included ampere-turns. Since the armature conductors are uniformly spaced the magnetomotive force is represented by the straight line curve H_a as shown in Fig. 2 (*c*); the development of Fig. 2 (*a*). The resulting flux ϕ_a is at each point on the armature directly proportional to the magnetomotive force and inversely proportional to the reluctance of the corresponding magnetic circuit. The armature

magnetomotive force, expressed in ampere-turns, is given by equation (1).

$$H_a = \frac{1}{2}\frac{ZI_a}{pa} \text{ ampere-turns per pole.} \tag{1}$$

H_a = m.m.f. of armature reaction in ampere-turns.
Z = total number of conductors in armature.
p = number of poles.
a = number of paths in armature.
I_a = total armature current.

It should be noted that the direction of the armature flux, as represented by the arrow heads in Fig. 2 (*a*) the vector ϕ_a in Fig. 2 (*b*) and by the ordinates of the curve ϕ_a in Fig. 2 (*c*) is at right angles to the direction of the main field ϕ_f (Fig. 1).

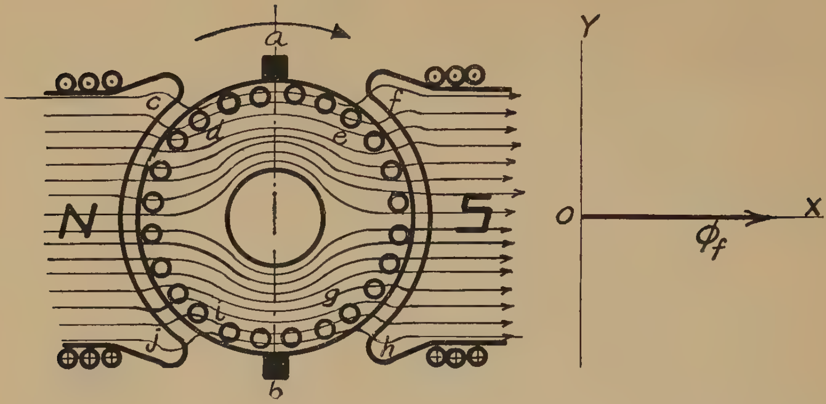
In Fig. 3 is shown, for the same machine, the magnetic flux distribution when the fields are excited and currents flowing in the armature; that is, operating under load conditions. The vector ϕ (Fig. 3 (*b*)) is the resultant of vectors ϕ_f from Fig. 1 (*b*) and ϕ_a from Fig. 2 (*b*). The development in Fig. 3 (*c*) shows the distribution of the total flux by curve ϕ, which, in essence, is the sum of curve ϕ_f in Fig. 1 (*c*) and ϕ_a in Fig. 2 (*c*). Two factors of importance should be noted in Fig. 3.

(1) The flux density is increased at (*c*) and (*h*) (Fig. 3) the *leading* pole tips, and decreased at (*f*) and (*j*), the *trailing* pole tips. The increase, however, is not necessarily equal to the decrease so that the change in flux distribution usually affects the total value.

(2) The armature reaction shifts the neutral plane and, therefore, in order to secure good commutation, the position of the brushes must be shifted or devices be included in the design of the machine to compensate for the armature reaction.

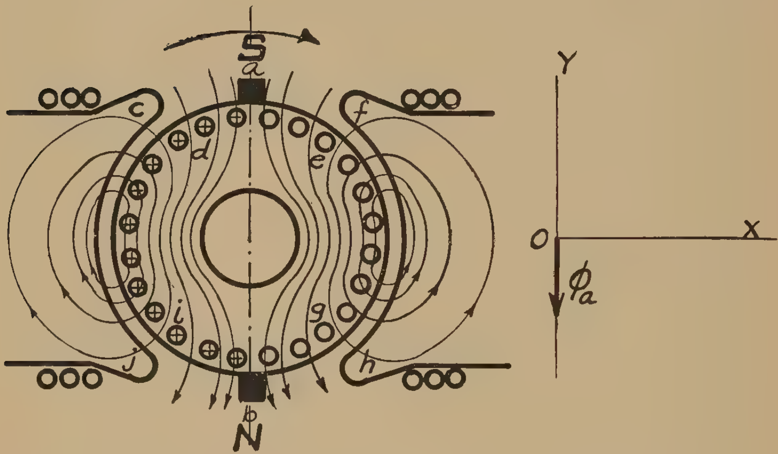
The armature flux crowds the field from the symmetrical position shown in Fig. 1 (*a*) into greater density at *c* the upper tip of the north pole and at *h* the lower tip of the south pole. With the clockwise rotation of the armature, as indicated by the arrow, the magnetic flux is crowded into the *trailing* pole tips while in the two *leading* pole tips the flux density is decreased.

In Fig. 4 is shown the development for one pole, or one half of Fig. 3 (*c*), drawn to larger scale and corrected for unequal flux distribution. It should be noted that although the armature m.m.f. H_a is represented by a straight line crossing the X axis



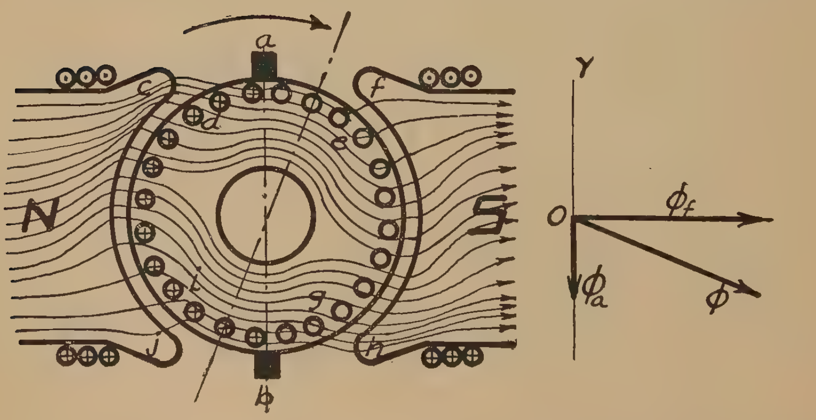
(a) (b)

Fig. 1.—Flux distribution



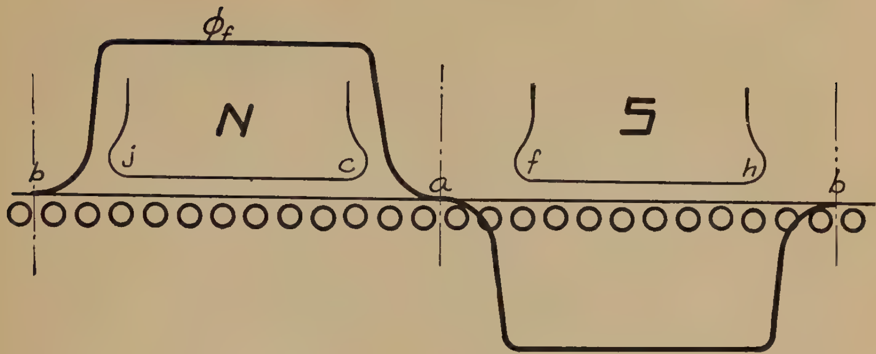
(a) (b)

Fig. 2.—Flux distribution



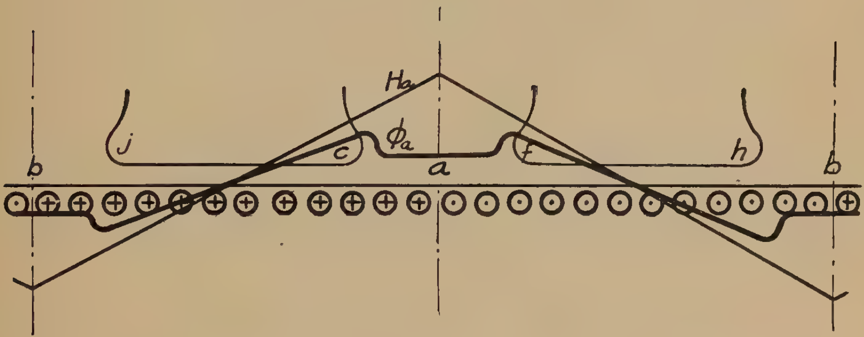
(a) (b)

Fig. 3.—Flux distribution from both



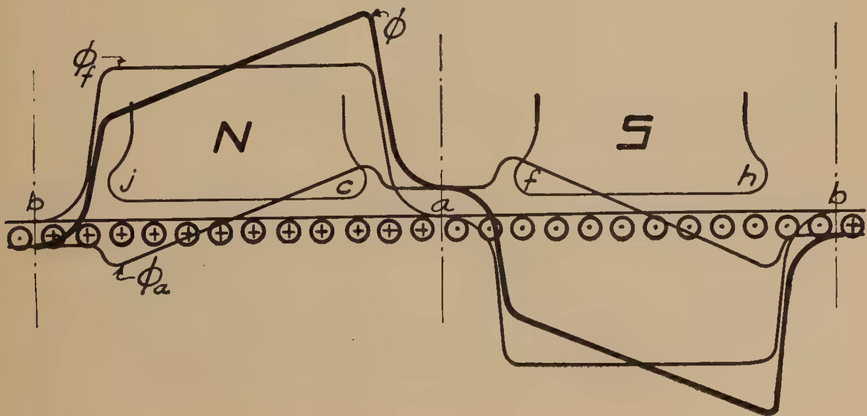
(c)

from field excitation.



(c)

from armature currents.



(c)

field excitation and armature currents.

at the middle point m of the pole, the armature flux curve ϕ_a is not balanced with respect to the center of the pole face. This is due to both of the factors referred to above. If properly designed the pole tips and, more particularly, the armature teeth operate at flux densities near the saturation point. The main field excitation may produce a flux density in the armature teeth as high as 140,000 lines per square inch. This is near or even beyond the saturation point and hence further increase in the flux density will require a much greater proportionate increase in m.m.f. On the other hand, at the leading pole tips, the flux density is less and hence the permeability greater. Hence the armature reaction produces cross-magnetization that is, a demag-

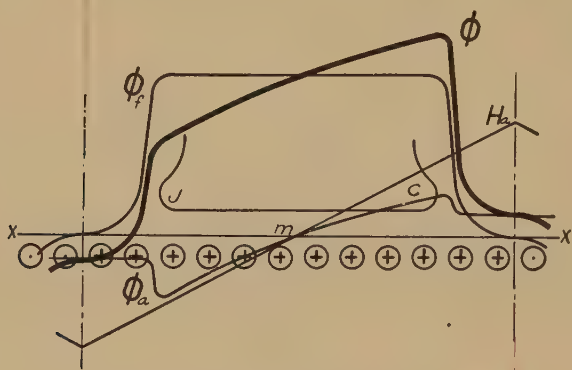


FIG. 4.—Resultant flux-distribution diagram.

netizing component of the armature m.m.f., opposite in direction to the main magnetic field. Part of the armature flux in the g-h sectors, as shown in Fig. 3a, passes through the field yoke and returns to the starting point through the c-d sector and does not pass through the g-h sector. Hence the area between the ϕ_a curve and the X axis on the left side of the point m is greater than the corresponding area included by the same curve on the right side of the middle point m.

Shifting Brushes.—In order to avoid sparking at the contact of the brushes with the commutator; that is, to secure satisfactory commutation, the brushes must be located essentially on the magnetically neutral zone between the poles. From the discussion in the preceding paragraph it is evident that in order to keep the brushes in the neutral zone as the load changes they must be shifted in position. From no load to full load a forward

shift of $\alpha°$ (approximately 18 deg.) as indicated in Fig. 5, would be required.

Selection of brushes adapted to the machine, compensating windings, commutating poles, and short-pitch windings are the

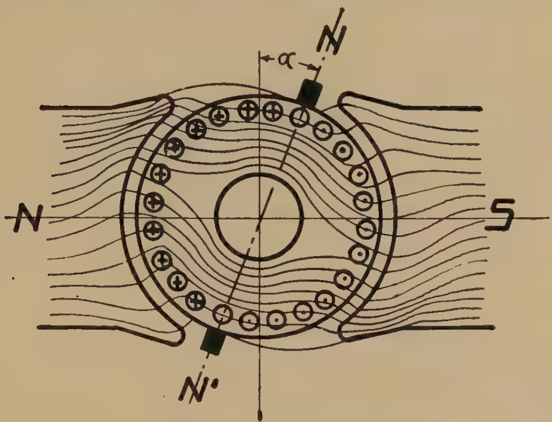


FIG. 5.—Brushes shifted.

more important means for reducing or eliminating the shifting of brushes on both generators and motors.

Demagnetizing and Cross-magnetizing Components of Armature Reaction.—The shifting of the brushes is not merely a move-

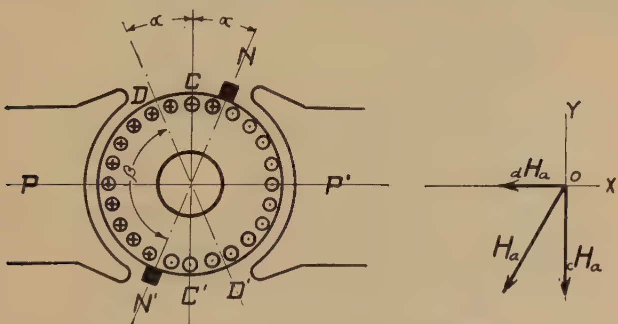


FIG. 6.—Cross- and demagnetizing ampere-turns.

ment of mechanical parts but also a shift or angular turn in the armature field thereby changing the direction as well as the magnitude of the armature reaction. The armature field is no longer in quadrature with the main field but at an angle depending on the load and the position of the brushes. The simplest

way to gain insight into the magnetic effects produced by the shift in brush position is to resolve the armature reaction into two components, one in opposition to the main field, on the X axis, and the other in quadrature, along the Y axis.

In Fig. 6 let the line NN', in the neutral plane, connect the center points of brushes and be at an angle α from the line CC', which represents the neutral plane under no load conditions. Draw the line DD' through the center of the armature and at an angle α on the other side of the center line CC' (Fig. 6). The m.m.f. produced by the ampere-turns in the 2α belt between the NN' and DD' lines in Fig. 6 is the *demagnetizing component* ${}_dH_a$ of the armature reaction H_a.

$$ {}_dH_a = \frac{I_aZ2\alpha}{a360} = \frac{I_aZ\alpha}{a180}, \text{ ampere-turns per pair of poles.} \quad (2) $$

α = brush shift in degrees.
Z = number of armature conductors.
I_a = total armature current.
a = number of paths in armature.

The ampere-turns of the remaining belt of armature conductors, marked β in Fig. 6, give the cross-magnetization component ${}_cH_a$ of the armature reaction.

$$ \beta = 180° - 2\alpha. \quad (3) $$

$$ {}_cH_a = \frac{I_aZB}{a360}, \text{ ampere-turns per pair of poles.} \quad (4) $$

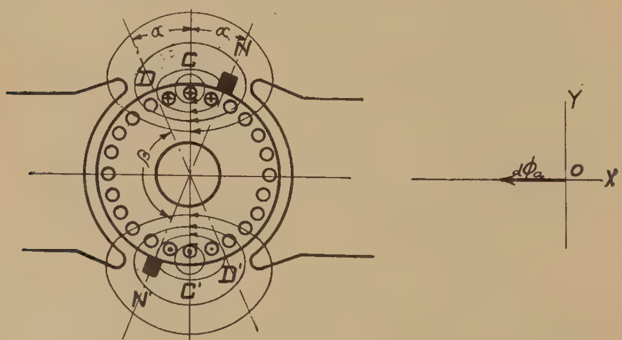


Fig. 7.—Demagnetizing component of armature reaction.

The demagnetizing and cross-magnetizing components of the armature reaction produce corresponding magnetic fields as illustrated by Figs. 7 and 8.

The above discussion relates to full-pitch windings. For machines having short-pitch armature windings the separation of the cross- and demagnetizing ampere-turns is more difficult.

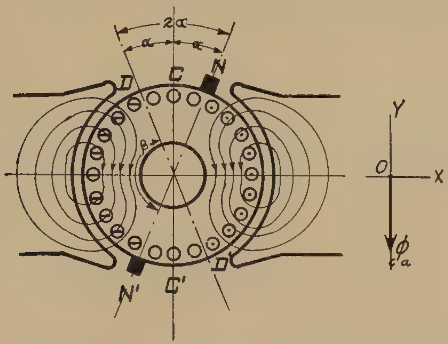


FIG. 8.—Cross-magnetizing component of armature reaction.

In Fig. 9 is shown a cross-section of an armature having 44 conductors. If the winding were full pitch the belt of conductors between c and d and, e and f would produce demagnetizing turns

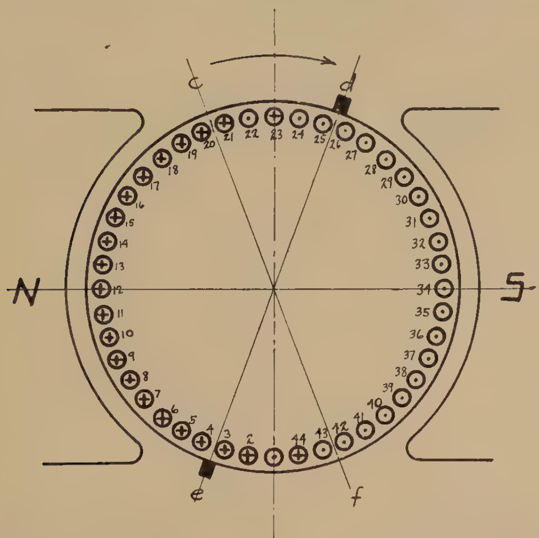


FIG. 9.—Effect of short pitch on demagnetizing ampere-turns.

in the same manner as explained for Fig. 7. In Fig. 9, however, the winding pitch has been made short. The effect of this on the demagnetizing turns is found by developing the winding of

Fig. 9 as shown in Fig. 10. The back connection connects conductor 1 to conductor 20. If the brushes were left in the geometrical neutral zone, conductor 1 would connect directly to a segment in contact with one of the brushes such as the point a. The brushes, however, are advanced in the direction of rotation by an angle α. With the position of the brushes thus located the correct direction of current is indicated on each conductor as shown in Fig. 10. Having thus found the direction of currents they may be marked correctly on Fig. 9 and the true demagnetizing effect may be computed for the belt of conductors between c and d, and e and f.

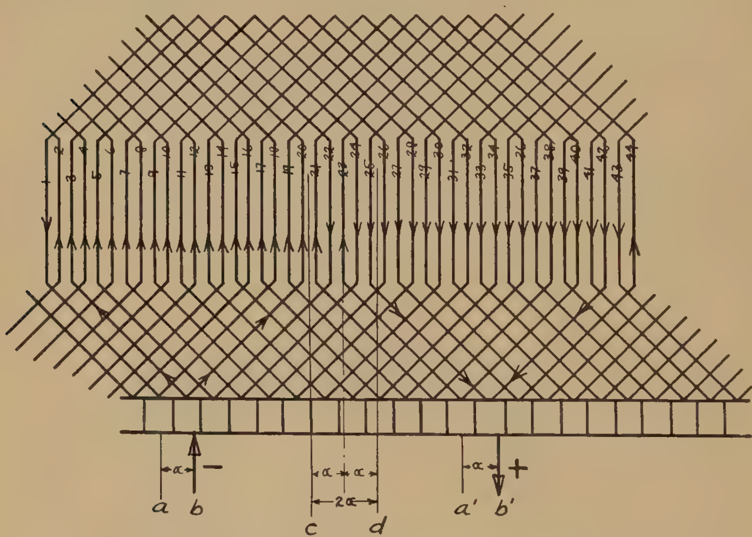


Fig. 10.—Development of winding in Fig. 9.

In Fig. 10 it will be noted that the brushes are assumed as a point contact. This obviously disregards the effect of the short-circuiting current during the commutation period when the current in the coil is being reversed.

Armature Reaction in Motors.—In Figs. 1, 2, and 3 in explanation of armature reaction in generators it has been assumed that the armature was driven in a clockwise direction which caused currents to flow as indicated. It is evident if this same machine be used as a motor having the same field direction, that the armature current must be in the opposite direction from that of the generator if clockwise rotation is to be maintained.

The armature reaction as illustrated by Fig. 2 will thus be just opposite from that of the generator and as a consequence the resultant flux will be distorted in the reverse direction of that shown in Fig. 3. This condition is illustrated in Fig. 11.

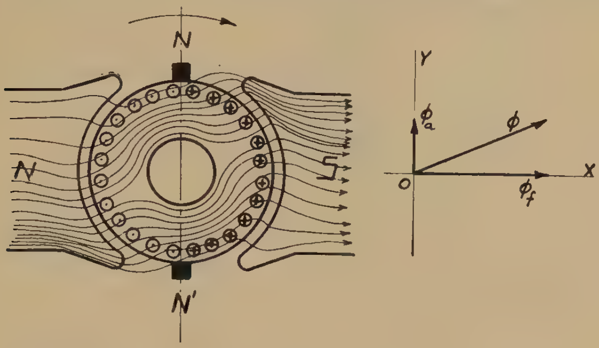


FIG. 11.—Flux distortion due to armature reaction in a motor.

Figure 11 shows that the leading pole tips are highly saturated and that the trailing pole tips are weakened, causing the flux neutral position to shift *backwards* from the direction of rotation, which in comparisón with Fig. 3 is just opposite to generator conditions.

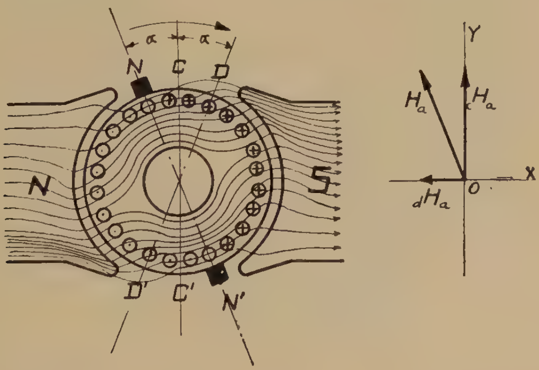


FIG. 12.—Brush shift of motor due to armature reaction.

The correct brush position in motors is therefore as indicated in Fig. 12. In a manner similar to that explained for generators, the brush shift enables the armature m.m.f. to be divided into two parts; namely, the demagnetizing and cross-magnetizing

ampere-turns. The vector diagram of Fig. 12 shows the direction of these components of armature reaction.

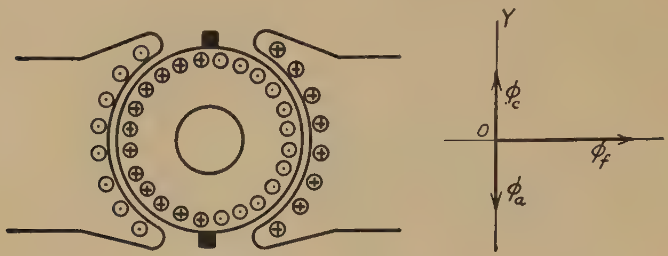


FIG. 13.—Compensating winding.

Compensating Windings.—In order to reduce the magnetic flux produced by the armature reaction a second or compensating

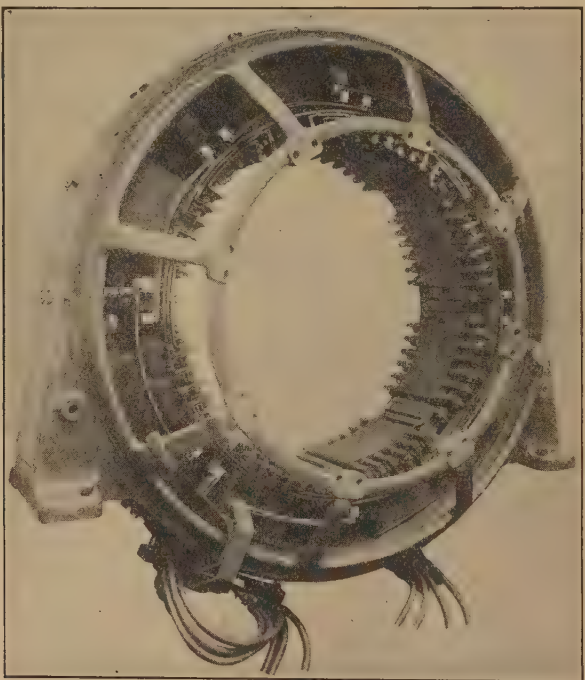


FIG. 14.—Photo showing compensating winding. (*General Electric Company.*)

winding may be placed in slots in the pole face with conductors parallel to the armature conductors as illustrated in Fig. 13. The compensating winding is connected in series with the armature

so that in adjacent bands of conductors the currents flow in opposite direction. Thus the current in the compensating winding largely reduces or neutralizes the m.m.f. of the armature ampere-turns. This prevents or at least greatly reduces the distortion of the field by the armature reaction. The importance of the compensating winding as a means for securing satisfactory commutation is discussed in Chap. XVII. The compensating winding is also an important factor in preventing flashovers between brushes.

From Fig. 13 it is evident that the compensating winding greatly reduces the cross-magnetization. Since every change in load current causes a corresponding change in the cross-flux it is important that the magnitude of the change be kept at a low value. A sudden variation in load, as the interruption of a short circuit by the operation of a circuit breaker, causes a rapid collapse of the cross-flux. Without a compensating winding the sudden change in the cross-flux would produce a high voltage between the commutator segments in addition to the normal operating voltage. This may become high enough to cause a flashover resulting in serious damage to the machine. In machines equipped with compensating winding the tendency to flashover is greatly reduced or entirely eliminated.

PROBLEMS

1. A generator having 420 armature conductors has a forward brush shift of 9½ deg. There are four poles and four paths. Find the demagnetizing ampere-turns per pair of poles due to brush shift when the total armature current is 300 amp.

2. A six-pole generator is wound with a duplex lap winding. There are 504 armature conductors and two turns per element. The brushes are shifted forward two commutator segments. If the total armature current is 384 amp., find the demagnetizing ampere-turns per pair of poles due to brush shift.

3. A four-pole, long-shunt, compound generator has 1,500 shunt-field turns per pair of poles, 20 series-field turns per pair of poles, 780 armature conductors, simplex lap winding, two turns per element. The brush shift is three-commutator segments forward. The shunt-field resistance is 27 ohms. What is the total effective excitation of this machine in ampere-turns per pair of poles when it delivers 100 amp. at 220 volts to a load? Neglect the effect of demagnetization caused by flux distortion.

4. The ampere-turns arising from cross-magnetization in an armature act on a magnetic circuit made up of the airgaps, teeth, armature core, and pole faces. By far the larger part of the m.m.f. is consumed in the airgaps and armature teeth so that the armature core and pole faces may be neglected. It is desired to find the ratio of effective flux at full load to that at no load of the following 10-pole, 550-kw., 250-volt machine. The winding has

720 conductors, simplex lap wound. The ratio of pole-face-to-pole pitch is 65 per cent. The ampere-turns per pole required for the airgap and the teeth are given below. Assume that the shunt-field current remains constant and neglect fringing at pole edges. Data for saturation curve follows:

Ampere-turns per pole required by airgap and teeth	Volts
650	40
1,350	80
2,100	120
2,750	160
3,800	200
5,300	240
5,900	250
6,300	260
7,200	280
9,200	320
11,500	360

CHAPTER XV

GENERATOR CHARACTERISTICS

In the three preceding chapters the structural form of dynamos and the interaction of the electric and magnetic circuits in the machines have been discussed quite apart from the specific nature of the service to be rendered. In general, the basic principles that underlie the design and construction of generators also apply to motors, and hence the preliminary discussion could, to best advantage, be made for both under the common title of dynamos. The operating characteristics of dynamos must of necessity conform to the requirements of the service to be rendered, however, and as a consequence generators and motors must receive independent treatment. An analysis of the operating characteristics of generators is made in this chapter, while the corresponding discussion of motors is found in Chap. XVI.

Generator characteristics are generally shown graphically in the form of curves drawn in rectangular coordinates. When the machine is operating at constant speed these curves show the relation of the terminal voltage of the generator to the load current, under varying load, generally from zero to the maximum permissible load. The size of the machine, number of poles, normal operating speed, and other factors relating to the amount of power, which the generator is designed to deliver, are of little moment as regards its operating characteristics. The factors on which generator characteristics depend are: (*a*) the type of circuit connections used for producing the field excitation, and (*b*) the saturation curve of the machine, which relates to the dimensions and permeability of the component parts of the magnetic circuit.

The discussion of generator characteristics will, therefore, be made under the following subtitles:

1. *Saturation curves:*
    (*a*) Field characteristic.
    (*b*) Process of building up voltage.
    (*c*) Critical field resistance.

2. *Characteristic of separately excited shunt generators:*
    (*a*) External characteristic.
    (*b*) Load characteristic.
3. *Characteristic of self-excited shunt generators:*
    (*a*) Saturation curve.
    (*b*) External characteristic.
    (*c*) Total characteristic.
    (*d*) Components of voltage drop.
4. *Characteristic of series generators:*
    (*a*) External and internal characteristic.
    (*b*) Stability of operation.
5. *Characteristic of compound generators:*
    (*a*) External characteristic, long shunt.
    (*b*) External characteristic, short shunt.
6. *Parallel operation of generators.*

Saturation Curves.—The diagram in Fig. 1 illustrates the composite nature of the magnetic circuits of generators. Thus, by tracing the closed path of the magnetic flux shown in the

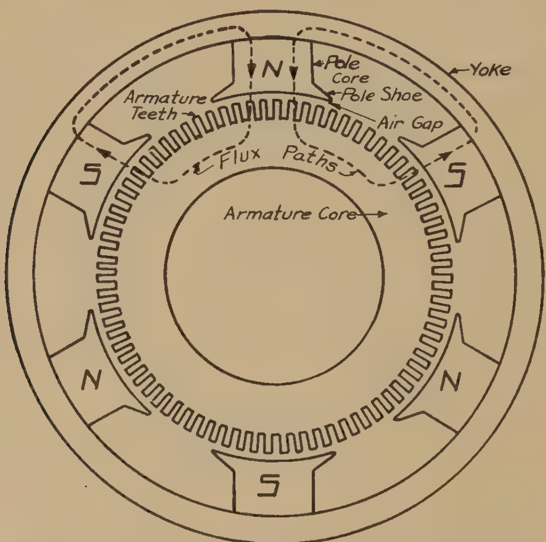


Fig. 1.—Magnetic circuits of multipolar generators.

figure from any starting point, as in the yoke, it is seen that the following series of parts compose the magnetic circuit: yoke, N pole core, N pole shoe, airgap, armature teeth, armature core,

armature teeth, airgap, S pole shoe, S pole core and back to the starting point in the yoke. It is evident that the generated voltage, which is directly proportional to the number of lines of force cut by the armature conductors, depends both on the m.m.f. provided by the field excitation and the reluctance of the magnetic circuit; that is, the paths taken by the magnetic flux. This reluctance depends on the dimensions and permeabilities of the materials in the component sections. The *saturation curve* is a graphical representation of the relation between the field excitation in amperes or ampere-turns and the generated e.m.f. in volts for constant speed of operation.

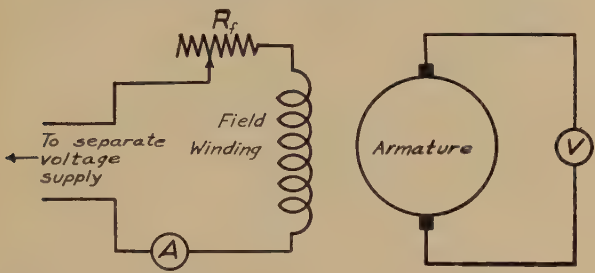


FIG. 2.—Circuit diagram of separately excited generator.

Data for drawing saturation curves may be obtained both experimentally and by design computations. When experimentally determined the power for field excitation must be provided for some outside source. That is, the machine is connected as a separately excited generator, for which the circuit diagram is shown in Fig. 2, and operated at a specified constant speed. By means of an ammeter A in the field circuit and a high resistance voltmeter V connected across the commutator brushes, simultaneous readings may be obtained of the field current and the generated e.m f. Let the field excitation be varied in steps from zero to any desired maximum value and a series of readings taken of the field current and the corresponding armature voltage. By plotting the field-current readings as abscissæ and the corresponding voltages as ordinates and connecting the resulting points, the saturation curve of the generator at the given speed is obtained.

In the same manner data may be obtained from saturation curves at any specified speed of armature rotation. In Fig. 3 are shown the saturation curves of a generator for four different speeds. It should be noted that for any given field excitation

the generated voltage is directly proportional to the speed. The bend, however, or the knee in the saturation curve shows that the flux density is not directly proportional to the field current.

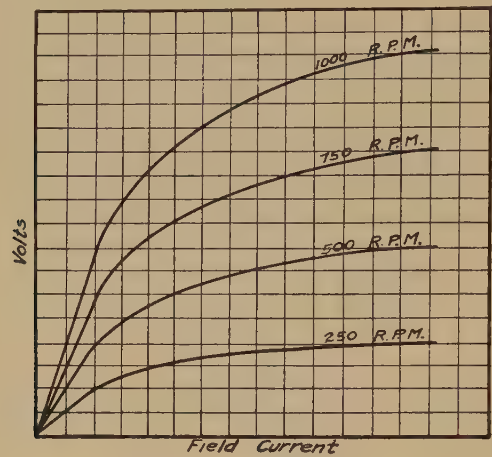


Fig. 3.—Saturation curves.

This is due to the variation in the permeability in the iron and steel sections of the magnetic circuit and also to the corresponding change in the magnetic leakage and in the fringing on the pole pieces and the armature teeth.

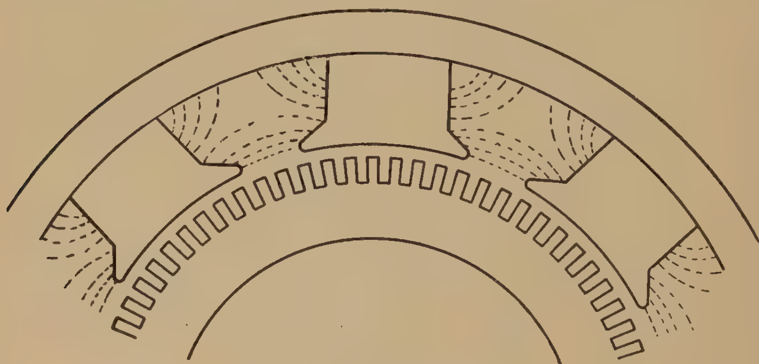


Fig. 4.—Magnetic leakage.

Magnetic leakage between pole cores, pole shoes, yoke, and adjacent poles is illustrated by the broken lines in Fig. 4. Fringing at the pole tips is illustrated by the curved lines in Figs. 5 and 6 and on the armature teeth in Fig. 7. If the permeability

of the iron and steel sections were constant for all flux densities, the leakage and fringing would be directly proportional to the field excitation. But as the permeability decreases with increase in flux density the fringing must necessarily have a correspondingly greater increase at higher field excitations.

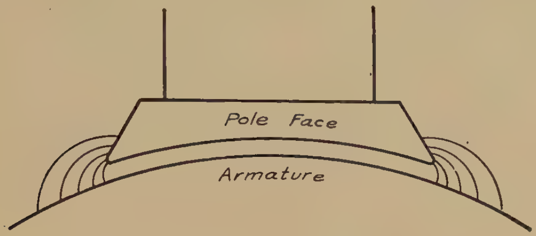


Fig. 5.—Fringing at pole tips, end view.

To show the composite nature of the saturation curve, the corresponding component magnetization curves for the sections of the field magnetic circuit, through which the field flux passes in series, are shown in Fig. 8, which have been computed from design data and not obtained directly from experimental observations.

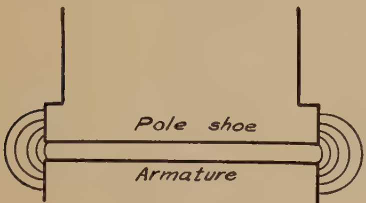


Fig. 6.—Fringing at pole tips, sectional teeth.

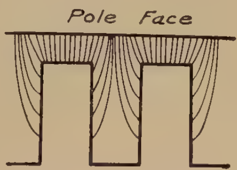


Fig. 7.—Fringing on armature view.

The quality of the iron and steel used and more particularly the flux density which is inversely proportional to the cross-sectional areas of the magnetic circuit in the several sections in series, produce magnetization curves that do not reach the bend of the curve for the same field excitation. This is shown quantitatively by the component magnetization curves in Fig. 8.

The airgap magnetization curve is a straight line through the origin as the permeability is constant and equal to unity at all flux densities. It is well to note that the airgap consumes by far the greater portion of magnetomotive force until the flux density approaches the saturation point. At the normal field excitation

of the machine the airgap requires from 75 per cent to 80 per cent of the total excitation. The magnetization curves for the pole core, yoke, teeth, and armature core, as shown in Fig. 8, differ widely and indicate the field excitation required in each case. By taking, at successive flux density values, the sum of the field exciting current required by the several sections of the magnetic circuit, as given by the respective magnetization curves, the saturation curve of the generator may be obtained.

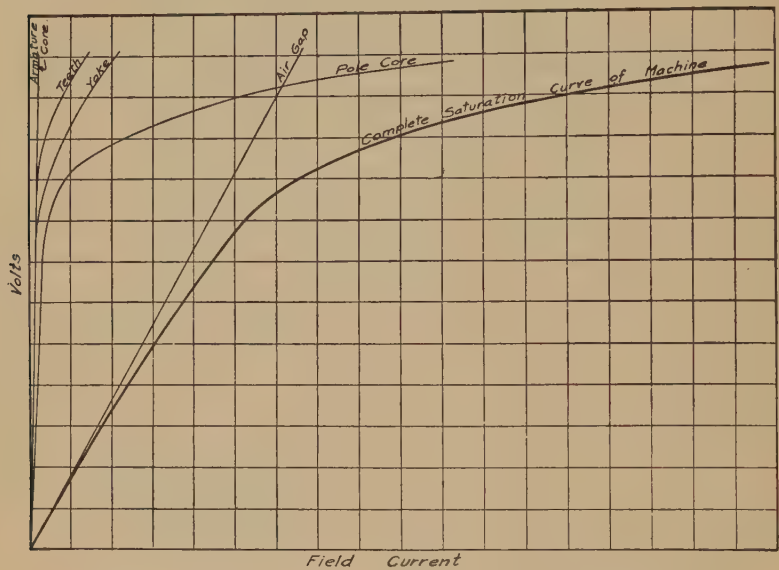


FIG. 8.—Saturation curve and component magnetization curves.

In the design of any particular machine the shape of the individual magnetization curves for the component parts of the magnetic circuit such as yoke, pole core, armature core, etc., may vary considerably from that shown in Fig. 8 by using different quality of material and by varying the dimensions of the several sections. The resulting saturation curve may therefore be varied within wide limits and as a consequence the operating characteristics of the machine must also vary. In fact, by the proper proportioning of the magnetic circuits of a machine the operating characteristics may be made to comply with the special requirements of the given load circuit. After a machine is constructed the saturation and operating characteristic curves may usually be determined by direct test. It is of prime importance however,

that the operating characteristics be quite accurately obtainable before the machine is constructed. The saturation curve is of course obtained as part of the design computations and after the saturation curve is known the operating characteristic curves can be graphically determined for all types of generators.

Field Characteristic Curve.—Since the operating characteristics of generators are obtained under constant speed (determined entirely by the governor of the prime mover) and constant field rheostat setting, it is evident that the operating characteristics will vary considerably, depending on the actual value of the total field resistance.

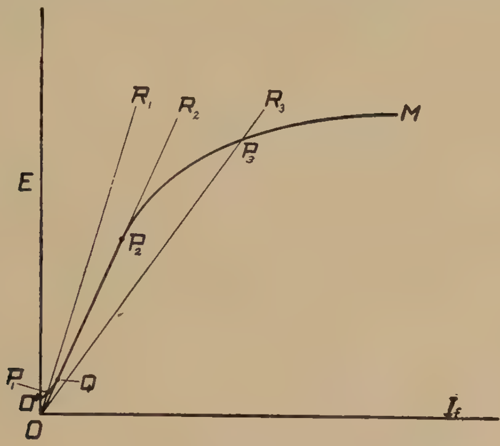


Fig. 9.—Field characteristic curve.

In Fig. 9 let $O'M$ represent the saturation curve of a generator with ordinates representing volts, and the abscissæ, field currents. The ordinate OO' represents the voltage generated at zero field current due to the residual magnetism in the field poles and yokes remaining from the preceding excitation. Let the field rheostat be so adjusted that the total field resistance is equal to R ohms. If the voltage across the field circuit be varied the field current will vary in direct proportion and if plotted as in Fig. 9, will give a straight line relationship as shown by OR_1. The *straight line* OR_1 is called the *field characteristic curve.* If the field rheostat setting be changed so as to reduce the field circuit resistance additional field characteristic curves may be obtained such as OR_2 and OR_3. As the resistance is decreased the slope of the line becomes less.

Generated Voltage.—The voltage generated in a dynamo is directly proportional to the time rate of cutting lines of force by the armature conductors as explained in Chap. V. For direct-current generators the rate of cutting lines of force will depend on the total flux per pole, the number of poles, the number of armature conductors in series and the speed as expressed by equation (1).

$$E = \frac{\phi p Z n}{60 a 10^8} \text{ volts.} \tag{1}$$

ϕ = magnetic flux per pole in maxwells.
p = number of poles.
Z = total number of armature conductors.
a = number of parallel paths in armature.
n = speed in r.p.m.

Equation (1) is basic and applies to all types and sizes of generators as well as motors.

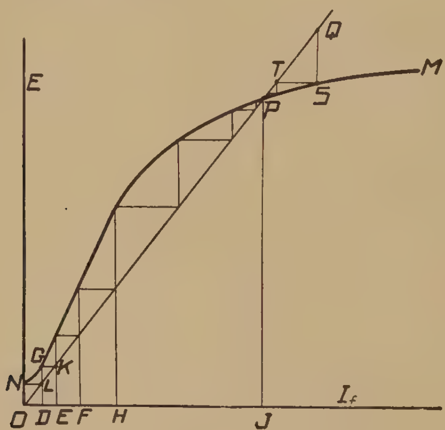


FIG. 10.—Building up voltage.

Process of Building up Voltage.—The importance of the field characteristic curve of a machine is shown in Fig. 10 which illustrates the process of a generator building up its terminal voltage when the field is excited from the armature terminals, as is the case in shunt machines. *NM* represents the saturation curve and *OQ* the field characteristic for a given setting of the field rheostat. The voltage *ON* due to residual magnetism is impressed on the field terminals when the field circuit is con-

nected to the armature producing a field current of OD as determined by the point L on the field characteristic line. The field current OD, however, causes a voltage of GD to be generated, which, acting on the field circuit, produces a current of OE. This step-by-step process continues until the point P is reached and a stable condition results. It is evident that the voltage will not increase any further. If it be assumed that the voltage actually builds up to the point Q, then the voltage at Q will produce field current merely sufficient to generate the voltage at S. The voltage at S, however, will only produce a field current corresponding to T and as a consequence the voltage necessarily comes to that of the point P and is measured by the ordinate PJ.

Critical Field Resistance.—Since the maximum voltage to which the machine will build up is determined by the intersection of the saturation curve and field characteristic line, the point of operation on the saturation curve can be controlled by varying the field resistance. In fact, if the field resistance is so large that the field characteristic falls above the saturation curve as OR_1 in Fig. 9, then the voltage cannot possibly build up. If the field resistance is decreased until the field characteristic would be as represented by the line OR_2 (Fig. 9) parallel with a portion of the saturation curve from Q to P_2, then the terminal voltage is in an unstable condition. This is called the *critical resistance of the field circuit.* For satisfactory stable operation the field resistance must be less than the critical resistance. From the foregoing discussion it follows that a machine may fail to build up for severa reasons:

(a) The field resistance may be too great, the remedy for which is obvious.

(b) There may be no residual magnetism in the field poles and yoke. The remedy in this case is to separately excite the field coils, either by means of separate supply circuit or by batteries.

(c) A third reason for failure in building up may be field windings so connected that they buck the residual magnetism.

Characteristics of Separately Excited Generators.—The circuit diagram of a separately excited generator, carrying load, is represented by Fig. 11.

Let E = generated voltage.
V = terminal voltage, indicated by voltmeter V.
R_l = load resistance.

R_a = armature resistance including the brushes.
R_f = field resistance.
I_a = armature current, also load current.
I_f = field current.
N_f = number of turns in field winding per pair of poles.
n = speed in r.p.m.

Under no-load conditions the generated voltage E is equal to the terminal voltage V. It is evident that under no-load conditions the external characteristic and the saturation curve are identical. Under load conditions the terminal voltage will be less than the generated voltage on account of the armature resistance drop as expressed by equation (2).

$$V = E - R_a I_a \tag{2}$$

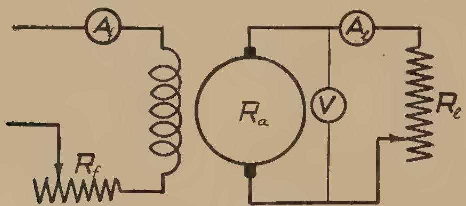


FIG. 11.—Circuit diagram of separately excited generator.

The resistance R_a includes the resistance of the armature conductors, the brush-contact resistance and the resistance of the brushes. The resistance of the armature conductors and the brushes is affected by the temperature but do not change after the temperature reaches a constant value. The brush-contact resistance is not constant, but, as is shown in Chap. XVII, varies practically inversely as the current density, decreases with the brush pressure and increases with the peripheral velocity of the commutator.

(*a*) *External Characteristics.*—In Fig. 12 let $O'M$ be the saturation curve of the generator at normal rated speed. Let the abscissæ represent the field excitation in ampere-turns per pair of poles, $N_f I_f$, and the ordinates the generated voltage E. The data for the saturation curve is obtained by design computations or running the machine at constant speed without load and taking readings as specified in the preceding paragraph.

Let the field rheostat be adjusted until the field ampere-turns are represented by OF_o and the corresponding generated voltage

by the ordinate F_oG_o. Assume a second origin A_o and let the line A_oP_o be the axis of ordinates and A_oA_1 the axis of abscissæ. Draw the broken line G_oP_o parallel to F_oA_o. Therefore the voltage represented by A_oP_o is equal to F_oG_o. Let a load be impressed on the generator which causes a current I_a to flow in the armature and load circuit, and let I_a be represented to scale by the line A_oA_1 (Fig. 12). Likewise, draw the line F_oD_1 of any convenient length through the point F_o and at right angles to the axis of

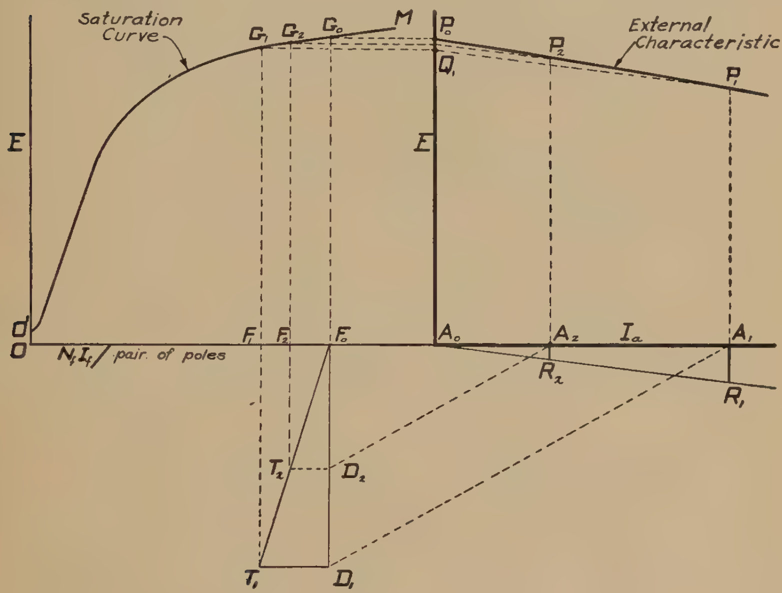


FIG. 12.—External characteristic curve of separately excited generator.

abscissæ; that is, as an extension of the ordinate G_oF_o. Connect the points A_1 and D_1. Through the point D_1 draw the line D_1T_1 parallel to OF_o and let the length of the line D_1T_1 represent the demagnetizing ampere-turns per pair of poles of the armature reaction to the same scale as the field excitation N_fI_f for the saturation curve. D_1T_1 is the demagnetizing ampere-turns due to brush-angle shift as was described under armature reaction in Chap. XIV, but does not include the demagnetizing effect caused by flux distortion, which is difficult to compute for the general case. The demagnetizing ampere-turns per pair of poles are equal to $\alpha ZI_a/180a$ in which α is the brush shift in degrees. Because the demagnetizing effect of flux distortion is neglected, the derivation of the characteristic curve will not be strictly

correct but should give fair results since the effect of flux distortion is in most cases comparatively small. Complete the triangle $F_oD_1T_1$ and erect the ordinate $T_1F_1G_1$. Since the demagnetizing ampere-turns of the armature reaction oppose the field excitation, the net field excitation under the given load conditions is represented by OF_1 and the corresponding generated voltage by the ordinate F_1G_1 or by the line A_oQ_1 equal in value to F_1G_1, as the line G_1Q_1 is drawn parallel to F_1A_o.

Let the ordinate A_1R_1 represent to scale the R_aI_a voltage drop. Draw A_oR_1. The generated voltage corresponding to the load current A_oA_1 has been found equal to A_oQ_1. The terminal voltage obviously equals the generated voltage minus the R_aI_a drop. The terminal voltage is therefore found by subtracting A_1R_1 from A_oQ_1; hence, drawing Q_1P_1 parallel to A_oR_1 locates P_1, which is a point on the external characteristic curve.

For any other load current as A_oA_2 the corresponding value A_2P_2 of the terminal voltage is obtained by making a similar graphical construction. By drawing A_2D_2 and D_2T_2 parallel to A_1D_1 and D_1T_1, respectively, it is evident that the demagnetizing ampere-turns of the armature reaction produced by the current A_oA_2 would be represented by D_2T_2 and the net excitation by OF_2. The corresponding armature-resistance drop is represented by A_2R_2, the generated voltage by F_2G_2, and the terminal voltage by A_2P_2. The line connecting the points P_0, P_1, P_2, etc. giving the relation of the terminal voltage to the armature or load current is the desired *external characteristic of the generator*. From the above analysis it is evident that the form of the external characteristic depends on the form of the saturation curve, the armature reaction, and the armature resistance. Besides neglecting of the effect of flux distortion, minor factors, like the demagnetizing action of the short-circuit currents in armature coils undergoing commutation, have been omitted. The external characteristic relates primarily to operation at the speed for which the machine is designed. For operation at any other speed the corresponding saturation curve must be used, which may greatly affect the form of the external characteristic.

Separately excited, direct-current generators are frequently used for commercial testing and in laboratory experiments. They are, however, seldom used in power plants as equally or more satisfactory results can be obtained by using shunt or compound-wound generators.

(b) *Load Characteristic.*—The load characteristic is the relation between the field excitation and the terminal voltage of the generator when the load current is kept constant in magnitude and the machine operated at constant speed. The load characteristic is represented graphically by a curve as in Fig. 13, for which the ordinates represent the terminal voltages under the above stated constant load current and speed conditions.

In Fig. 13 let OM be the saturation curve of the generator at its rated speed. Let OF be the terminal voltage required for constant load current I through some given value of the variable load resistance R. Draw the line FL parallel to the axis of

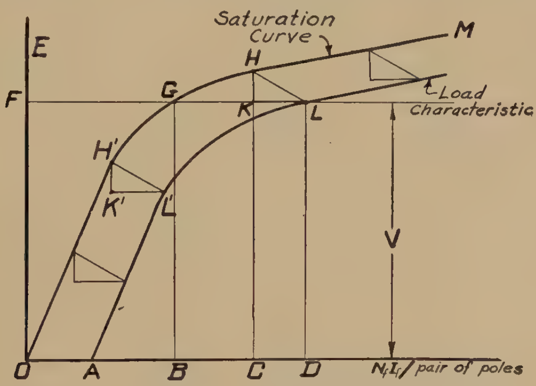


FIG. 13.—Load characteristic of separately excited generator.

abscissæ and intersecting the saturation curve at G. In order to give a terminal voltage equal to BG sufficient additional field excitation must be provided to overcome both the armature resistance drop and the demagnetizing effect of the armature reaction. Find the point H on the saturation curve where the distance HK drawn parallel to OF is equal to the resistance voltage drop. Draw the ordinate HKC. The length of the intercept BC on the axis of abscissæ represents the ampere-turns field excitation required to overcome the armature voltage drop. Lay off KL to represent the ampere-turns field excitation per pair of poles required to overcome the demagnetization of the armature reaction. Draw the ordinate LD. Therefore, OD represents the total required field ampere-turns and L is a point on the load characteristic curve.

For constant-load current the loss of voltage due to armature resistance and armature reactance is constant and represented

by the voltage vector HL. If at any other point as H' on the saturation curve a line $H'L'$ be drawn equal in length and parallel to HL then L' must be a point on the load characteristic. A line joining L, L', etc. forms the desired load-characteristic curve.

Characteristics of Shunt Generator.—For the circuit diagram of shunt generators shown in Fig. 14, let the following notation apply:

E = generated voltage.
V = terminal voltage, indicated by voltmeter V.
R_l = load resistance.
R_f = field resistance.
R_a = armature resistance.
I_l = load current.
I_f = field current.
I_a = armature current.
N = shunt-field turns per pair of poles.
n = speed in r.p.m.

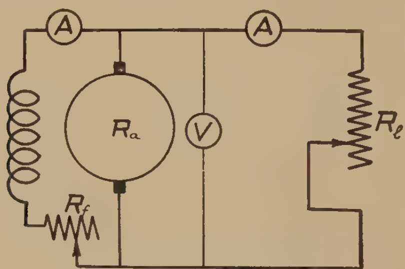


FIG. 14.—Circuit diagram of shunt generator.

The shunt generator is self-exciting with the field circuit connected across the brushes in parallel with the external or load circuit. That is, the field winding shunts the load circuit. A resistance R_f in the field circuit permits regulation of the field excitation independent of the load. Since the field winding is connected directly across the brushes and the field excitation will require only a small part of generated electric energy, the resistance of the field circuit must be large in comparison to that of the load circuit. Hence the shunt-field winding consists of many turns of small-size wire.

(a) *Saturation Curve of Shunt Generator.*—To obtain experimentally the saturation curve the shunt field may be disconnected and the machine changed into a separately excited generator.

By running the machine idle at constant speed and varying the field excitation the saturation curve may be found as explained under separately excited generators.

A very close approximation to the actual saturation curve may be obtained, however, without making any change in field connections or use of outside source for field current. Let the shunt generator run idle, that is, without load at the desired constant speed. The machine is self-exciting and the field current only flows in the armature circuit. The field current is, comparatively, very small so that both the armature drop and the armature reaction may be considered negligibly small. The readings taken on the voltmeter V may therefore, under the given conditions, be considered equivalent to the generated voltage E, or if greater accuracy is desired, a correction for the armature resistance drop can readily be made.

(*b*) *External Characteristic of Shunt Generator.*—In Fig. 15 let $O'M$ be the saturation curve of the shunt generator at rated speed. With the machine running at normal speed but without load, let the field-circuit rheostat be adjusted so that the straight line OL intersecting the saturation curve $O'M$ at K represents the field-circuit characteristic. That is, the slope of the line OL is proportional to the field resistance so that the ordinate $F_oK = E$, the generated voltage, and the abscissæ $OF_o = N_fI_f$, the field excitation in ampere-turns. Hence the coordinates at any point on the line OK represent the corresponding terminal voltage and field excitation of the generator when running at the given constant speed.

Let the point B on the line OF extended be taken as the origin of a second set of coordinates with BE parallel to OE. Let the armature current be represented by the abscissæ, and the terminal voltages by the corresponding ordinates. Draw the straight line BR at such an angle to the axis BC that for any armature current BU the armature resistance drop is represented, to scale, by the corresponding ordinate UV.

Let a load of such value be placed on the generator that the terminal voltage will be represented by the line BS_1. The problem is to locate the point on the external characteristic of the generator. Through S_1 draw the line QS_1D parallel to the axis of abscissæ, intersecting the line OL at H_1. An ordinate drawn from H_1 to OF_o determines the field ampere-turns equal to OF. Let $DH_1 = \alpha zI_a/180a$, the demagnetizing ampere-turns per pair of

poles for any assumed value of armature current. Draw vertically from D the line DG equal to the armature drop for the same value of current. Connect G and H_1 by the line GG_1H_1 and from the point G_1 draw the ordinate G_1D_1A.

Since the armature drop and the demagnetizing ampere-turns, equal to $\alpha z I_a/180a$, are both directly proportional to the armature current for all loads, it is evident that the slope of the line GH_1 is a constant for all loads. The sides D_1G_1 and D_1H_1 of any right triangle having the slope of GH_1 as hypotenuse, therefore, represent simultaneous values of armature drop and demagnetizing ampere-turns respectively for the corresponding armature

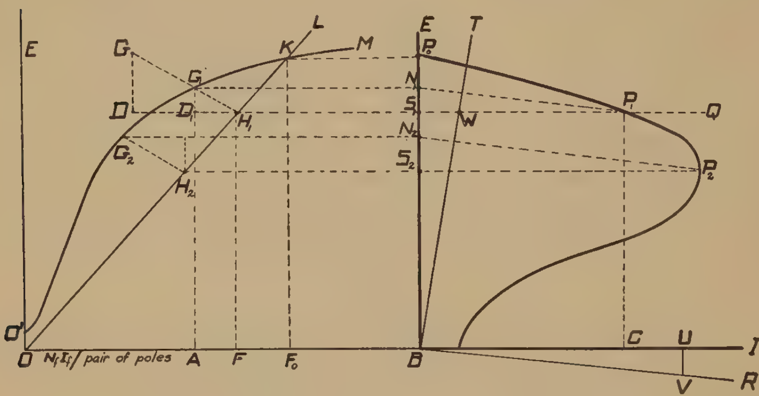


FIG. 15.—External characteristic of shunt generator.

current. That the triangle $D_1G_1H_1$ corresponds to the given value of terminal voltage BS_1 may be shown as follows: The field excitation due to the field current is equal to OF as previously stated. If from this excitation the correct demagnetizing ampere-turns AF or D_1H_1 be subtracted the resultant or net field excitation is given by OA. The generated voltage is therefore AG_1. The terminal voltage and generated voltage differ by the armature voltage drop; that is, H_1F or D_1A added to the armature voltage drop should equal AG_1, the terminal voltage. By construction G_1 falls on the saturation curve and D_1G_1 and D_1H_1 are simultaneous values of the armature voltage drop and demagnetizing ampere-turns produced by the armature current, although not yet determined, corresponding to the terminal voltage BS_1. The armature current corresponding to the armature drop D_1G_1 is readily obtained by drawing through G_1 a line

parallel to D_1S_1 intercepting the axis of ordinates at N_1. Also from N_1 draw a line parallel to BV intercepting the line S_1Q at P_1. From P_1 draw the ordinate P_1C. This determines the current BC corresponding to the voltage drop N_1S_1; and P_1 is therefore a point on the external characteristic. Any other point such as P_2 on the external characteristic curve may be located by assuming some other value of terminal voltage and following the same procedure as outlined above. G_2H_2 is drawn parallel to G_1H_1 and the corresponding armature voltage drop found equal to N_2S_2. N_2P_2 drawn parallel to BV and intersecting the line $H_2S_2P_2$ at P_2, which is a point on the external characteristic curve.

On the line S_1Q let the point W be located so that $S_1W = OF/N_f = I_f$ for the given terminal voltage BS_1. Draw the straight line BT through the point W. The values of the field, load, and armature currents, therefore, are given by the lengths S_1W, WP_1 and S_1P_1, respectively, for the machine operating under load with BS_1 as its terminal voltage.

$$S_1P_1 = BC = I_a \tag{3}$$

$$S_1W = I_f \tag{4}$$

$$WP_1 = I_l \tag{5}$$

Similarly, for any other point P_2 on the external characteristic the values of field, load, and armature currents may be obtained graphically by repeating the above described construction.

It is apparent from the external characteristic curve that the load current increases as the load resistance is decreased, up to a certain point and then decreases. A line drawn parallel to BT and tangent to the external characteristic curve determines the maximum load current the generator can produce. If the resistance in the load circuit is decreased beyond this point the excitation and the load current rapidly decrease to small values; that is, the generator ceases to function or breaks down. If the resistance in the load circuit is increased sufficiently the excitation will build up and terminal voltage will be automatically adjusted to meet the new load conditions. Due to hysteresis the return characteristic curve differs somewhat from the original curve, however, being displaced toward the left. In Fig. 15 it will be noted that under short-circuit conditions when the terminal voltage is zero and no field current is flowing, there is still a fairly large

armature current. The voltage causing this current is obviously produced by the residual magnetism of the machine.

It might be inferred that the shunt generator may be short circuited with impunity. This is, however, not the case, except for very small machines, because the magnitude of the armature current before reaching the maximum critical point on the external characteristic would be far greater than the permissible current-carrying capacity of the armature. The heat generated in the armature by the very large currents would quickly burn out the insulation of the armature windings and the excessive magnetic and mechanical forces may also cause damage in the machine.

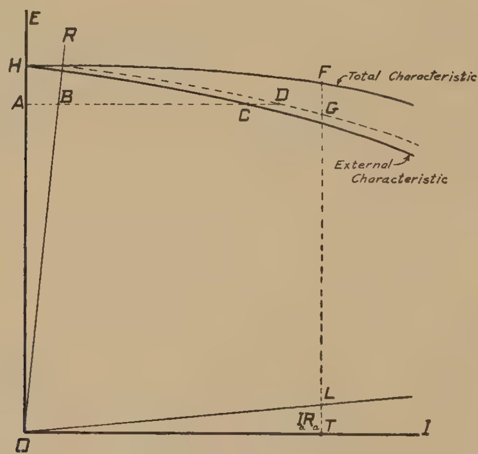


FIG. 16.—Total characteristic of shunt generator.

(c) *Total Characteristic of Shunt Generator.*—In the total characteristic curve the ordinates represent the generated voltages and the abscissæ the corresponding armature currents. In Fig. 16 let the curve HC be the part of the external characteristic of a shunt generator that covers the operating range of the machine. For a given load let OA be the terminal voltage and let the line AD be drawn parallel to OT the axis of abscissæ. Using the same construction as in Fig. 15, let AB represent the field current under the given load conditions. Draw the line OR through the point B. Let $CD = AB$ and the line AD, therefore, will represent the armature current corresponding to the terminal voltage OA. By repeating the above construction for

the terminal voltages of other loads the locus of the armature current plotted against terminal voltage is determined.

Let the abscissæ OT represent the armature current I_a and the ordinate TL the corresponding armature voltage drop R_aI_a. Let the ordinate TF intercept the armature current terminal voltage curve at G. If GF on the ordinate at T is made equal in length to TL, then F is a point on the total characteristics of the generator. By similar construction for other terminal voltages the corresponding points on the total characteristic curve may be located. By drawing the locus thus determined the curve HF or total characteristic of the shunt generator is obtained.

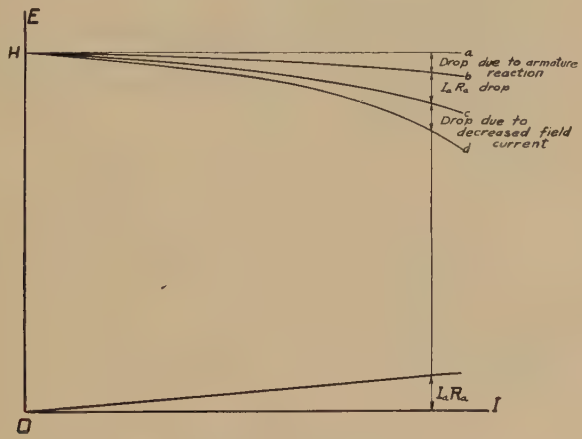


Fig. 17.—Curves showing components of voltage drop in shunt generator.

In Fig. 17 the drop in terminal voltage for changes in load has been separated into its components. These are determined experimentally in the following manner: Curve d is the external characteristic of the machine obtained by running the self-excited generator at normal speed and noting the terminal voltages and the corresponding armature currents under varying load increase. As the load is increased the voltage decreases due to three causes. These are, (a) armature R_aI_a voltage drop; (b) loss of voltage due to armature reaction, and (c) lowering of voltage generated due to decreased field current. The decrease in field current is caused by the voltage drop from (a) and (b), because the machine is self-excited and therefore the field current must be affected by changes in voltage. Obviously, the loss in voltage due to decreased field current may be eliminated during

the test by operating the generator as a separately excited dynamo, keeping the field current constant. Readings of terminal voltage and armature current therefore provide data for locating curve *c*. Curve *b* may be obtained from curve *c* by adding to the ordinates the R_aI_a voltage drop. The remaining drop in voltage between curves *a* and *b* must be caused by the demagnetizing effect of the armature reaction.

A properly designed shunt generator run at its rated speed and operating over the permissible range of variation in load regulates fairly well for approximately constant voltage. Shunt generators are suitable for operation on constant potential circuits where the load is nearby so that there is little line drop. Thus, shunt generators are used for field excitation of alternators. Shunt generators are particularly suitable for charging storage batteries. They will automatically reduce the charging current as the battery voltage increases when nearing the condition of being fully charged.

Change in speed of operation will produce corresponding changes in the saturation curve which in turn affect both the external and total characteristics. In order to obtain the characteristics for any given speed of operation, the saturation curve for the same speed must be used.

Characteristics of Series Generators.—A circuit diagram of a series generator is shown in Fig. 18.

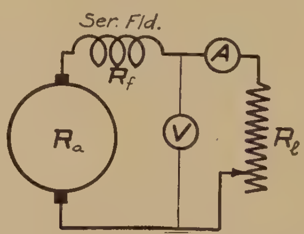


Fig. 18.—Circuit diagram of series generator.

Let E = generated voltage.
V = terminal voltage indicated by voltmeter V.
R_l = load resistance.
R_a = armature resistance, including the brushes.
R_f = field resistance.
I = Current in the circuit; armature, load and field current.
N_s = series field turns per pair of poles.
n = speed in r.p.m.

Since the armature, field, and load circuits are in series, any variation in the load produces a change in the field excitation and likewise a proportionate change for both the armature resistance drop and the demagnetizing action of the armature reaction, assuming that the rotation is held constant at the rated speed.

(a) *External and Internal Characteristics.*—Let it be desired to obtain both the internal characteristic and external characteristic curves of a series generator from the saturation curve. The internal characteristic curve shows the relation between the generated voltage and load current.

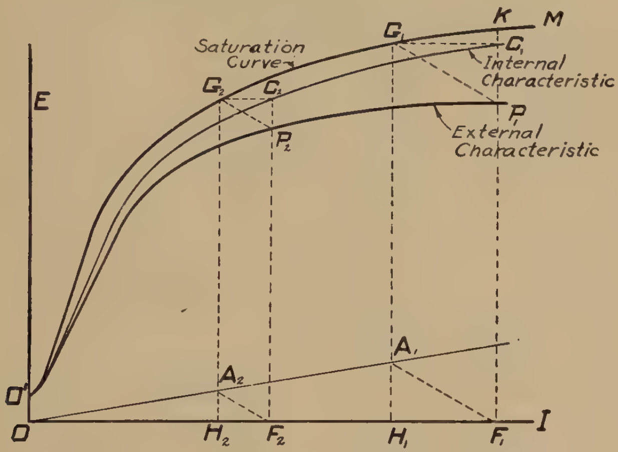


FIG. 19.—Characteristic curves of series generator.

In Fig. 19 let $O'M$ be the saturation curve plotted with volts as ordinates but with series-field current as abscissæ along OF_1 in place of ampere-turns as used in Fig. 15 for the shunt generator. Since the field current for a series machine is the same as the armature and load current, the current values will also represent the abscissæ for the characteristic curves. Let it be required to find points on the internal and external characteristic curves for the load current OF_1 corresponding to the point K on the saturation curve. The distance H_1F_1 is made equal to the equivalent demagnetizing effect of the armature reaction in terms of the current in amperes instead of in ampere-turns field excitation. H_1F_1, therefore, represents the quotient of the demagnetizing ampere-turns per pair of poles divided by the series field turns per pair of poles. An ordinate erected at H_1

intersects the saturation curve at G_1 which determines the generated voltage as equal to H_1G_1. Let G_1C_1 be drawn parallel to H_1F_1. Therefore, C_1 is a point on the internal characteristic curve corresponding to the load current OF_1. If C_1P_1 is drawn equal to the armature and series-field voltage drop for the current OF_1, point P_1 will be a point on the external characteristic curve. Since G_1C_1 and C_1P_1 are directly proportional to load current, additional points such as C_2 and P_2 may be obtained by means of similar triangles, such as $G_2C_2P_2$; the sides of which are made proportional to the load current. This is accomplished graphically by drawing A_1F_1 parallel and equal to G_1P_1, thus locating the point A_1. Connect A_1 to the origin by the line OA_1. For any desired load current such as OF_2 let the line A_2F_2 be drawn parallel to A_1F_1 and ordinates erected at both points, A_2 and F_2. The ordinate G_2H_2, therefore, represents the generated voltage and C_2 the corresponding point on the internal characteristic curve. A line drawn through the point G_2 parallel to A_2F_2 intercepting the ordinate F_2C_2 at P_2 determines a second point P_2 on the external characteristic curve.

(b) *Stability of Operation of Series Generators.*—When starting series generators it is sometimes difficult to build up the voltage. In order that the machine may be self-exciting two conditions must be met. First, the residual magnetism must be in the right direction so that when the current begins to flow the excitation must add to and not reduce the residual magnetism in the field. Second, the resistance in the external circuit must not exceed a critical value that may be determined from the external characteristic curve in much the same manner as was described for the critical resistance value of the field resistance in shunt generators.

The form of the external characteristic of the series generator, if the operation were well beyond the bend or knee of the saturation curve, will show that the machine has an inherent tendency to supply a constant current to the load. However machines are generally, designed to operate near the bend on the saturation curve so that an auxiliary device is necessary. With the addition of some type of regulating device the series generator will deliver a constant direct current for wide variation in load. One form of constant current regulating device for series generators is shown in Fig. 20.

In the diagram R_l represents variable load resistance and R_s a variable resistance shunted across the series field winding. The shunt R_s provides a by-pass for part of the load current. By automatically varying the value of R_s with the changes in load the generator will produce a constant current for variable loads over a wide range.

The constant-current series generator is adapted to supplying power for series arc lighting circuits, but even for this purpose has become almost obsolete, as it has been found more economical to supply series arc-lamp circuits with constant direct current by means of mercury arc rectifiers. The constant-current regulation is accomplished by constant-current transformers supplying power to the mercury arc rectifiers.

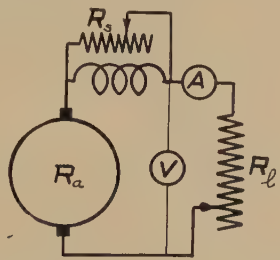


Fig. 20.—Circuit diagram of constant-current generator.

The Thury system using series generators for constant current transmission of power is described in Chap. XX. Also for booster circuit see Chap. XX.

The booster is designed to operate well below the knee of the saturation curve. Under this condition the external characteristic, for the operating range, becomes essentially a straight line and as a consequence the booster will produce a terminal voltage practically directly proportional to the load current.

Characteristics of Compound Generators.—In compound generators the shunt-field current may be connected either directly across the brushes or from one brush to the terminal of the series-field winding. Machines having the first form of field connection are termed *short-shunt generators* (Fig. 24), and the second form, *long-shunt generators*, for which the circuit diagram is shown in Fig. 21.

Let E = generated e.m.f.

V = voltage across the terminals of the shunt field, indicated by voltmeter V.

R_l = load resistance.
R_f = shunt-field resistance.
R_s = series-field resistance.
R_a = armature resistance.
I_l = load current.
I_f = shunt-field current.
I_a = armature current.
N_f = shunt-field turns per pair of poles.
N_s = series-field turns per pair of poles.
n = speed in r.p.m.

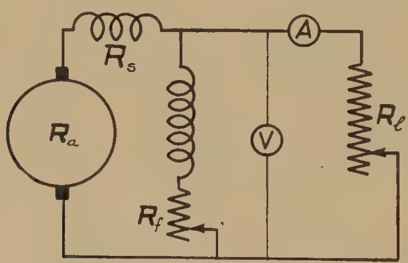


FIG. 21.—Circuit diagram of long-shunt compound generator.

(a) *Saturation Curve for Compound Generator.*—In order to obtain data for the saturation curve the compound generator may be run at rated speed carrying no load and the readings taken in the same manner as explained for shunt generator. This method gives approximate values so that if greater accuracy is necessary the shunt-field winding may be disconnected and the field excitation supplied from some outside source. Data for the saturation curve can be taken with the machine operating as a separately excited generator, using the shunt field only.

(b) *External Characteristic of Long-shunt, Compound Generator.*—In Fig. 22 let $O'M$ be the saturation curve, with the field excitation in ampere-turns per pair of poles along the axis of abscissæ. Let a second set of coordinates be drawn with A_0 as origin along the straight line OA_0A_1. On the axis of abscissæ A_0A_1 the load currents are represented in per cent of full-load current. Through the origin O draw the line OR so as to represent the shunt-field characteristic of the generator. The intersection of OR with the saturation curve at point T, therefore, determines the no-load voltage A_0P_0; obtained by drawing TP_0 parallel to OA_0.

The problem is to determine other points on the external characteristic curve. Let it be desired to find the point corre-

sponding to 100 per cent full-load current A_0A_1. Lay off OC equal to the ampere-turns per pair of poles due to the series field for 100 per cent full-load current. Since the armature reaction opposes the field excitation lay off DC equal to the demagnetizing effect in terms of equivalent ampere-turns per pair of poles for the same value of load current, namely, for 100 per cent full load. The net gain in field excitation, represented in ampere-turns, is therefore represented by OD instead of OC. At the point D lay off BD on the ordinate at D equal to the armature and series-field voltage drop caused by the 100 per cent full-load current. Draw BB_1 parallel to OR intersecting the saturation

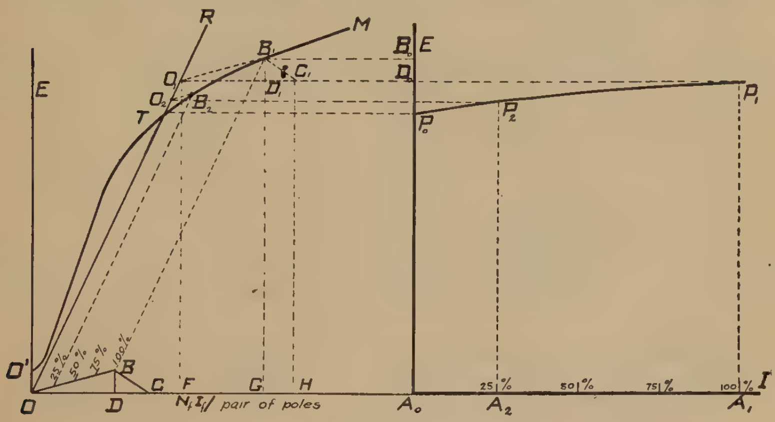


FIG. 22.—External characteristic, long-shunt compound generator.

curve at B_1. Draw B_1O_1 parallel to OB intersecting OR at O_1. Draw $O_1D_oP_1$ parallel to the axis of abscissæ, intercepting the ordinate through the point A_1 at P_1. Thus P_1 is the point on the external characteristic curve corresponding to 100 per cent full-load current. To prove that point P_1 is the desired point on the external characteristic curve draw the ordinates C_1H, B_1G and O_1F through the points C_1, B_1 and O_1.

If the terminal voltage as represented by the point P_1 has been correctly determined, then the field excitation due to the shunt field will be represented by OF in ampere-turns. If to this excitation the series-field ampere-turns O_1C_1 at 100 per cent full-load current be added, the excitation will be represented by OH. The total net excitation will be reduced, however, by the demagnetizing ampere-turns for 100 per cent full load which are represented by $D_1C_1 = HG$, thus leaving OG as the actual effective

excitation. The generated voltage corresponding to OG is GB_1. In the case of compound generators the generated and terminal voltages should differ by the combined armature and series-field voltage drop. Since the point B_1 falls on the saturation curve, this condition is, therefore, satisfied by the triangle $O_1B_1C_1$ which by construction is identical to triangle OBC.

For determining any other point on the external characteristic curve the line OB may be divided into parts measured in per cent of its total length. Let it be desired to locate the point on the external characteristics corresponding to 25 per cent load. Let A_0A_2 be the load current at 25 per cent full load. At the 25-per cent point on the line OB draw a line parallel to OR intersecting the saturation curve at B_2. Through B_2 draw a line parallel to BO intersecting the line OR at O_2. Draw a line through the point O_2 parallel to the axis of abscissæ intersecting the ordinate A_2P_2 at P_2.

The point P_2 is therefore the point on the external characteristic curve and the ordinate A_2P_2 represents the terminal voltage at 25 per cent full load.

Other points in the curve may be located graphically in like manner by repeating the above process. The curve drawn through P_0, P_1, P_2, etc., is the external characteristic of the generator.

The construction in Fig. 22 shows the dependence of the external characteristic on the component factors, as the series-field excitation and the saturation curve. An examination of the above construction makes it evident that the amount of series-field excitation determines whether the terminal voltage shall increase with the load, remain essentially constant for all loads or decrease in value with increasing load.

Increasing the speed of rotation has a marked effect on the compounding produced by the series field, as shown in Fig. 23. At the lower speed the shunt-field ampere-turns NF are required to produce the necessary voltage as compared to NK at the higher speed. For the same load current the series-field excitations FO and KL would be equal in value. But the increase in voltage BC due to the series field at the higher speed is much greater than the corresponding increase EH at the lower speed. The higher the speed the lower will be the point of operation on the saturation curve. Hence, by changing the speed of operation the external characteristic will be modified in the same manner

as if the flux density in the field magnetic circuit were correspondingly lowered. In order to operate at a different speed without changing the external characteristic the ratio of the shunt and series-field ampere-turns at full load must be kept at a fixed ratio. This can be accomplished by means of an adjustable

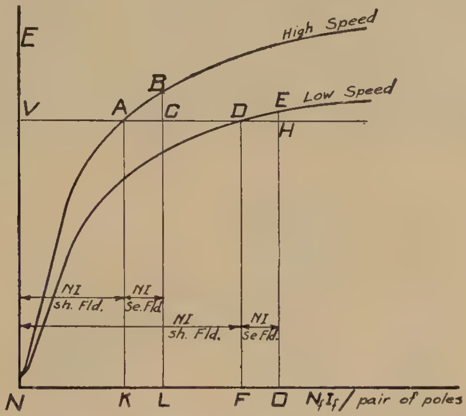


FIG. 23.—Effect of change in speed. Compound generator.

shunt across the series field windings as illustrated by the circuit diagram in Fig. 25.

(*c*) *External Characteristic of Short-shunt Compound Generator.* The circuit diagram of a short-shunt compound generator is shown in Fig. 24.

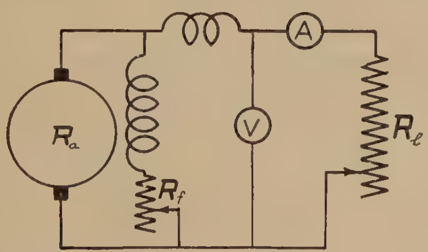


FIG. 24.—Circuit diagram for short-shunt compound generator.

Since the shunt-field current is small in comparison to the load current and the number of turns in the shunt-field circuit is large in comparison to the series-field turns, the external characteristics of short-shunt compound generators is essentially the same as for long-shunt compound generators.

(*d*) *The Series-field Shunt.*—In the design and manufacture of compound-wound generators the series-field winding is given

sufficient number of turns to produce the maximum degree of compounding that may reasonably be expected in service. If less compounding is required in any specific case the magnetizing effect of the series field is reduced by means of an adjustable shunt across the terminals of the series winding, as illustrated in Fig. 25. The shunt consists of german-silver bands or wire and serves as a by-pass for part of the load current. For steady load conditions, that is, for constant load current, the load current divides between the shunt over the series-field winding inversely as the ratio of the resistances in the two

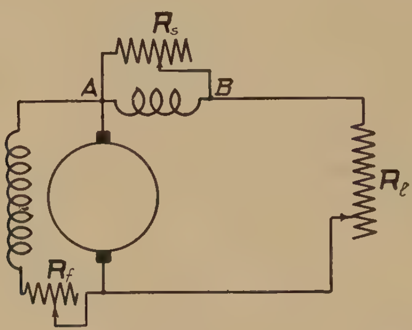


Fig. 25.—Series-field shunt on compound generator.

paths. The division continues nearly in the same ratio for slow changes in load current, but if the load fluctuates quite rapidly the inductance of the field winding must be taken into consideration. This difficulty may be eliminated by inserting an inductance in the shunt across the same field-winding terminals. In order to hold a constant ratio between the current in the shunt and the current in the series-field winding the inductance in the shunt path must be in the same proportion to the inductance of the series-field winding as the resistance of the shunt is to the resistance of the series-field winding.

From the above discussion it is evident that the compound generator is more flexible than the shunt or series generator. By properly adjusting the series-field excitation the external characteristic of the compound generator can be made to conform to practically all load requirements. The increase of voltage from no load to full load can be made to compensate completely for the potential drop, not only inside the generator itself, but for the transmission and distribution circuits as well.

By far the greater part of direct-current power is produced by compound generators.

Parallel Operation of Generators. (*a*) *Series Generator.*—When the load exceeds the rating of a single generator it is necessary to operate two or more machines in parallel. This appears as the natural procedure but makes it necessary to provide means for insuring a proper division of load between the several machines operating in parallel. In order that series generators may operate in parallel and properly divide the load an equalizing connection must be provided between the series fields, as illustrated for two machines in Fig. 26.

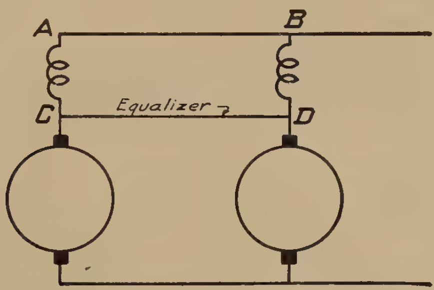


FIG. 26.—Circuit diagram, two series generators in parallel. Equalizer connection.

Without the equalizer, series generators cannot operate in parallel. It is evident by considering the external characteristics that without an equalizer the machine would be in unstable equilibrium, there being no inherent tendency to properly divide the load between the units. If for any cause one of the machines has a slight reduction in power output the machine in parallel increases its load and thereby raises its terminal voltage. This increase in voltage will cause a further increase in current, and as a consequence the current and hence the voltage in the first machine is reduced still further. This process proceeds rapidly until not only will the second machine carry all the load but the first generator would become a motor reversing its direction of rotation.

The effect of the equalizer (Fig. 26) is to bring the series-field windings in parallel with each other whereby the operation becomes stable. Any slight difference in power output or terminal voltage will be automatically corrected as the current in

the two field windings will divide so as to raise the low voltage machine and thereby insure proper division of load.

(b) *Shunt Generators.*—From an examination of the external characteristics of shunt generators it is apparent that the operation of two or more connected in parallel will be stable. An increase in load lowers the terminal voltage and hence if a machine tends to carry more of the load its voltage drops below that of the other machine. Consequently, the second machine, having a tendency for having higher terminal voltage, will automatically increase its load so that the terminal or busbar voltages will be the same for both generators. The division of load will depend on the relative forms of the external characteristics of the

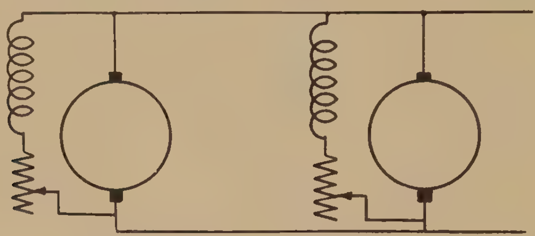


Fig. 27.—Circuit diagram of two shunt generators operating in parallel.

machines operating in parallel and not on their full-load rating. In order that the machines shall divide the load in proportion to the respective ratings their characteristics must be identical, letting load current in each case be plotted in terms of per cent of full load.

(c) *Compound Generators.*—The parallel operation of compound generators may be divided into two groups, depending on whether the terminal voltage decreases or increases with the load. Compound generators with drooping external characteristics will operate in parallel and for the same reason as stated for shunt generators in the preceding paragraph. Any tendency to drop part of the load would tend to raise the voltage and, by the same token, any tendency to take a greater share of the load would also tend to lower the voltage; therefore, the operation in parallel would be stable. To assure division of load proportionate to the rating of the generators the external characteristics must be identical in form. To meet load conditions practically all compound generators are overcompounded, however, so that the terminal voltage increases with the load. That is, overcompounded generators have characteristics like the series

generators, and in order to operate in parallel an equalizer must be provided with circuit connections as shown in Fig. 28. A

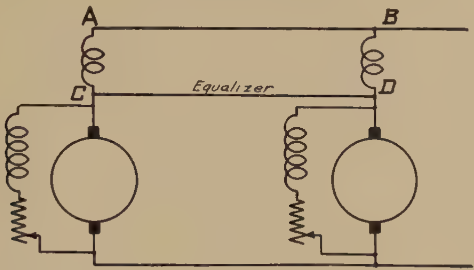


FIG. 28.—Circuit diagram showing equalizer connection for compound generators operating in parallel.

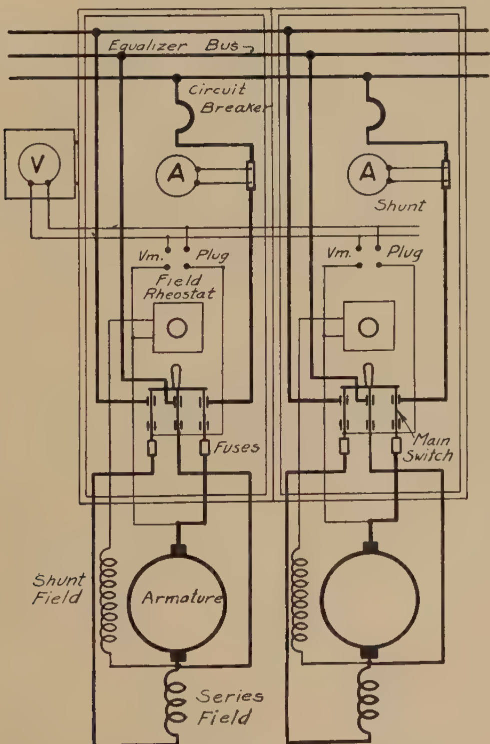


FIG. 29.—Switchboard wiring diagram. Parallel operation of compound generators.

typical switchboard wiring diagram for two compound generators operating in parallel is shown in Fig. 29.

In order to gain a clear insight into the division of load between two compound generators, with equalizer connections operating in parallel, consider the load and external characteristics shown in Figs. 30, 31, and 32. In Fig. 30 assume that at no load the terminal voltage of two shunt generators or two undercompounded generators operating in parallel is the same; that *both have drooping characteristics* and that the curve for generator No. 2 is the flatter. For the given load voltage E_l in Fig. 30, the current in each machine and the load current must be I_1, I_2, and I_l, respectively. That is, generator No. 2, having the flatter characteristic, will carry the greater part of the load.

$$I_1 + I_2 = I_l \tag{6}$$

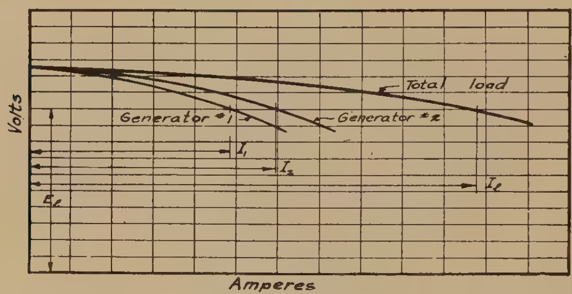


Fig. 30.—Parallel operation of shunt generators or undercompounded generators.

Let the shunt-field excitation or the speed of generator No. 1 be increased until the characteristics will cross, as shown in Fig. 31. For the station load voltage E_l the two generators will share the load equally, that is $I_1 = I_2$ and $I_1 + I_2 = I_l$. For larger loads machine No. 2 will carry the greater share of the load. If the station load is decreased below that corresponding to the value of E_l, for which the terminal voltages of the two machines are equal, the load taken by machine No. 2 will decrease rapidly and, for station loads less than I_3, will take no load but will reverse and operate as a motor.

The form of the characteristics of the two machines, (Fig. 30) can be modified by varying the resistance in the series-field circuit, as the equalizer connection will cause the series-field currents to divide in the inverse ratio of the series-field resistances. A more nearly equal division of load may be obtained, therefore, by having the no-load voltage the same for both machines; and then, by adjusting the ratio of the resistances in the series fields,

make the characteristics of the two machines to practically coincide. Consider the two overcompounded generators represented by the characteristic curves in Fig. 32. By adding resistance to the series-field circuit of generator No. 1, which has the more rapidly rising characteristic curve, the series-field excitation

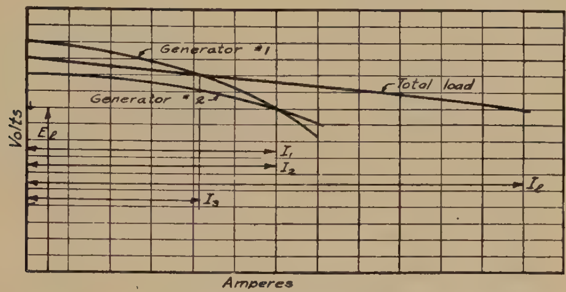


FIG. 31.—Parallel operation of shunt generators or undercompounded generators.

would be decreased, thereby causing a more nearly equal division of load. Or f the rate of rise in voltage as indicated by the curve for generator No. 1 would be desirable, a proper division of load can be obtained by decreasing the resistance in the series field of generator No. 2. As the two series fields are in parallel due to the equalizer connections, any change in the resistance of

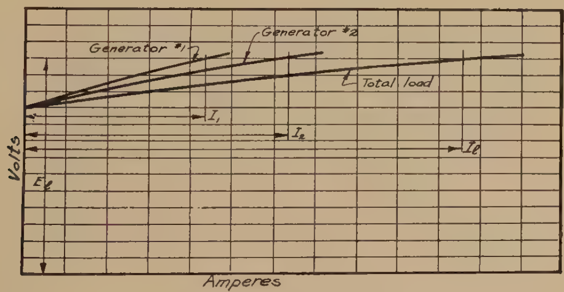


FIG. 32.—Parallel operation of overcompounded generators.

either machine will affect the series-field excitations and, therefore, the external characteristics of both generators. If the load changes rapidly the division of the field currents will be inversely as the impedance of the series-field circuits, which under steady conditions will be in the same proportion as the ratio of the corresponding resistances.

Under constant load conditions and with the series-field resistances constant, the series-field excitation, on account of the equalizer connection, will also be constant for both machines regardless of the division of load between the two machines, that is, the currents in the two armatures. For if the governor of the prime mover of one of the generators tends to increase its speed the *effect on the division of load is essentially determined by the shunt-field excitation.* Basically, for the same reason as for two shunt generators operating in parallel the load would shift towards the generator whose speed was increasing, thereby checking the rise in speed and insuring stable operation.

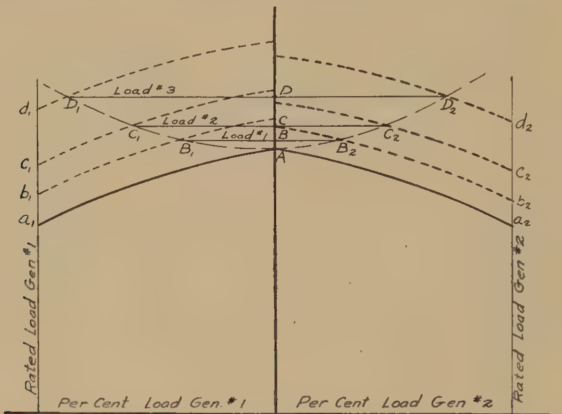


FIG. 33.—Division of load of overcompounded generators operating in parallel.

It is a tedious problem to determine the exact load division between the two generators for all load currents particularly if the saturation curves of the two machines, from no load to full load, differ markedly in form. One method of procedure is to first plot the shunt characteristics Aa_1 and Aa_2 of the two generators (Fig. 33). With the *resistances of the series-field circuits constant and of known values* the series-field excitation for any given load current is readily determined. From the saturation curve and the series-field excitation the increase in field flux in each generator for given increases in load current is obtained. Hence, external (constant-load current) characteristic curves for any given load current may be obtained by adding the excitation produced by the series-field to the shunt-field characteristics. In Fig. 33 a set of these *constant-load current external characteristic curves* (b_1, b_2; c_1, c_2; d_1 d_2) are shown for two

overcompounded generators operating in parallel. From these curves the approximate division of load between the two machines may be obtained as indicated in the diagram for any specified load current. The actual external characteristics for changes in load current are shown in the broken lines $AB_1C_1D_1$ and $AB_2C_2D_2$, intersecting the sets of constant-load characteristics. Any change in the relative amount of resistance in the series-field circuit will affect both the constant-load current characteristic and the external characteristics of the two machines.

Regulation.—Due to armature-resistance voltage drop, armature reaction, variation in speed of prime mover, and other causes, the voltage at the terminals of a generator varies for changes in load. The regulation of a generator is the ratio, expressed in percent, of the difference of the rated-load and no-load terminal voltages to the rated-load voltage. The regulation of a direct current generator is usually stated by giving the numerical values of the voltage at no load and at rated load, and in some cases it is desirable to state regulation at intermediate points. The regulation of direct-current generation refers to changes in voltage corresponding to gradual changes in load and not to comparatively large momentary fluctuations in voltage that sometimes accompany sudden changes in load. The conditions under which the regulation is determined can best be expressed by quoting rule 5-501 of the A.I.E.E. Standards.

5-501 Conditions for Tests of Regulation. (*a*) *Method of Applying Load.*—In making regulation tests, unless otherwise specified, the load shall be increased to the rated load and the regulation observed with decreasing load.

(*b*) *Speed.*—(1) Generators shall be tested at rated speed allowing the specified drop in speed inherent in the prime mover.

(2) Synchronous motor-generator sets shall be tested at constant speed.

(3) Induction motor-generator sets shall be tested at their inherent speed.

(*c*) *Excitation.*—The regulation shall be determined under such conditions as to maintain the field adjustment constant at a value which gives a rated-load voltage at rated-load current. These conditions are as follows:

In the case of separately excited fields, constant excitation.

In the case of self-excited machines, constant resistance in the field circuits.

(*d*) *Temperature.*—The regulation test shall be made at the final temperature attained by the generator at operation under full load for the time specified in the rating, and the temperature shall be maintained as nearly constant as possible during the test. In self-excited generators, correction for change in temperature of the field circuit shall be made by necessary changes in the rheostat, as under (*c*).

(*e*) *Tests of Three-wire Generators.*—The tests of regulation with respect to the outside mains shall be made as with two-wire generators (see paragraphs *a, b, c, d*). The difference in voltage between each outside main and the neutral shall be measured with rated current in the heavier loaded main, rated voltage between the outside mains, and the specified percentage of rated current in the neutral.

PROBLEMS

1. Plot the external characteristic curve of a separately excited shunt generator having the following constants:

Shunt field turns = 1,500 per pair of poles.
α = 7.50° brush-angle lead.
a = 4 paths.
Z = 780.
R_a = 0.2 ohms.
I_f = 3 amp. (constant) (No load V = 209)
n = 1,000 r.p.m.

Find E_T for I_a = 20, 40, 60, 80, and 100 amp.
Data for saturation curve at 1,000 r.p.m.

Shunt Field I_f	E(1,000 r.p.m.)
0.0	7.2
0.3	90.0
0.6	137.0
0.9	164.0
1.2	177.5
1.5	188.5
1.8	194.5
2.1	199.5
2.4	203.5
2.7	207.0
3.0	209.0

2. Find the external characteristic curve of the generator in Problem 1 if the machine is run at 1,110 r.p.m. and the field current is reduced to 1.5 amp. and held constant by separate excitation. Compare the shape of the external characteristic obtained in this problem with that of Problem 1 and give reasons for the difference.

3. Plot the load characteristic curve of the generator in Problem 2 (n = 1,110 r.p.m.) for I_a = 100 amp. Label triangle used in construction and give computations for it.

4. Plot the external characteristic curve of a self-excited shunt generator, given the same saturation curve as in Problem 1 except that the machine is to be driven at 1,110 instead of 1,000 r.p.m.

$p = 4$	R_a = 0.2 ohms
$\alpha = 8.5°$	R_f = 110 ohms (constant)
a = 4 paths	n = 1110 r.p.m.
$Z = 780$	Shunt-field turns per pair of poles = 1,500

5. Plot the external characteristic curve of a series generator having the following constants:

$p = 6$	R_a = 0.21 ohms
$\alpha = 8°$	R_{se} = 0.08 ohms
$a = 6$	N_{se} = 108 (total)
Z = 1,200	

Find E_T for I_a = 10, 30, 50, 65, and 75 amp.

Data for saturation curve as follows:

I	E
0.0	8.0
7.5	100.0
15.0	152.0
22.5	182.0
30.0	198.0
37.5	209.0
45.0	216.0
52.5	221.5
60.0	226.0
67.5	229.5
75.0	232.0

6. Plot the external characteristic of a long-shunt compound generator, assuming that the same saturation curve applies as in Problem 1 when the machine is driven at 1,000 r.p.m.

$\alpha = 8.5°$
a = 4 paths
$Z = 780$
R_a = 0.2 ohms
R_f = 244.5 ohms
R_{se} = .08 ohms
n = 1,342 r.p.m.
N_{se} = 20 per pair of poles
N_f = 1,500 per pair of poles

Find E for I_a = 20, 40, 60, 80, and 100 amp.

7. Plot the external characteristic curve of a long-shunt compound generator having the same data as in Problem 6 except that the shunt-field

resistance is 366.7 ohms and the r.p.m. is increased to 1,608. Compare the curve obtained with that of Problem 6 and explain the difference.

8. Two shunt generators operating in parallel are rated at 100 and 175 kw., respectively. The external characteristics in terms of kilowatt output and terminal voltage may both be represented by straight lines. The first machine has a 5½ per cent regulation and the second generator has 2½ per cent regulation. Both machines are adjusted so that at no load they each have a terminal voltage of 250 volts. Find the load on each machine in kilowatts when they supply a total load of 250 kw. Find the terminal voltage at this load.

9. A simplex wave-wound 900-r.p.m. generator has 8 poles. If the flux is 48,000 lines per square inch and the pole faces are 6 in. × 8 in., find the terminal voltage of the generator when it is delivering 600 amp. The armature resistance is 0.028 ohms and there are 720 armature conductors.

10. Laboratory tests of a four-pole shunt generator show that the field current must be increased from 16.3 to 21.8 amp. in order to maintain constant terminal voltage when the load is varied from zero to full load. There are 500 shunt-field turns per pole. The rated load is 370 amp.

(*a*) If the machine is to be changed to a short-shunt flat compound generator, how many series-field turns must be used per pole? (Select the nearest higher half turn.)

(*b*) If nine series turns per pole are added having a resistance of 0.012 ohms per pole, what should be the resistance of a shunt or diverter around the series field to give a flat compound machine?

11. It is desired that the voltage of an eight-pole 250-kw. 550-volt compound generator should increase from 500 to 550 volts under load increase from no load to full load. Using the shunt field alone for excitation, laboratory tests show that the field current must be increased from 7.7 to 15.8 amp. The machine has 300 shunt-field turns per pole.

(*a*) How many series-field turns per pole will be required?

(*b*) If eight series-turns per pole are added, having a resistance of 0.016 ohms per pole, what should be the resistance of a diverter around the entire series field in order that the generator shall produce 550 volts at full load?

CHAPTER XVI

MOTOR CHARACTERISTICS

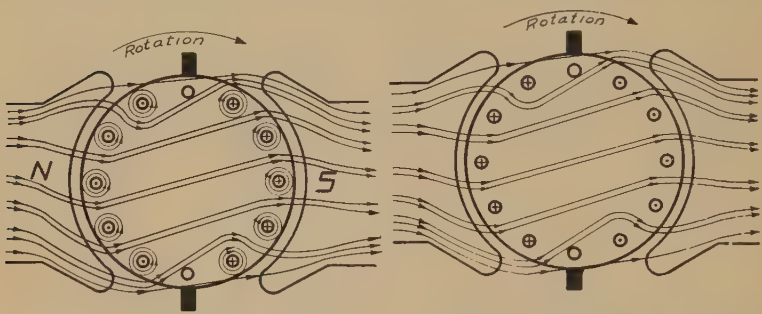
The basic principles of motor action and the general form of the motor as a machine for converting electric energy into mechanical energy are discussed in the preceding chapters on dynamos, armature windings, and armature reaction. In essence, the motor is merely an inverted generator; the fundamental principles are the same while the chief difference is that the conversion as well as the flow of the energy are reversed. The preceding discussions deal with the principles on which electric energy is converted into mechanical energy by the motor without any relation to the characteristics of the mechanical load to be carried; that is, the nature of the service to be rendered. Motors, like generators, must be so designed that they will be adapted to the type as well as to the size of the mechanical load they must carry. It has been found that, aside from the amount of power the machine must deliver, two factors, speed and torque, are of prime importance. Therefore, the *speed* and *torque characteristics* are of fundamental importance in the design and operation of electric motors.

On the basis of the speed factor the industrial application of motor drive may be divided into three classes: (*a*) *constant speed,* (*b*) *multiple speed,* and (*c*) *variable speed.* A large share of industrial loads require motors that operate at constant speed. For this type of service shunt motors connected to constant potential mains are well adapted. In some fields, as, for example, motors for individual drive in machine shops, the service requires that the same motor must be able to operate at several speeds but that for any given speed within the permissible load range the motor must run at a constant speed. In the variable speed division, for which the street railways offer an excellent illustration, the speed varies with changes in load, even though the impressed voltage is approximately constant.

The characteristics of motors are generally stated graphically in the form of curves showing the interdependence of speed,

torque, and power with the circuit connections and the corresponding electric and magnetic quantities.

Counter Electromotive Force.—Figure 1 (*a*) gives a diagrammatic representation of the turning moment produced in a motor by the reaction of the armature current with the magnetic field. If the direction of the field flux and the armature currents is assumed as indicated by the arrows on the lines of force (Fig. 1 (*a*)) a clockwise turning moment is produced, which, if of greater magnitude than required to overcome the friction in the bearings and at the brushes, causes the armature to rotate. When the armature rotates the armature conductors cut the field



(*a*) Direction of motor armature currents.

(*b*) Direction of induced or counter-e.m.f. in armature conductors.

Fig. 1.

flux, thereby generating a voltage, the direction of which is indicated by the dots and crosses on the armature conductors in Fig. 1 (*b*). The lines of force are drawn of the same form in Fig. 1 (*b*) as in Fig. 1 (*a*).

Let V = impressed terminal voltage.
E_a = induced armature voltage (counter-e.m.f.).
I_a = armature current.
R_a = armature resistance.
R_{se} = Series-field resistance.
R_f = Shunt-field resistance.
Z = total number of conductors in armature.
a = number of paths in armature.
ϕ = total flux lines per pole.
p = number of poles.
n = speed in r.p.m.

For shunt motors or separately excited motors,

$$V = E_a + R_a I_a \tag{1}$$

For series motors and compound-wound motors,

$$V = E_a + R_a I_a + R_{se} I_{se} \tag{2}$$

From equations (1) and (2) the current in the armature in any type of direct-current motor is given by equation (3).

$$I_a = \frac{V - E_a}{R_a} \text{ or } \frac{V - E_a - R_{se} I_{se}}{R_a} \tag{3}$$

By referring to Fig. 1, it will be observed that for the same direction of field flux and armature rotation the voltage generated by the cutting of lines of force in the several armature conductors is in opposition to the impressed voltage which causes currents to flow in the armature conductors. That is, the induced voltage E_a is a *counter-e.m.f.* with respect to the impressed voltage V. Hence, the net voltage that causes currents to flow in the armature, which produce motor action, is the difference of the impressed and induced voltages or of the impressed voltage and counter-e.m.f. as expressed by equation (3).

In a properly designed motor the resistance drop must be small in comparison to the impressed voltage and therefore the counter-e.m.f. cannot be much less than the impressed voltage. In the shunt generator the induced voltage equals the sums of the terminal voltage and the armature drop. In the shunt motor the induced voltage, that is counter-e.m.f., is the difference of the terminal impressed voltage and the armature resistance voltage drop.

It is evident that the counter-e.m.f. in the motor can be expressed by the same form of equation as the voltage induced in a generator, as in equation (4).

$$E = \frac{p\phi Zn}{a60 \cdot 10^8} \text{ volts} \tag{4}$$

Since the counter-e.m.f. must be nearly equal to the impressed voltage, it follows from equation (4) that direct-current motors obtaining power from constant potential mains must operate at essentially constant speed unless the flux ϕ is varied. If the field flux ϕ varies for changes of load, either by reason of armature reaction or in ampere-turns of field excitation, a corresponding change of speed will follow automatically.

Mechanical Power.—In a generator the mechanical power supplied to the machine is equal to the sum of the losses in the machine and the electrical power delivered to the outside circuit. In the motor the same relation, but in the reverse order, must be true. That is, the electrical input must be equal to the sum of the losses and the mechanical power delivered to the load. To express this relation for a shunt motor, in the form of an equation, multiply both sides of equation (1) by the armature current and add the term representing the power in the shunt field to both sides of the equation.

$$VI_a + R_fI_f^2 = E_aI_a + R_aI_a^2 + R_fI_f^2 \qquad (5)$$

The sum of $VI_a + R_fI_f^2$, on the left side of the equation represents the power input of the motor and $R_aI_a^2 + R_fI_f^2$ on the right side the copper losses in the machine. Hence, the third term E_aI_a must be an expression for the mechanical power developed by the motor. The greater part of this power forms the power output and is available at the shaft or pulley. The difference between the mechanical power developed in the motor E_aI_a and the power actually transmitted by the shaft to the load, called the mechanical losses, is transformed into heat by friction in the bearings and on the commutator, by windage, core losses, etc., as more fully explained in Chap. XVIII.

Torque.—Equation (5) serves well in stating concisely the basic power relations of motor action but is not applicable for direct measurement of the mechanical output. The counter-e.m.f. E_a is not readily measured when the motor is in operation, particularly under rapidly varying load conditions.

In order to make quantitative comparisons between the power transmitted by shafts, belts, and pulleys, of different dimensions, the term *torque* has been introduced. Torque is the turning moment of the tangential force acting on a pulley or shaft. Torque is expressed as the product of the tangential force and the radial distance in pound-foot, kilogram-meter, dyne-centimeter, etc. Numerically, it is equivalent to the force that at unit distance from the axis of rotation would produce a turning moment of equal magnitude.

Let the pulley in Fig. 2 be 4 ft. in diameter and let the tangential forces in the belt on the two sides be 50 lb. and 125 lb. respectively. A tangential force of 75 lb. is therefore applied to the surface of the pulley. The torque is therefore the product

of the tangential force into the radial distance; that is, 75 lb. × 2 ft. = 150 lb.-ft. This is equivalent to a force of 150 lb.

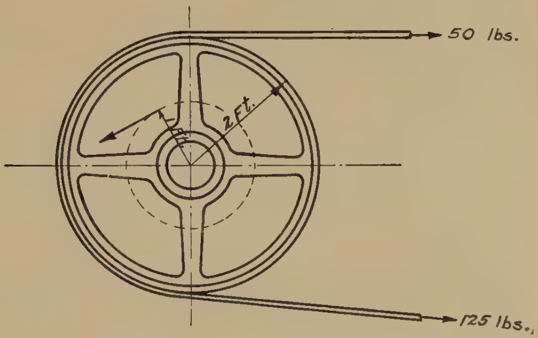


FIG. 2.—Torque illustrated by belt and pulley.

applied tangentially on a pulley of a 1-ft. radius. If expressed in other units, the numbers would be changed. Thus if the tangential force were measured in grams and the radius in centi-

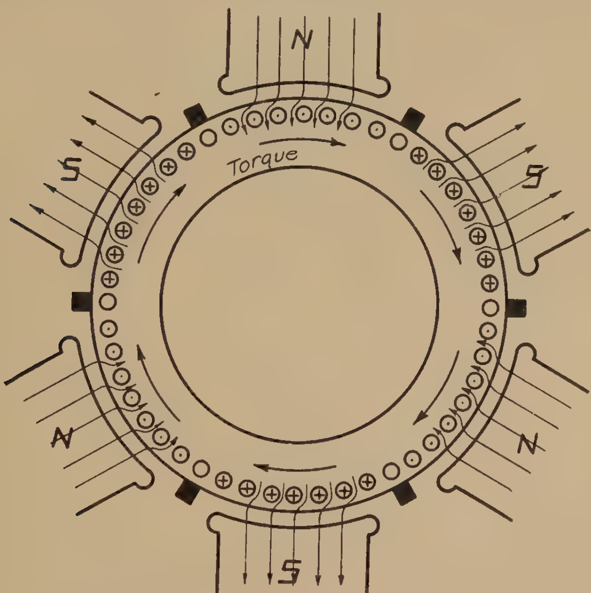


FIG. 3.—Torque in a six-pole motor.

meters the torque would be expressed in gram-centimeters. Therefore, in Fig. 1 the torque T = 150 lb.-ft. = 2,033,600 g.-cm. = 20.336 kg.-meter.

Figure 3 illustrates diagrammatically the torque or turning moment in a six-pole motor. The reaction between the armature currents and the field flux is in the same direction for the six poles. The resulting turning moment represents the torque of the motor, which may also be expressed in terms of the tangential force applied at unit radial distance from the axis of rotation that would produce the same turning moment.

By means of a prony brake the torque can be measured experimentally and used as a factor in computing the mechanical output of the motor.

Let P = mechanical power output expressed in foot-pounds per minute, kilogram-meters per second, etc.

T = torque in pound-foot, kilogram-centimeters, dyne-centimeters, etc.

n = speed in revolutions per minute.

Therefore, if P and T are expressed in consistent units the relation of the mechanical power, torque, and speed is expressed by equations (6), (7), and (8).

$$P(\text{ft.-lb. per minute}) = 2\pi n T(\text{lb. ft.}) \tag{6}$$

$$P(\text{kg.-meter per second}) = \frac{2\pi n T(\text{kg. meter.})}{60} \tag{7}$$

$$P(\text{watts}) = EI\ (\text{watts}) = \frac{2\pi n T(\text{dyne cm.})}{60 \cdot 10^7} \tag{8}$$

Hence, from equations (4) and (8) torques may be expressed in terms of field flux, armature current, and armature conductors as in equation (9).

$$\begin{aligned} T(\text{dyne-cm.}) &= \frac{60 \cdot 10^7 E_a I_a}{2\pi n} \\ &= \frac{60 \cdot 10^7 p\phi Z n I_a}{2\pi n a 60 \cdot 10^8} \end{aligned} \tag{9}$$

$$\text{Let } z' = \frac{pZ}{a60 \cdot 10^8} \tag{10}$$

$$T(\text{dyne-cm.}) = \frac{60 \cdot 10^7 \phi Z' I_a}{2\pi} \tag{11}$$

$$T(\text{kg.-meter}) = \frac{60 \cdot 10^7 \phi Z' I_a}{2\pi \cdot 980 \cdot 10^3 \cdot 10^2} = 0.975\phi Z' I_a \tag{12}$$

$$T(\text{lb.-ft.}) = \frac{60 \cdot 10^7 \phi Z' I_a}{2\pi \cdot 980 \cdot 453.6 \cdot 30.48} = 7.05\phi Z' I_a \tag{13}$$

From the above equations it appears that the torque in any given medium depends only on the product of field flux and the

armature current and is independent of the speed. Indirectly the speed enters in as the current depends on the difference in the impressed voltage and the induced counter-e.m.f. Hence with constant impressed voltage the speed will automatically adjust itself so that the counter-e.m.f. will permit sufficient current to flow in the armature to produce the required torque.

Any change in the mechanical load on the motor must produce a corresponding change in the electric power input. Increasing the mechanical load will require more current to flow in the armature, assuming the impressed voltage to be constant. To cause more current to flow the counter-e.m.f. must decrease, which requires a decrease in the product of the field flux and the speed. If the motor field remains constant the speed must decrease. Under steady load conditions the speed is constant and the current flowing in the armature is of the magnitude that will produce the required torque. In differentially wound compound motors the load current flowing through the series field produces ampere-turns in opposition to the shunt-field current. Hence with increase in load the field flux is reduced and as a consequence the counter-e.m.f. is decreased, causing a greater armature current to flow. If the series- and shunt-field ampere-turns are in correct proportion the differential motor will operate at constant speed for wide variations in load. If the series field decreases the field flux at a greater rate than would be required for constant speed operation the armature speed will increase with the increase in load. This condition is rarely desired, but differential compound windings, adjusted to produce constant speed for a wide range of loads, is of great practical importance.

Characteristics of the Shunt Motor and the Separately Excited Motor. (*a*) *Speed Characteristic.*—In the upper left-hand corner and the lower right-hand corner of Fig. 4 are shown the circuit diagrams of the separately excited and shunt motor, respectively. Under the condition that both the impressed voltage V and the field excitation current are constant the speed and torque characteristics for the two types are alike. From equations (1), (4), and (10) an expression for the speed may be derived as given in equation (14).

$$n = \frac{V - R_a I_a}{\phi Z'} \tag{14}$$

If there were no armature reaction the field flux ϕ would be constant and hence the speed-load characteristic would be a straight line, the speed decreasing somewhat with the increase in load. The armature reaction is, however, a factor that cannot be neglected and therefore the value of ϕ, under the assumed

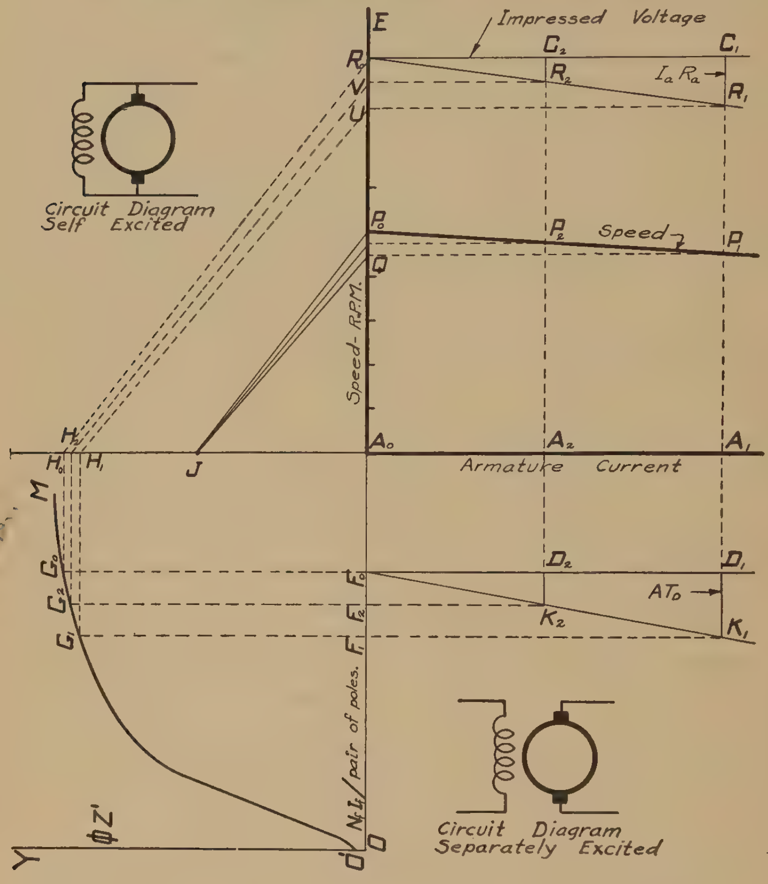


FIG. 4.—Speed characteristics of shunt motor and of separately excited motor.

condition of constant-field excitation, will vary with the load; the amount depending largely on the saturation curve of the machine and the flux density of the field magnetic circuit.

The demagnetizing effect arising from flux distortion must of necessity be neglected as was done in the determination of the characteristic curves of generators because it cannot be evaluated

for use as a general case. As previously mentioned, however, under generator discussions, the error should be comparatively small. The demagnetizing effect caused by the brush-angle shift is, therefore, the only one considered.

In Fig. 4 let the coordinate axes A_0A_1 and A_0E be extended to H_0 and O, respectively, and let the line OY be drawn parallel to A_0A_1. With the point A_0 as the origin let the abscissæ represent armature current and the ordinate speed in r.p.m. Also let the voltage be represented on the A_0E axis.

Let O be a second origin with the coordinates turned 90 deg. from the above set and let $O'M$ be the saturation curve of the motor at its rated speed, plotted with ampere-turns per pair of poles along OA_0 and with values of $\phi Z'$ along OY, in place of the usual value of generated voltage.

Let A_0R_0 represent the constant impressed voltage V, and A_0A_1 the full-load armature current. Through A_1 draw the ordinate $C_1A_1K_1$ and let $A_1C_1 = A_0R_0$. Draw C_1R_0 and lay off C_1R_1, on C_1A_1 equal to the armature voltage drop R_aI_a at full-load current I_a. Draw the straight line R_1R_0. For any other armature current as A_0A_2 the intercept C_2R_2 on the corresponding ordinate of the lines C_1R_0 and R_1R_0 represent the armature voltage drop similar to C_1R_1 for the current A_0A_1. Through R_1 draw a line parallel to A_0A_1 intercepting the axis of ordinates at U. Let OF_0 represent the constant-field excitation, N_fI_f per pair of poles, of the motor. Through F_0 draw a line parallel to OY intercepting the ordinate A_1K_1 at D_1 and the saturation curve $O'M$ at G_0. On the ordinate K_1A_1 let the distance K_1D_1 represent to scale the demagnetizing ampere-turns per pair of poles = $\alpha ZI_a/180a$. Draw the straight line K_1F_0. Hence intercepts on other ordinates similarly drawn for other currents would represent the corresponding demagnetizing ampere-turns. Thus the intercept D_2K_2 on the ordinate drawn through A_2 represents the demagnetizing ampere-turns for the armature current A_0A_2.

Through K_1 draw a line parallel to OY and intercepting OA_0 at F_1 and the saturation curve at G_1. Then OF_1 represents the net field excitation at full load which is represented by the armature current A_0A_1. Through G_0 and G_1 draw lines parallel to OA_0 intercepting the A_0A_1 axis at H_0 and H_1, respectively. Draw the straight lines H_0R_0 and H_1U.

Let A_0P_0 represent the theoretical no-load speed of the motor in r.p.m. This value may be readily computed from equation

(14) and a suitable speed scale thus chosen along the ordinate A_0P_0.

$$n = \frac{V - R_aI_a}{\phi Z'}$$

where $V = A_0R_0$, $I_aR_a = 0$, and $\phi Z' = H_0A_0$, all of which correspond to the theoretical no-load condition. The term *theoretical no load* corresponding to zero armature current is here used to differentiate from the actual no load condition where a small armature current flows in order to overcome friction, windage, etc. Through P_0 draw a line parallel to H_0R_0 intercepting the axis of abscissæ at J. Through J draw a line parallel to H_1U, intercepting the axis of ordinates at Q. Through Q draw a line parallel to the axis of abscissæ, intercepting the full-load current ordinate at P_1. Then A_1P_1 represents the full-load speed in r.p.m. on the same scale as A_0P_0 represents the no-load speed.

This statement may be proved by the following consideration: From equation (14) and by construction,

$$n_0 = \frac{V}{\phi Z'} = \frac{A_0R_0}{A_0H_0} = \frac{A_0P_0}{A_0J} \tag{15}$$

$$n_1 = \frac{V - I_aR_a}{\phi Z'} = \frac{A_0P_0 - R_0U}{A_0H_1} = \frac{A_0U}{A_0H_1} = \frac{A_0Q}{A_0J} \tag{16}$$

Dividing (16) by (15)

$$\frac{n'}{n_0} = \frac{A_0Q}{A_0P_0} = \frac{A_1P_1}{A_0P_0} \tag{17}$$

Since the scale of speeds was so chosen that A_0P_0 is numerically equal to the no-load speed it follows that A_1P_1 is also numerically equal to the value of speed at the load A_0A_1.

By a similar construction the speed for any other armature current may be obtained graphically. Thus A_2P_2 represents the motor speed for a load current A_0A_2. Connecting the points P_0, P_1, P_2, etc. the speed-load characteristic curve is obtained for all loads under the given conditions of constant impressed voltage V and constant-field excitation. The armature reaction decreases the effective field excitation and hence tends to increase the speed. Therefore, the armature reaction will, in a measure, compensate for the slowing-down effect produced by the armature voltage drop R_aI_a. With field excitation near to or even below the bend in the saturation curve the demagnetizing effect of the armature reaction may overcompensate the effect of the armature voltage drop. Under these conditions the speed will

rise as the load is increased. In practice the field excitation OF_0 is considerably above the bend in the saturation curve. In order to obtain essentially constant speed operation the armature of the motor is designed so as to give stronger armature reaction than for an otherwise identical dynamo to be operated as a generator.

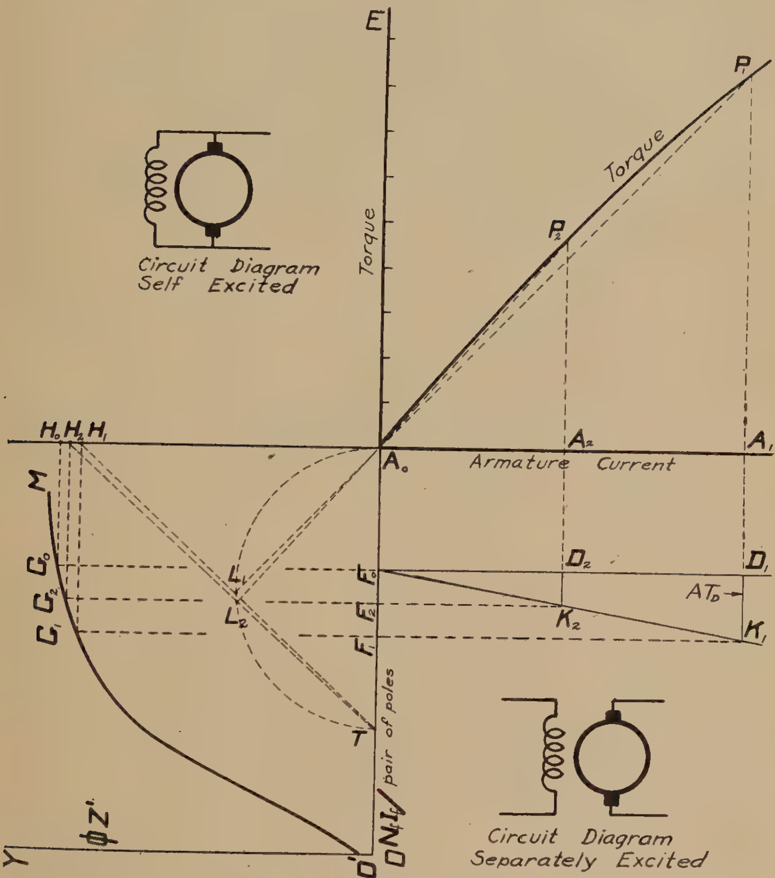


FIG. 5.—Torque characteristic of shunt motor and of separately excited motor.

(b) **Torque Characteristics.**—In Fig. 5 let two sets of coordinates $A_0E - A_0A_1$ and $OA_0 - OY$ be drawn as described for Fig. 4. Assume that the field excitation current I_f and the impressed voltage V are constant. Let A_0A_1 represent the full-load current and erect the ordinate A_1P_1. In the same way as

in Fig. 4 let $O'M$ be the saturation curve of the motor at rated speed and let OF_0 represent the normal field excitation. Through F_0 draw a line parallel to A_0A_1 intercepting the ordinate A_1P_1 at D_1 and the saturation curve at G_0. Let D_1K_1 represent the demagnetizing effect of the armature reaction in ampere-turns per pair of poles and drawn to the same scale as the field excitation OF_0. Through K_1 draw a line parallel to D_1F_0 intercepting the axis OA_0 at F_1 and the saturation curve at G_1. Through G_0 and G_1 draw lines parallel to OA_0 and intercepting the axis A_0A_1 at H_0 and H_1, respectively. Let a convenient length A_0T on the A_0E axis be the diameter of a semicircle. Draw the straight line H_1T intersecting the circumference of the circle at L_1. Through L_1 and A_0 draw a straight line, intersecting the ordinate for A_1 at P_1. Then A_1P_1 is proportional to the full-load torque.

From equation (13),

$$T(\text{lb.-ft.}) = 7.05\phi Z'I_a \tag{18}$$

Hence from construction of Fig. 5,

$$\begin{aligned} T(\text{lb.-ft.}) &= 7.05(F_1G_1)(A_0A_1) & (19)\\ &= 7.05(A_0H_1)(A_0A_1) & (20)\end{aligned}$$

The triangles A_0TH_1 and $A_0A_1P_1$ are similar by construction, and therefore,

$$A_1P_1 = \frac{(A_0H_1)(A_0A_1)}{A_0T} \tag{21}$$

But A_0T is a constant and hence A_1P_1 is proportional to the torque. It may be more desirable to have A_1P_1 exactly equal to the torque, corresponding to a chosen scale along A_0E rather than simply proportional to the torque. This may be accomplished in the following manner: For any given value of armature current such as A_0A_1 the actual value of $\phi Z'$ is obtained by the construction method already outlined. The torque corresponding to A_0A_1 may then be computed from equation (13). A suitable scale is marked off along A_0E and the torque P_1A_1 plotted. From P_1 the line $P_1A_0L_1$ is drawn and at right angles to it the line H_1T. That is, the point T is located by the reverse process of that outlined previously. The semicircle $A_0L_1L_2T$ is drawn the same as before. The diameter of the semicircle is thus determined of such a length as to give direct values of the torque instead of being merely proportional to the torque.

For any other load current as A_0A_2 the corresponding point on the torque curve may be located by the first method of construction utilizing the same semicircle. When the location of a sufficient number of points on the torque curve have been determined the torque curve $A_0P_1P_2$ etc. can be drawn as shown in Fig. 5. The torque characteristic curve, although bending slightly, is essentially a straight line passing through the origin. From the graphical construction in Fig. 5 it is evident that the torque curve will approach a straight line the more nearly the field flux ϕ remains constant over the working range of the motor. The above discussion applies to both shunt and separately excited motors under the assumption that the impressed voltage V and the field excitation current I_f are constant. If the motor is started cold a rise in temperature will occur which will change the resistance of both the field and armature circuits. The assumed conditions of constant-impressed voltage and constant-field current will not be complied with, unless an adjustable rheostat is inserted in the field circuit. For consistent readings in taking data for the speed and torque characteristics the motors should be operated under load until a constant temperature is reached and the readings taken when the temperature conditions of the field and armature circuits are essentially constant.

It should be noted that in the operation of shunt motors on constant potential mains the speed perceptibly increases as the temperature of the machine is rising. The increase in resistance in the field due to the rise in temperature reduces the field current and hence decreases the flux ϕ, and, as a consequence, causes an increase in the speed. The rise in temperature in the armature, however, increases the armature voltage drop, and this tends to decrease the speed. The effect produced by the change in field flux, however, is greater than that of the armature current, and hence the motor speed increases with a rise in temperature.

Characteristics of a Series Motor. (*a*) *Speed-load Characteristics.*—Let it be assumed that the impressed terminal voltage is constant; that is, for obtaining the speed characteristics the series motor receives power from constant potential mains. The graphical construction in Fig. 6 for obtaining the speed-load characteristic is similar to that of Fig. 4 for the shunt motor. The axes for two sets of coordinates $A_0A_1 - A_0E$ and $OA_0 - OY$

are drawn as described for Fig. 4. Let A_0R_0 represent the impressed voltage V; A_0A_1 full-load current; C_1R_1 on the ordinate passing through A_1, the resistance voltage drop $(R_a + R_{se})I_a$ in the motor. Through R_1 draw a line parallel to A_0A_1 intercepting A_0E at N_1; and draw the straight lines R_0R_1 and R_0C_1. Let $O'M$ be the saturation curve of the motor and OF the ampere-

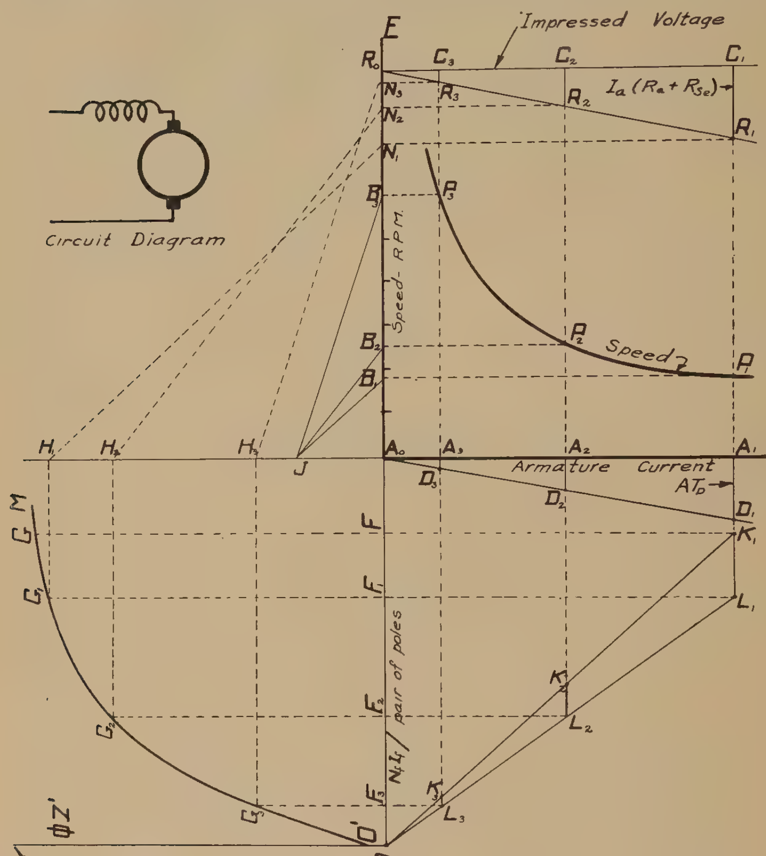


FIG. 6.—Speed characteristic for the series motor.

turns of field excitation per pair of poles N_fI_f. Let the length A_1D_1 on the ordinate at A_1 represent the demagnetizing effect of the armature reaction of the current A_0A_1 drawn to the same ampere-turns scale as the field excitation along OA_0. Along OA_o lay off OF equal to the ampere-turns per pair of poles produced by the field current, which in a series machine is also the armature

current A_0A_1. OF is therefore equal to the armature current A_0A_1 multiplied by the series-field turns per pair of poles. Draw the line GFK_1 parallel to A_0A_1. Lay off K_1L_1 equal to A_1D_1, the demagnetizing ampere-turns, and connect both L_1 and K_1 to the point O by straight lines. The actual field excitation produced at any load such as A_0A_1 may then be found by drawing an ordinate through A_1 to the intersection point with OK_1 at point K_1. The net excitation, however, is obtained by subtracting the demagnetizing ampere-turns K_1L_1 from the field excitation giving OF_1 as the effective excitation and F_1G_1 as the corresponding value of $\phi Z'$. Through G_1 draw a line parallel to OF_1 intersecting A_0A_1 at H_1 and connect H_1 and N_1 by a straight line. By means of equation (14) the speed corresponding to the current A_0A_1 may be computed by measuring the value of $\phi Z'$ as determined by the line A_0H_1. A convenient scale may be chosen along A_0E and the speed corresponding to the current A_0A_1 plotted as A_1P_1. The distance A_0B_1 is made equal to A_1P_1 and through B_1 a line is drawn parallel to N_1H_1 intercepting the axis of ordinates at J. To find other points on the speed characteristic curve, the same procedure is followed as was outlined in the construction of the external characteristic for shunt motors. After the location of a sufficient number of points has been determined the speed curve $P_1P_2P_3$ etc. can be drawn in full.

It should be noted that as the load decreases the speed increases. The rate of increase in speed depends on the operating point on the saturation curve. For points well above the bend in the saturation curve as G and G_1 the change of speed is slight; but for points as G_2 and still more for points below the bend as at G_3 the increase in speed is much greater for the same decrease in load. At small loads the speed rapidly becomes dangerously high and at no load, with only the residual magnetism in the field, the speed would quite likely increase until the machine were wrecked. From the speed characteristic it is apparent that series motors must at all times be directly connected to the load by gears or direct coupling and not by belting so as to protect the motor against excessive speeds that would occur under no load conditions.

(*b*) *Torque Characteristic.*—The construction in Fig. 7 for obtaining the torque characteristic follows the same procedure as in Figs. 5 and 6. The full-load armature current A_0A_1, the saturation curve $O'M$, the field excitation OF_1, the demagnetizing effect

of the armature reaction $A_1D_1 = K_1L_1$, and the location of G_1 and H_1 are identical with the construction in Fig. 6. The distance A_0T is a constant selected for construction purposes and a circle drawn with A_0T as diameter. Connect H_1 and T by a straight line intercepting the circumference of the circle at N_1.

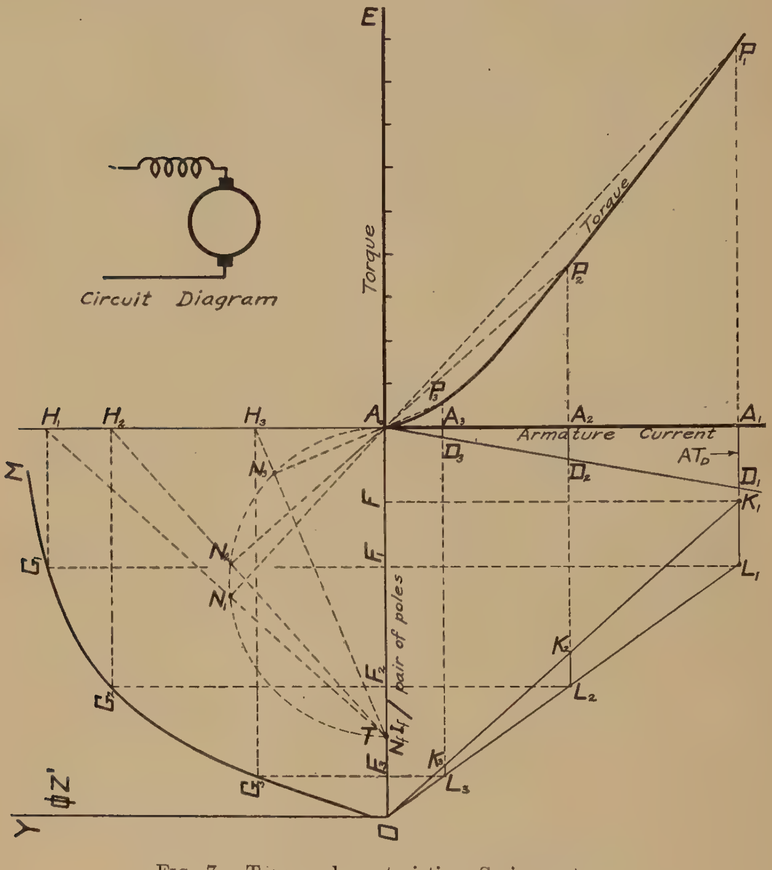


FIG. 7.—Torque characteristic. Series motor

Let the line through N_1 and A_0 be extended until it intercepts the ordinate A_1P_1 at P_1. Then P_1 is a point on the torque characteristic curve. This may be proved by expressing the torque factors in equation (13) in terms of lines in Fig. 7.

$$T = 7.05\phi Z'I_a = 7.05(A_0H_1)(A_0A_1) \tag{22}$$

From the similar triangles $A_0A_1P_1$ and A_0TH_1,

$$A_1P_1 = \frac{A_0H_1 \cdot A_0A_1}{A_0T} \tag{23}$$

Therefore, A_1P_1 is proportional to the torque and P_1 is a point on the torque characteristic curve. I it be desired to obtain the torque so as to numerically correspond to a chosen speed scale on the A_0E axis, the same procedure may be followed in determining the diameter A_0T of the semicircle as was outlined for shunt generators in Fig. 5.

For any other load current as A_0A_2 the construction may be repeated and the location of the corresponding point P_2 determined. After locating a sufficient number of points the complete torque characteristic $P_1P_2P_3$ etc. may be drawn as in Fig. 7.

As the operating range of the series motor covers the whole saturation curve, the torque characteristic necessarily deviates from a straight line much more than the torque curve for the shunt motor. In the series motor at light loads the field flux is nearly directly proportional to the load current and as a result the torque varies approximately as the square of the load current. As load is increased, however, the magnetic circuit approaches saturation and the torque decreases to a more or less direct proportion to the load current as illustrated by Fig. 7.

The torque at the pulley as would be measured in a prony brake test would necessarily be less than the total developed torque by an amount equivalent to the torque that would be required to overcome friction, core loss, and windage in the motor. (See Chap. XVII.)

Characteristics for the Differential Compound Motor. (*a*) *Speed Characteristic.*—From the circuit diagram in Fig. 8 it is evident that the series-field excitation opposes the main-field excitation produced by the shunt-field winding. The effect of the series field is, therefore, to reduce the effective flux ϕ in the same manner as the demagnetizing effect of the armature reaction. If the combined *effect* of the series field excitation and the demagnetizing ampere-turns of the armature reaction, D_1K_1 in Fig. 8, is greater than the resistance voltage drop, C_1R_1 in Fig. 8, the speed will increase with the load as indicated by the speed curve in Fig. 8.

The graphical construction for obtaining the speed characteristics for the differential compound motor follows the same

system as for the shunt motor in Fig. 4 with the single exception that the line D_1K_1 represents the sum of the demagnetizing effect of the armature reaction and the series field ampere-turns.

It is evident that the speed curve can be made a straight line; that is, constant speed for all loads by using the proper number

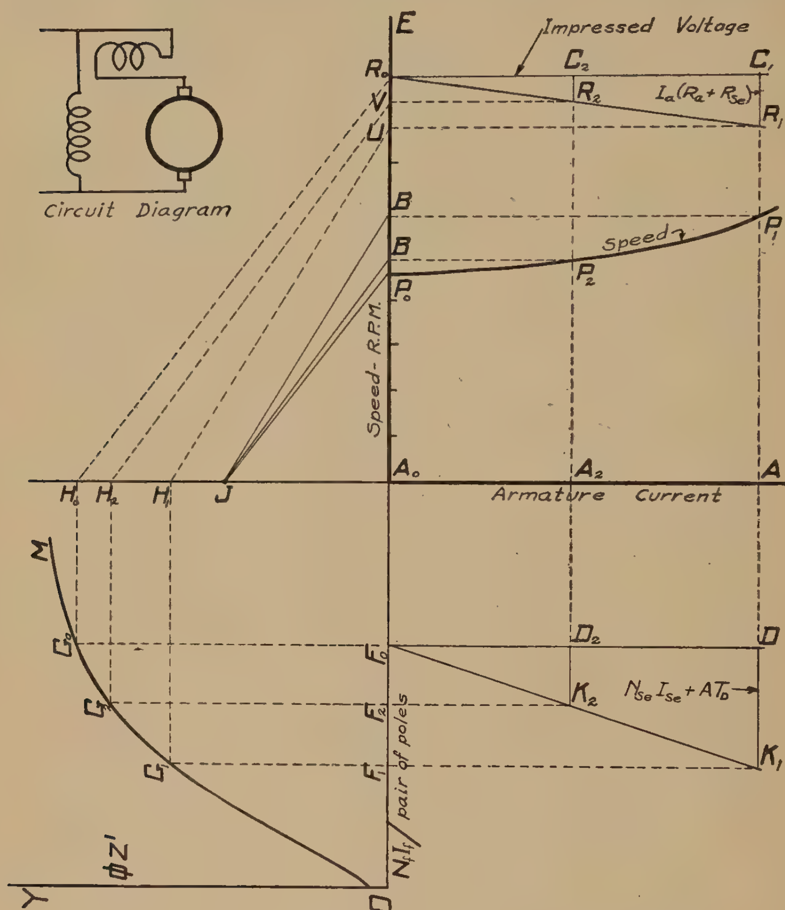


FIG. 8.—Speed characteristic. Differential compound motor.

of ampere-turns in the series-field winding. If a larger number of ampere-turns are used the speed will rise with an increase in load as shown in Fig. 8. If less ampere-turns than the number required to give constant speed are used the speed will decrease as the load increases.

The differential compound motor is generally used under service conditions that require constant speed at all loads.

(b) *Torque Characteristics.*—The circuit diagram and torque characteristics for a differentially wound compound motor is

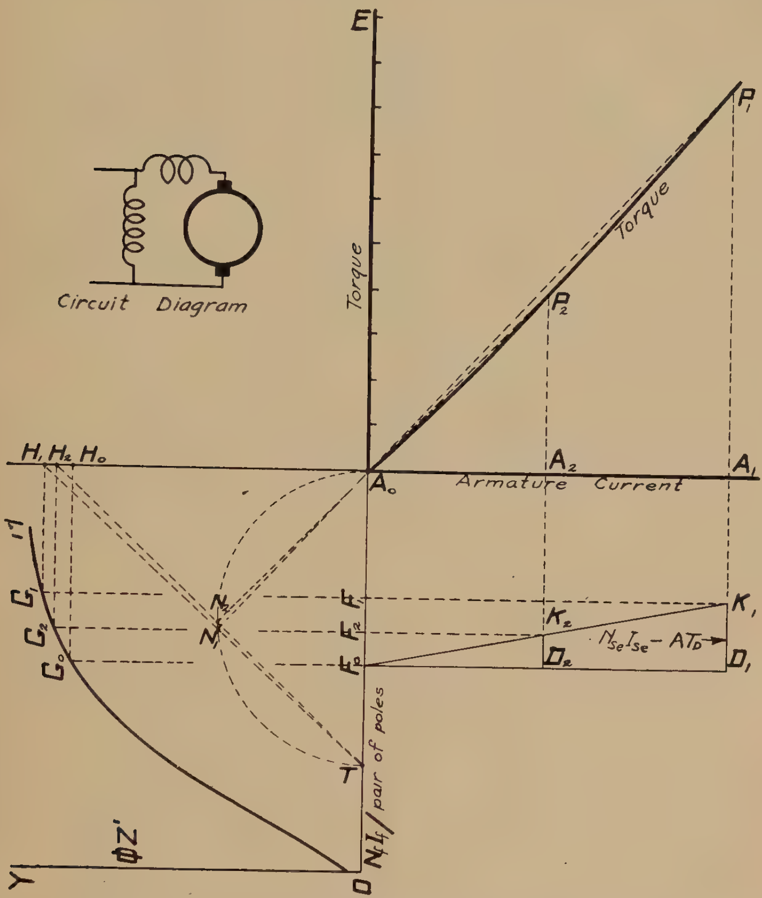


FIG. 9.—Torque characteristic. Differential compound motor.

shown in Fig. 9. The construction for graphically locating the speed curve is identical with that used in Fig. 5, for the shunt motor with the single modification that in Fig. 9 the line D_1K_1 represents the sum of the demagnetizing effect of the armature reaction and the series-field ampere-turns.

The downward bend of the torque curve becomes greater if the series-field ampere-turns are increased and more nearly a straight line if the series ampere-turns are reduced.

Characteristics of the Cumulative Compound Motor. (*a*) *Speed Characteristics.*—The circuit diagram and a typical speed-

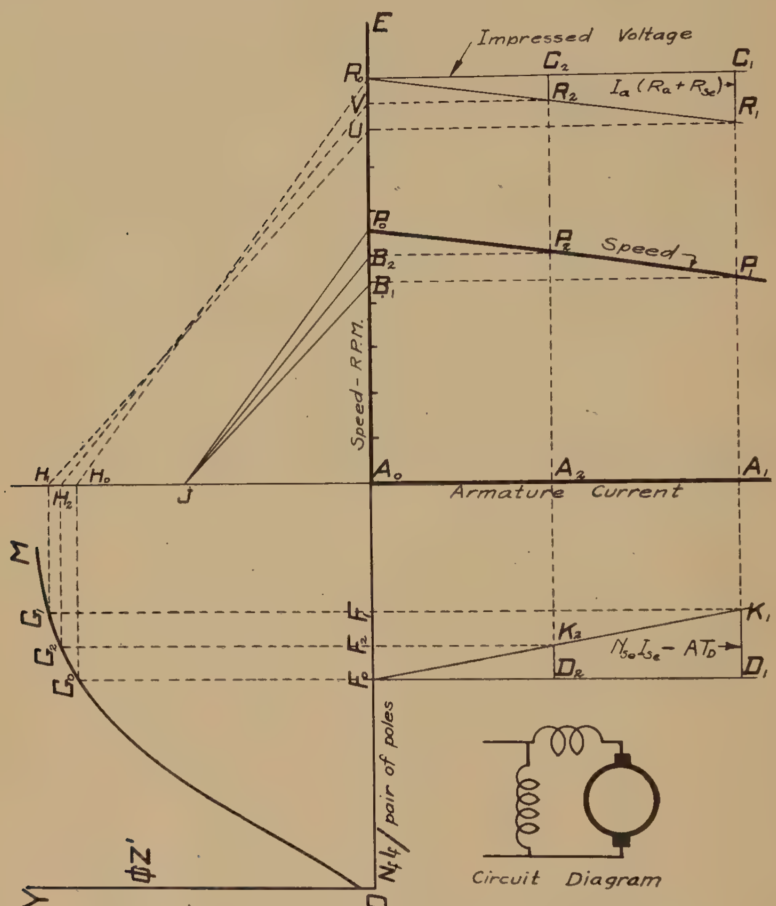


FIG. 10.—Speed characteristics. Cumulative compound motor.

load curve for a cumulative compound motor are shown in Fig. 10. The graphical construction is identical in procedure with that of Fig. 8, the only difference being that for the cumulative motor the series-field excitation adds to instead of subtracts from the main excitation by the shunt field. The series-field

ampere-turns are assumed to more than compensate for the demagnetizing effect of the armature reaction so that the line D_1K_1 is laid off in the opposite direction to the corresponding line in Fig. 8.

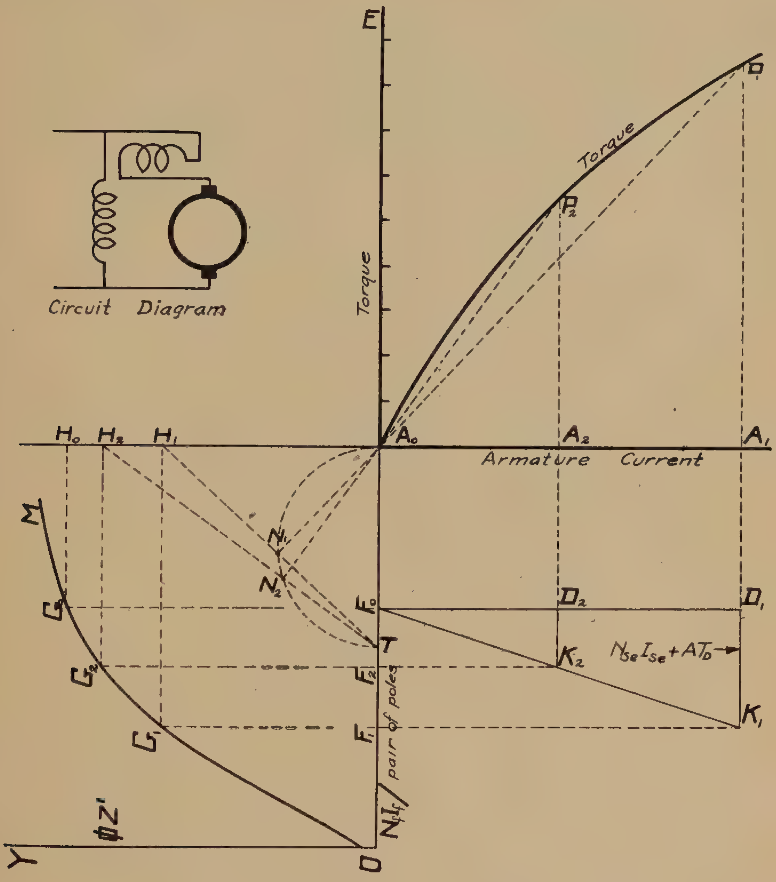


FIG. 11.—Torque characteristics. Cumulative compound motor.

The speed-load curve is essentially a straight line with the decrease in speed for increase in load greater than in the shunt motor.

(*b*) *Torque Characteristics.*—The torque-load curve for the cumulative compound motor is shown in Fig. 11. The graphical construction is the same as for Fig. 9. For the same reason as explained for Fig. 10 the line D_1K_1 represents the difference of

the magnetizing ampere-turns of the series field and the demagnetizing effect of the armature reaction. It is assumed that the ampere-turns of the series field are greater than the demagnetizing effect of the armature reaction and hence the line D_1K_1 is laid off in direction opposite to that of the armature reaction in Figs. 6, 7, 8, and 9. The torque curve is not far from a straight line bending upwards slightly for increase in load.

Sets of Typical Speed-load and Torque-load Characteristics.—In order to more readily compare the operation characteristics of the four basic types of electric motors a set of typical speed-load

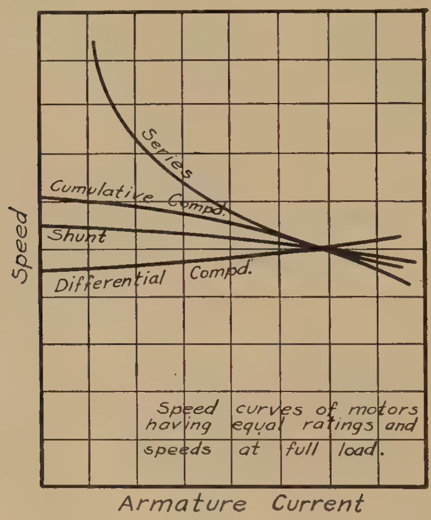


FIG. 12.—Typical speed-load curves.

curves are shown in Fig. 12 and the corresponding torque-load characteristics in Fig. 13.

Both the speed and torque characteristics of the shunt motor lie between the corresponding curve for the differential and cumulative compound motors. The series motor characteristics are markedly different from the other three. The shunt and compound motors are essentially constant speed machines and supply the motive power for industrial service requiring approximately constant speed at all loads. The series motor is primarily used for loads in which the speed varies over a wide range and especially for service in which rapidly increasing torque is required as the speed decreases. Elevators, hoists, and electric railways

are important fields in which the required torque and speed characteristics are met to best advantage by the series motor. The compound-motor characteristics, however, tend to approximate those of a series motor. Hence the compound motor is sometimes used in place of the series motor because the compound motor has a definite no-load speed. The application of the series motor to electric railways is discussed in a later section of this chapter.

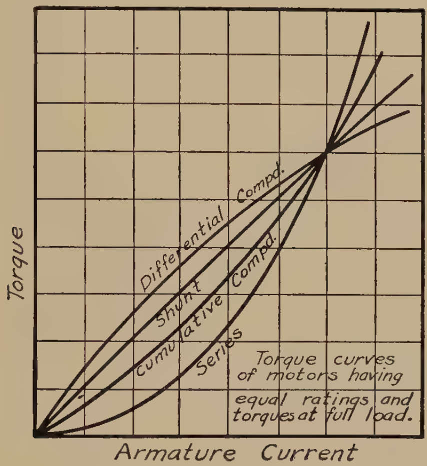


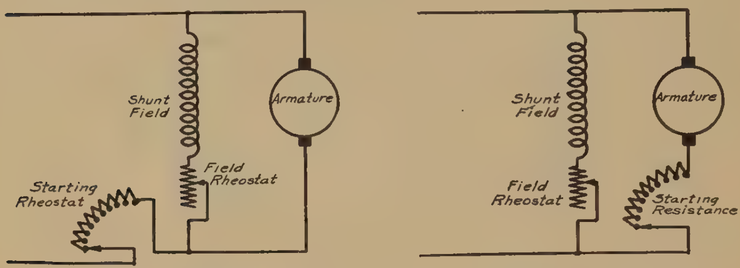
FIG. 13.—Typical torque-load curves.

The Starting of Motors.—When motors are running at no load the counter-e.m.f. is almost equal to the impressed voltage. In carrying a load the counter-e.m.f. is somewhat less and the difference between the induced and impressed voltages is absorbed by the armature resistance voltage drop. The counter-e.m.f. is proportional to the product of the speed and the effective field flux. Hence when the motor is starting the counter-e.m.f. at start is zero and then increases with the speed, reaching its maximum value at rated speed. In starting motors particularly under load, two requirements must be met.

(*a*) The field excitation should be as large as possible in order that the initial armature current will produce a torque as great as possible to overcome the starting load.

(*b*) Some means for limiting the flow of current during the starting period must be provided as the armature resistance is too small for this purpose.

Statement (*b*) does not apply to very small motors, because the armature resistance is larger in small motors and, more particularly, because the rotating armature having small mass requires little time to reach full speed as the energy stored kinetically is small, and, therefore, the temperature rise in the armature does not reach the danger point.



(*a*) Incorrect connections. (*b*) Correct connections.
FIG. 14.—Circuit diagram. Starting a shunt motor.

To gain full field strength at the starting point the shunt-field circuit should be connected directly across the line. To limit the flow of current during the starting period a rheostat or starting resistance is placed in series with the armature. The circuit diagrams in Fig. 14 illustrate the above stated methods.

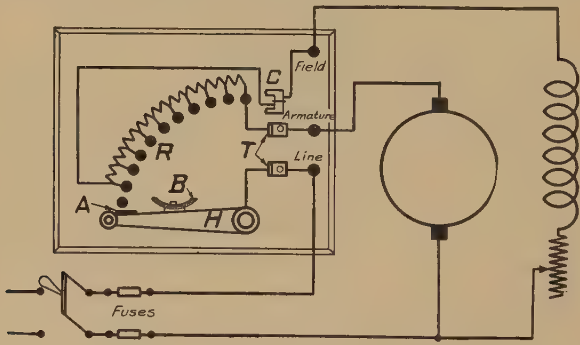


FIG. 15.—Three-point starting box for shunt motor.

The circuit diagram of a three-point starting box for a shunt motor, so named because of its three terminals, is shown in Fig. 15. When the motor is not in operation the contact lever arm *H* is in the position shown in Fig. 15 and the line switch is open.

To start the motor close the line switch, move the arm *H* to the second contact point which connects the shunt-field circuit

directly across the line but limits the current in the armature circuit by the resistance in the starting rheostat as well as the resistance in the armature circuit proper. The contact lever is then slowly moved toward the right until all of the resistance in the starting rheostat is cut out and the contact switch *B* shorts the rheostat at *T* thus bringing the armature terminals directly across the line. The small magnet *C* energized by the shunt-field current attracts the armature *A* and thus holds the lever arm *H* in the vertical position as long as sufficient current flows in the shunt-field circuit. To stop the motor pull the main switch. Usually the arm *H* does not immediately return to the

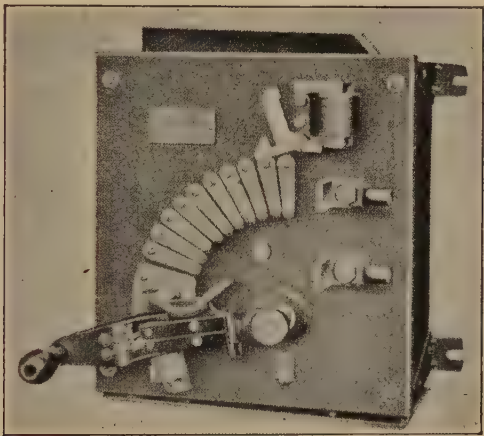


Fig. 16.—Cutler hammer, three-point starting box.

starting position due to the counter-e.m.f. of the armature being able to maintain sufficient current in the hold-up coil *C* so as to still hold the arm *H* in the running position. As the machine slows down, the counter-e.m.f. decreases until the spring acting against the arm *H* returns it to the starting position.

Motors should be stopped by opening the line switch and not by pulling down the arm. If the motor is not under load and therefore consuming a relatively small value of line current, no harmful effects may be encountered by pulling the arm *H*. The small line current is easily broken and the energy of the magnetic field is gradually dissipated through the closed armature circuit. If the motor is to be stopped when under heavy load, however, a large current must be disrupted which would

cause severe arcing across the rheostat contact studs. Due to the inductance of the armature winding high voltages may be

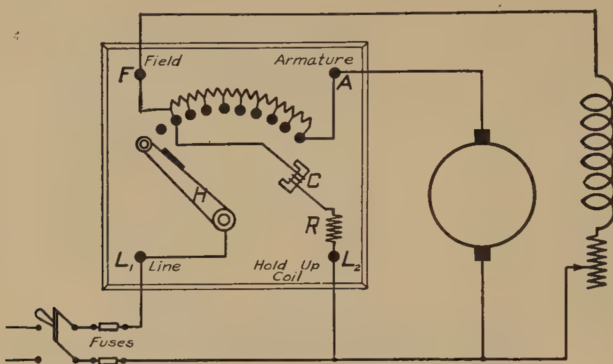


FIG. 17.—Four-point starting box. Shunt motor.

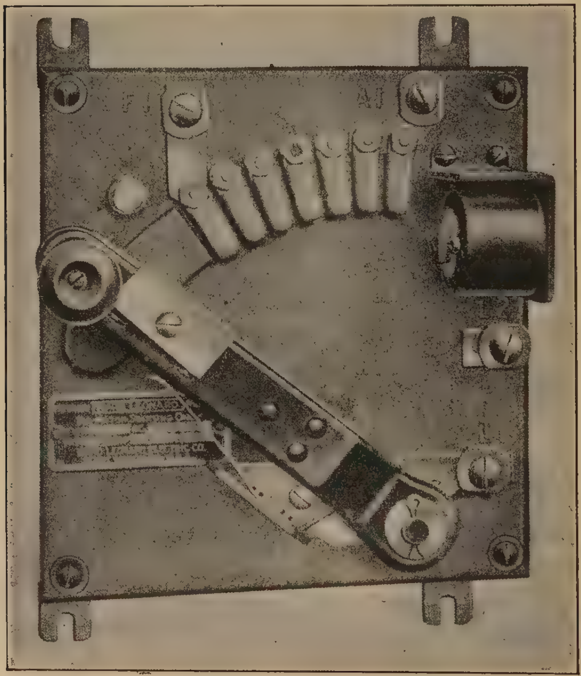


FIG. 18.—Ward-Leonard motor starter.

induced by an attempt to suddenly break the current flow and a flashover may occur between the arm *H* and the first contact

stud that may cause serious damage. The correct and safe method is, therefore, to use the line switch.

The advantage of the three-point box is mainly that if for any reason the field circuit be broken, which will tend to cause the machine to run away in speed, the coil *C* is simultaneously deenergized and the motor stopped by the arm *H* returning to the starting position. This feature is in addition to the function of the holdup coil *C* providing no-voltage release. If the line suddenly is without voltage the arm *H* returns to the starting position, thereby preventing the danger of burning out the armature when voltage is restored to the line.

The circuit diagram of a four-point starting box is shown in Fig. 17. The starting operations are the same as for the three-

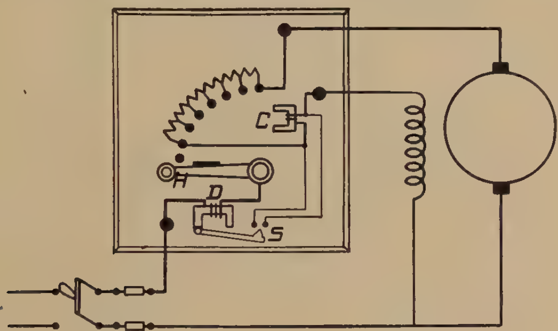


FIG. 19.—Circuit diagram of starting box with overload and no-voltage release. Shunt motor.

point starting box. The hold-up coil in series with the magnet *C* has high resistance and is connected directly across the line. When the main switch is opened the current eventually ceases to flow through the magnet *C* and the starting arm is released and by spring action is brought back to its initial position. It is well to note, in both the three- and four-point boxes illustrated in Figs. 15 and 17, that the armature current starting resistance is automatically inserted in the field circuit as the starting arm *H* is raised. The field current, however, is not seriously affected since the starting box resistance is very small compared to that of the shunt-field winding itself.

In order to protect the motor against overheating, an overload release may be placed in the starting box, the circuit diagram of which is shown in Fig. 19. Under normal operating conditions the starting arm *H* is in the vertical position in contact with the

holding magnet C. The main current to the motor passes through the magnet D, which under normal operating conditions remains inactive. If the load becomes excessive so that the current increases above the predetermined limit for overheating the motor the magnet D raises the armature S, which closes the contact that short circuits the winding of the magnet C. This releases the starting arm H, which by means of springs is brought back to its initial position and thereby opens the power supply circuit. In the starting of differential compound motors special care must be taken, because the high inductance of the shunt-field winding may so greatly retard the building up of the shunt-field flux that the series-field flux will overpower and reverse the

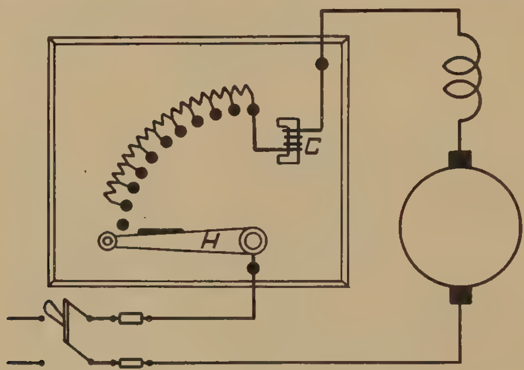


FIG. 20.—Circuit diagram of starting box for series motor. No-load release.

residual flux, thus causing the motor to actually start in the wrong direction. This may be prevented by shunting or short circuiting the series field during the starting period.

The circuit diagrams of two simple starting boxes for series motors are shown in Figs. 20 and 21. The resistance of the starting box is in series with both the armature and field circuit of the motor.

The starting box shown in Fig. 20 has a *no-load release magnet* and the diagram in Fig. 21 shows a somewhat similar arrangement which opens the circuit under *no-voltage conditions.*

The no-load release magnet has a few turns in series with the motor and hence releases the starting arm under no-load conditions, or if the load falls below a specified value thereby preventing the machine from running away.

The hold-up coil of the no-voltage release (Fig. 21) is connected through a high resistance directly across the line. When

the voltage drops to zero, or some predetermined low value, the magnet C releases the starting arm H, which by spring action moves back to the initial position and thereby breaks the motor circuit.

A large variety of starting boxes have been developed to meet the requirements imposed under widely different service conditions. Starting boxes should be selected on the basis of specific service requirements as well as on the type and size of motor to be used.

In some lines of service the load varies through a cycle of comparatively short period so that the time of starting the motor becomes a considerable part of the total period of operation.

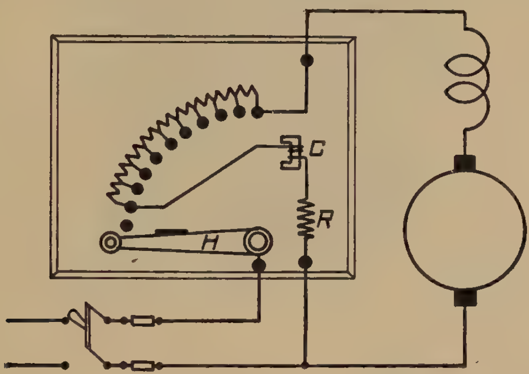


Fig. 21.—Circuit diagram of starting box for series motor. No-voltage release.

Speed control of the motor, moreover, may be necessary for a still greater part of the time. Thus motors providing the motive power in hoists, cranes, elevators, electric railways, etc., start under load, operate at one of several possible speeds for a short time, and then stop. The cycle repeats with variations in maximum value of current, length of period and speed of motor. The starting and speed-control mechanism are combined into a single unit, usually called the *controller*. The controller, necessarily, must be of more rugged construction than the ordinary starting box. The conductors and resistance units must be of sufficient size, and have ample heat-radiating surface to carry the load current for much longer periods than a starting box designed merely to bring a motor up to full speed with plenty of time to cool off between operations.

FIG. 22.—Controller for series motor, on a hoist Type S. (*Westinghouse Elec. and Mfg. Co.*)

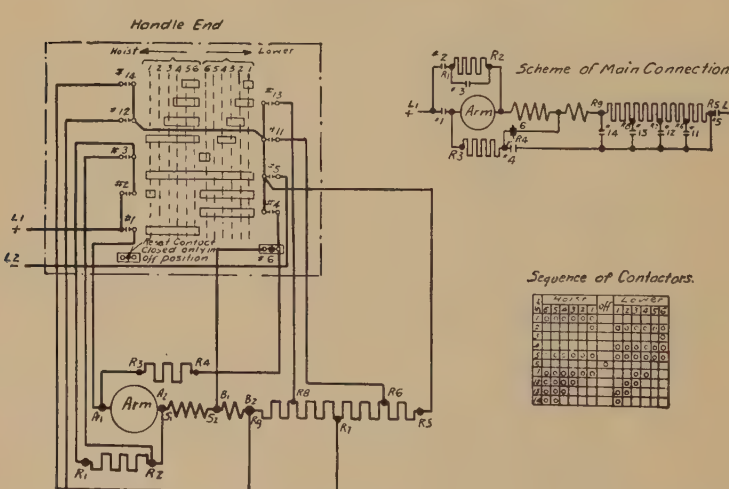


FIG. 23.—Circuit diagram of controller in Fig. 22 (*Westinghouse Elec. and Mfg. Co.*)

An open view of a crane motor controller is shown in Fig. 22 with the corresponding circuit diagram in Fig. 23.

Automatic starters are necessary for large motors and particularly where high acceleration is required as for steel mill motors.

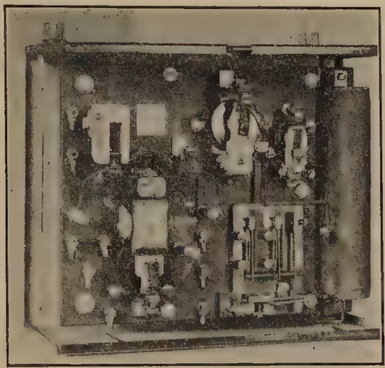


Fig. 24.—Automatic starter with overload protection and dynamic braking. (*Westinghouse Elec. and Mfg. Co.*)

A typical automatic starter is shown in Fig. 24 and the corresponding circuit diagram in Fig. 25. The starter is placed into operation by closing a snap switch or pushing the starting button. By using three-way and four-way switches in connection with the

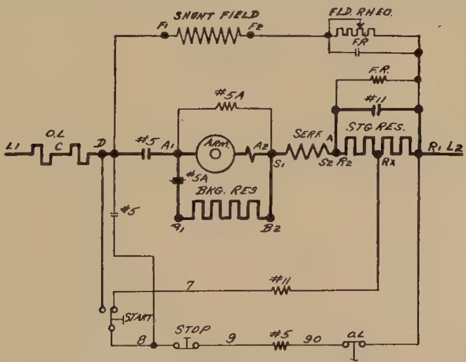


Fig. 25.—Circuit diagram for starter in Fig. 24. (*Westinghouse Elec. and Mfg. Co.*)

automatic starter the motor may be started from any distance and from widely separated points. After the starting switch has been closed the successive contacts in the starter close automatically and as rapidly as possible under the predetermined

current magnitude limits. Automatic starters are more reliable and give better service than can be obtained by hand operation. If properly installed an automatic starter will, under given circuit conditions, start the motor in the prescribed length of time and eliminate delays caused by the blowing out of fuses and opening of circuit breakers that are likely to occur when the starter is operated by hand. It is evident that automatic starters as well as hand-operated starters may be provided with low-voltage release, overload release or circuit breakers, and time-limit protection.

Electric Railway Motors.—Electric railways form so large and important a field for the application of direct currents that the type of motor required for this service should receive special attention. The basic factors in the service requirements of

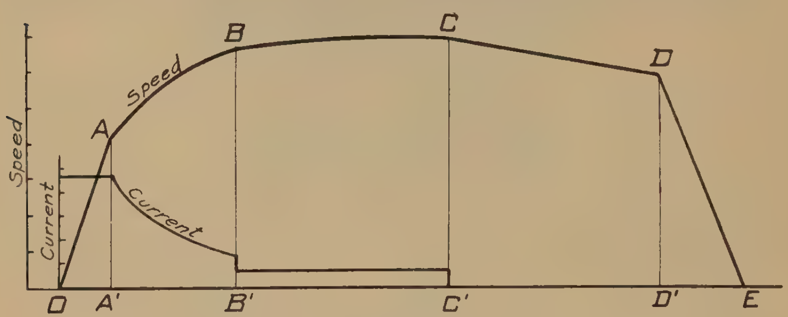


Fig. 26.—Typical speed-time, current-time characteristics of electric railway operating cycle.

railway motors can be observed in the street-car-operating cycle diagram shown in Fig. 26.

In starting a street car or electric train a heavy torque is required but as the speed increases the torque decreases. It is evident that this basic-load characteristic is practically identical with the torque characteristic of the series motor (Fig. 7). For this reason series motors are used exclusively for electric trains using direct-current motive power. The torque characteristics of the shunt and compound motors do not conform to the torque requirements in train service and hence cannot be used.

A typical operating cycle of a street car is illustrated by the speed- and current-time curves in Fig. 26. The car starts, accelerates rapidly at first and then at a slower rate, coasts at a slowly decreasing short period, and finally decelerates rapidly under the application of the brakes until the car stops.

During the starting period the current is held to a predetermined maximum value by means of the controller resistance which is cut out step by step. During this period the torque per motor is, therefore, essentially constant and a fairly uniform acceleration is obtained as indicated by the straight line section *O-A* of the speed-time curve in Fig. 26. When all the resistance in the controller has been cut out the acceleration will be less rapid but the speed increases *A-B* and at a still slower rate for *B-C*. Over the *B-C* period there is comparatively little increase in speed and the power is largely used to overcome friction and the grade of the track if it should not be level. At *C* the current is cut off and the car coasts to *D*, when the brakes are applied until the car stops at *E*.

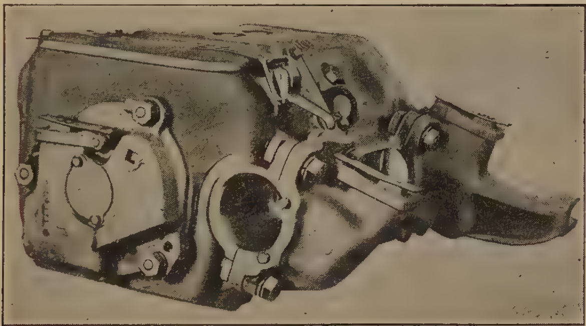


FIG. 27.—Box frame railway motor. 600-volt, direct currents. (*Westinghouse Elec. and Mfg. Co.*)

It is evident that the torque required for the operating cycle in Fig. 26 is essentially the same as the torque of the series motor. Necessarily, a large variety of railway motors must be provided to meet the various needs of electric railways under widely differing schedules, track and load conditions. But the basic torque-speed relations are essentially the same for all. The limitations of design, due to the conditions under which the motors must operate, are very severe so that railway motors have many special features that distinguish them from stationary motors.

A photograph of a typical railway motor is shown in Fig. 27.

The characteristics of two widely used electric railway motors are shown in Figs. 28 and 29.

The torque is expressed either in pounds at 1-ft. radius or in the tractive effort on the car. Speed is given in r.p.m. or in miles

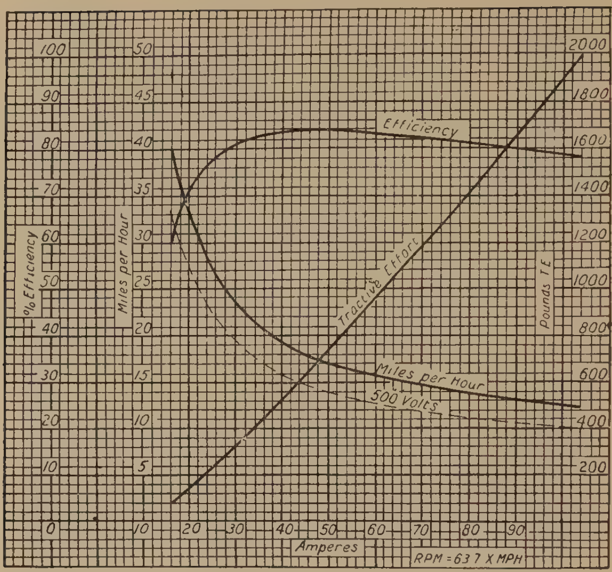


FIG. 28.—Performance curves of 600-volt, direct-current railway motor. 14:69 gear ratio. 26 in. wheels. (*Westinghouse Elec. and Mfg. Co.*)

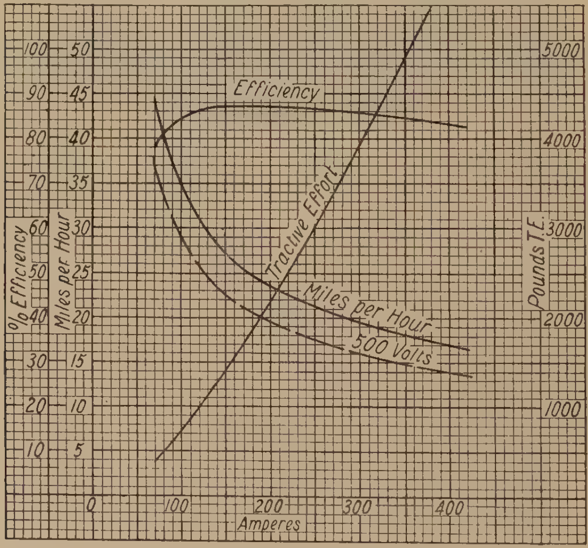


FIG. 29.—Performance curves of 600-volt, direct-current railway motor. 16:61 gear ratio. 33 in. wheels. (*Westinghouse Elec. and Mfg. Co.*)

per hour travel by the car. It is evident that in order to transform the torque in pounds-feet into tractive effort, or speed in r.p.m. into miles per hour the gear ratio and wheel diameters must be known. The curves would be of the same shape but the scales giving numerical values must be shown so as to conform with the units used. The efficiency and temperature characteristics as shown in Figs. 28 and 29 are discussed in Chap. XIX. The torque speed and temperature characteristics are inherent in the motor itself, but in order to meet service requirements effective speed control must be provided by means of accessory equipment.

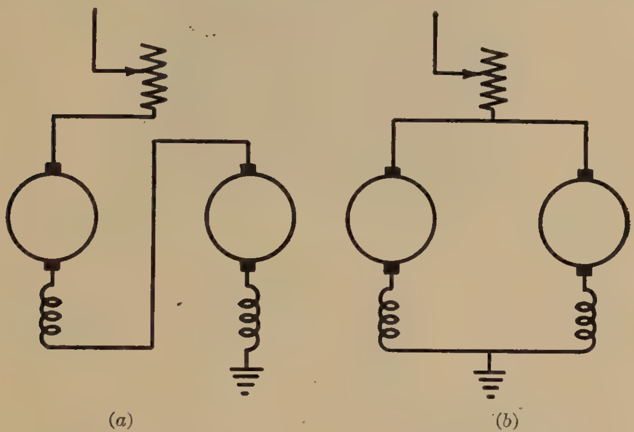


FIG. 30.—Circuit diagrams of series-parallel connections.

Series-parallel Control. *Series Motors.*—Electric railway cars equipped with two motors generally use the series-parallel method of speed control. At start the two motors and all of the controller resistance units are connected in series as illustrated by the circuit diagram in Fig. 30 (*a*).

As the car accelerates the controller resistance is cut out step by step until the two motors in series are directly across the line; that is, each motor has half of the line voltage. The connections are then changed so that the two motors are placed in parallel, but with all of the controller resistance in series, as illustrated by the circuit diagram in Fig. 30 (*b*). With still further increase in speed the controller resistance is again cut out step by step until the two motors, in parallel, are connected directly across the line. The successive steps taken in the cutting out of the controller resistance units and the change from

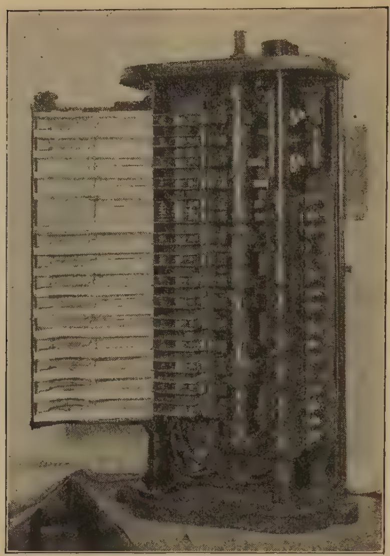
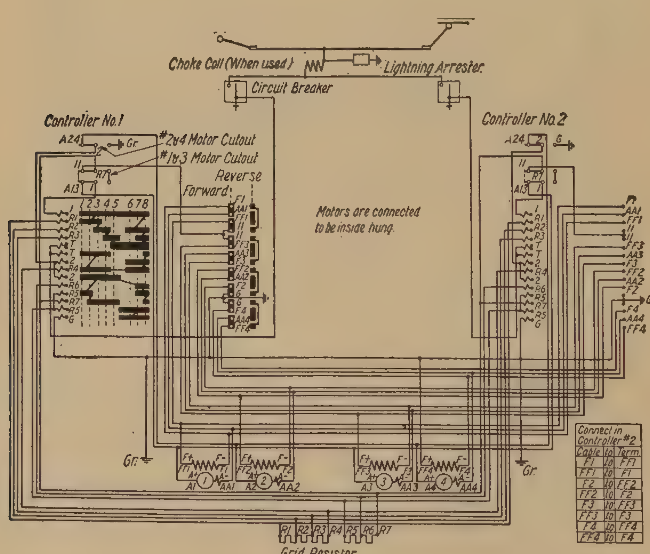


FIG. 31.—Two-motor railway controller. Type K-35. 600 to 750 volts, direct currents. (*Westinghouse Elec. and Mfg. Co.*)



NOTE: For single-end operation, No. 2 controller and its circuit-breaker are omitted

FIG. 32.—Circuit diagram of controller connections. Type K-35. (*Westinghouse Elec. and Mfg. Co.*)

series to parallel connections are based on the counter-e.m.f. generated in the motor as the speed increases. At all times the current flowing must not exceed in magnitude a predetermined maximum value. The difference in the impressed line voltage and the motor counter-e.m.f. divided by the resistance in the circuit determines the magnitude of the current.

This progressive change in the amount of resistance in series with the motors as well as the change of connections from series to parallel of the motors is accomplished by means of a controller. A typical two-motor railway controller is shown in Fig. 31 and a diagram of the circuit connection used in Fig. 32.

A set of successive circuit connections of a two-motor controller providing for series parallel operation but using the bridge transition method is shown in Fig. 33.

The successive steps in changing from the starting connections with the two motors and all the controller resistance in series to the final full speed connection with the two motors in parallel and directly across the line can best be observed by a careful study of the circuit diagram in Fig. 33.

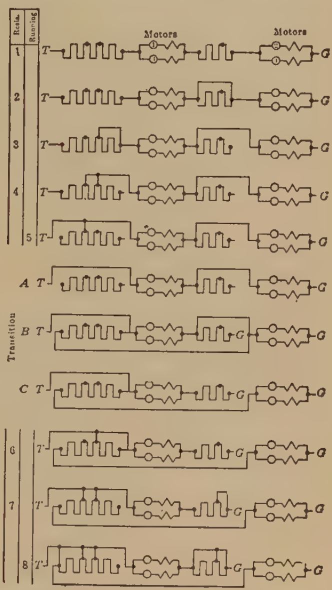


FIG. 33.—Circuit diagram of controller, using bridge-transition method.

For trains of several cars the controllers located in each car are operated from one point by means of a master controller.

Speed Control. *Shunt Motors.*—In principle the methods of speed control of shunt motors must relate to changes in (*a*) the resistance in series with the armature, (*b*) the field excitation, (*c*) the impressed voltage, and (*d*) the number of armature conductors in series. The truth of the above statement is evident from the speed equation for the shunt motor.

$$n = \frac{V - (R_a + R)I_a}{\phi Z'} \tag{26}$$

(*a*) *Rheostatic Control.*—By inserting a variable resistance R in series with the armature resistance, R_a, it is evident that speed

regulation may be obtained. The arrangement is, however, uneconomical, and also the speed regulation is very poor. A better method is to insert a variable resistance in the shunt-field circuit and thereby vary the field flux ϕ. The energy loss is much less and a much better regulation is obtained. A comparison of

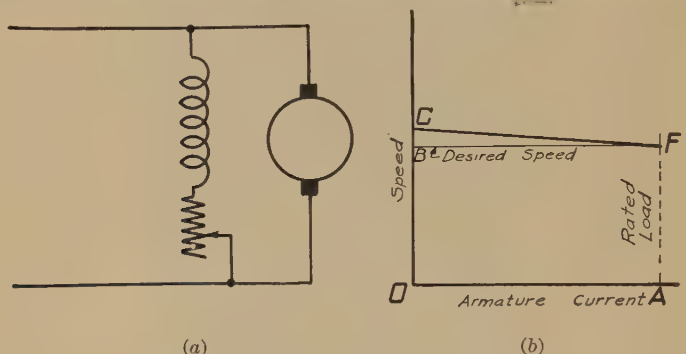


FIG. 34.—Speed control of shunt motor by means of a variable resistance in the field circuit.

the regulation resulting from placing the resistance in the armature or field circuits is shown in Figs. 34 and 35. In Fig. 34 (b) is shown the speed-current characteristic when the variable resistance is in the shunt-field circuit. For a given full-load speed AF the

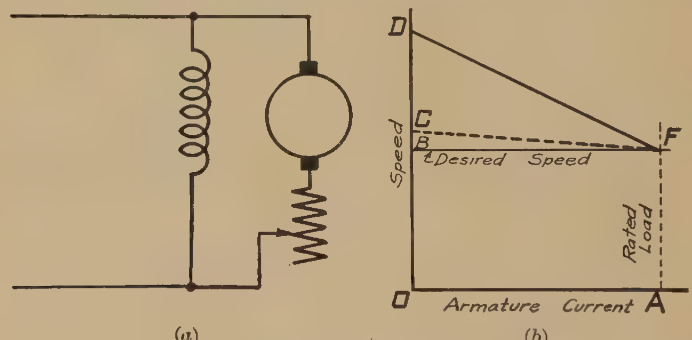


FIG. 35.—Speed control of shunt motor by means of variable resistance in series with the armature.

no-load speed without change in the field resistance would increase to the value represented by OC. In Fig. 35 (b) is shown the corresponding change from full-load speed AF to no-load speed OD without change in the resistance R in series with the armature. The broken line FC gives the speed regulation shown in Fig. 34 (b)

for the resistance in the field circuit. ~~Hence~~ the speed regulation for the resistance in the field circuit (Fig. 34) is decidedly better than when armature resistance is used.

The variable resistance in the field circuit method (Fig. 34) gives good regulation and is very effective over a moderate range of speed. At the original speed of the motor the field should be near the saturation point; that is, well above the bend of the curve. By inserting resistance in the field circuit the flux is decreased, and, as a consequence, the field intensity, especially at the pole tips becomes so weakened by armature reaction that the machine will not commutate properly. This limitation may, however, largely be overcome by the use of interpoles. It is evident the slowest speed obtainable by field control occurs when the field rheostat resistance is all cut out. If slower speeds are desired

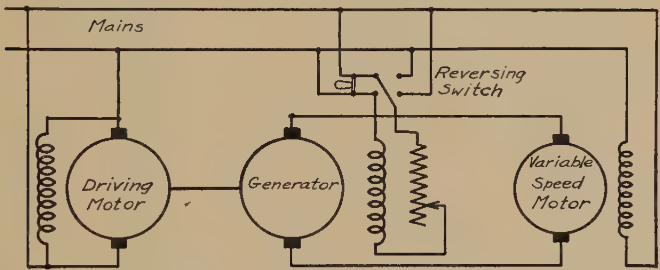


Fig. 36.—Circuit diagram of the Ward-Leonard system of speed control.

the voltage impressed must be varied or armature resistance control must be used.

(b) *Voltage Control.*—It is evident from the speed equation that changes in the impressed voltage will directly affect the speed. Any method by which the impressed voltage may be made to vary in proportion to the desired changes in speed should be effective. The Ward-Leonard system of speed control, a circuit diagram of which is shown in Fig. 36, operates on this principle. It is evident from the circuit diagram in Fig. 36 that the Ward-Leonard system provides the desired variable voltage to the variable speed motor by means of a self-contained motor-generator set. The motor driving the generator receives power from constant-potential mains and runs at constant speed but the generator field is provided with a variable field resistance and, in addition, a reversing switch so that the voltage generated can be varied from zero to full value in either direction. The main

objection to this method is the high cost of the auxiliary motor-generator set.

(c) *Field-flux Control.*—In the *Stow variable-speed* motor, the cores of the field poles are movable and can be made to slide in or out by turning a wheel on top of the frame. By means of bevel gears and rods the cores move uniformly in the several poles. If the poles are moved outward the airgaps between the field poles and the armature are increased. This increases the reluctance of the magnetic circuit and hence for constant-field excitation the field flux is decreased. As the flux ϕ decreases the speed increases and the desired speed control is realized.

In the *Lincoln variable-speed* motor the same end is gained by moving the armature endwise under the poles so as to bring one end of each armature conductor outside the path of the field flux. Moving the armature along the axis of rotation so that the conductors are partly outside the space occupied by the field flux reduces the rate of cutting lines of force, or in order to cut lines of force at the same rate the speed must be increased. Hence moving the armature endwise, in or out of the field flux, produces the same effect on the speed as varying the strength of the field flux.

Division of Load between Motors.—The division of load between shunt motors operating in parallel is based on the same principles as the division load of between shunt generators connected to the same busbars. If the speed-current characteristics, expressed in percentage of full load, are identical for the two motors, the machines will divide the load in proportion to their load capacities. For machines having different torque-current characteristics the division of load will depend on the same principles as discussed in Chap. XV for generators.

In the Thury system, having series motors connected in series and operating on a constant current basis, constant speed is obtained by changing the position of the brushes, thereby causing changes in torque instead of speed to follow changes in load.

Regulation.—The term *regulation* as applied to motors refers to changes in speed produced by changes in load. In constant-speed direct-current motors the regulation is the ratio of the difference between rated-load and no-load speeds to the rated-load speed. The voltage impressed on the motor must be constant and as specified in the rating. The temperature of the motor must be that attained at operation under rated load for the time specified in the rating.

PROBLEMS

1. Plot the speed characteristic curve of a shunt motor having the following data given:

$$\begin{aligned} p &= 4 \\ Z &= 194 \\ a &= 2 \\ N_{sh.f.} &= 1{,}600 \text{ per pair of poles} \\ I_a \text{ full load} &= 220 \text{ amp.} \\ \text{Impressed voltage} &= 120 \text{ volts} \\ R_a &= .0245 \text{ ohms} \\ R_f &= 18.5 \text{ ohms} \\ \alpha &= 7.42° = \text{brush-angle shift.} \end{aligned}$$

The relation between ampere-turns per pair of poles and the flux ϕ per pole as obtained from design data is as follows:

$N_f I_f$ per pair of poles	ϕ per pole
0	0
720	250,000
1,460	500,000
2,200	750,000
2,920	1,000,000
3,660	1,250,000
4,480	1,500,000
5,460	1,750,000
7,060	2,000,000
10,380	2,250,000
17,480	2,500,000

2. Plot the torque characteristic curve of the shunt motor in Problem 1.

3. Plot the speed characteristic curve of a series motor having the same saturation curve as in Problem 1.

$$\begin{aligned} p &= 4 \\ Z &= 194 \\ a &= 2 \\ N_{se.f.} &= 47 \text{ per pair of poles} \\ I_a \text{ full load} &= 221 \text{ amp.} \\ \text{Impressed voltage} &= 120 \text{ volts} \\ R_a &= 0.0245 \text{ ohms.} \\ R_{se.f.} &= 0.0055 \text{ ohms.} \\ \alpha &= 7.5° \end{aligned}$$

4. Plot the torque characteristic curve of the series motor in Problem 3.

5. Plot the speed characteristic curve of a cumulative compound motor having the same saturation curve as in Problem 1.

$$\begin{aligned} p &= 4 \\ Z &= 194 \\ a &= 2 \\ N_{se.f.} &= 8 \text{ pair of poles} \\ N_{sh.f.} &= 1{,}600 \text{ per pair of poles} \\ I_a &= 220 \text{ amp. at full load} \end{aligned}$$

$$\text{Impressed voltage} = 120 \text{ volts}$$
$$R_a = 0.0245 \text{ ohms}$$
$$R_{se.f.} = 0.0055 \text{ ohms}$$
$$R_{se.f.} = 27.2 \text{ ohms}$$
$$\alpha = 7.5°$$

6. Plot the torque characteristic curve of the compound motor in Problem 5.

7. Plot the speed characteristic curve of a differential compound motor having the same constants as the motor in Problem 5.

8. Plot the torque characteristic curve of the differential compound motor in Problem 7.

9. What should be the resistance in a starting box suitable for a 20 hp. 125-volt shunt motor if it is desired to produce 150 per cent full-load torque at the initial starting point? The armature resistance is 0.045 ohms.

10. Repeat Problem 9 for a 20 hp. 500-volt shunt motor having an armature resistance of 0.11 ohms.

11. A shunt motor has an armature resistance of 0.333 ohms. When running light the machine takes an armature current of 6 amp. at 125 volts and runs at 1,800 r.p.m. Let it be assumed that armature reaction is directly proportional to armature current and that at no load it reduces the flux by 1 per cent. Find the speed of this motor when $I_a = 10$, 20, 30, and 40 amp.

12. The mechanical force acting on a conductor located in a magnetic field and carrying a current is given by the following equation: $F = BH \sin \theta / 10$. In which F is in dynes, B in gausses, l in centimeters, I in amperes, and θ the angular position of the conductor to the direction of the lines of force is assumed to be 90 deg.

The average flux density under the poles of a 10-pole machine is 53,000 per square inch. There are 720 armature conductors forming a simplex lap winding. The total armature current is 2,200 amp. The cross-section of each pole face is 10 by 13½ in. The pole enclosure (ratio of span of pole face to pole pitch) = 65 per cent. Neglecting fringing, find the torque produced by the armature if the conductors are located 24 in. from the center of the shaft.

13. A shunt generator delivers a load of 116 amp. at 110 volts when running at 1,200 r.p.m. The armature resistance is 0.12 ohms. Without changing the field resistance the above generator is operated as a motor on 110-volt mains and loaded until the armature current is the same as above when operating as a generator. The shunt-field resistance is 25.6 ohms. Find the generated voltage when operating as motor and as generator.

CHAPTER XVII

COMMUTATION

When the armature of a dynamo rotates the conductors cut lines of force coming alternately from north- and south-field poles. The voltage generated in each armature conductor, therefore, reverses in direction directly after passing each pole. As a consequence, the flow of the resulting currents in the armature conductors alternate in direction, having two reversals for each pair of poles. The process of converting the alternating currents in the armature conductors into a unidirectional flow, that is, direct currents, in the circuit outside the armature is called *commutation*. The commutation process is accomplished by means of stationary brushes in sliding contact with the commutator. The process is quite complex and the problem of securing satisfactory commutation under various operating conditions presents difficult mechanical as well as electrical problems. Commutation is probably the most serious limitation encountered in the design and operation of direct-current machinery.

The commutator consists of radial sectors or bars of hard copper separated by sheets of mica insulation. The assembled commutator is of cylindrical form, and the brushes press against the smooth outside cylindrical surface. The mica insulation is generally slightly undercut, leaving a small groove at the surface between the segments, so that the brushes only come in contact with the copper bars. Under proper operating conditions the commutator surface acquires a glossy highly polished surface. This gives a low coefficient of friction between the moving commutator and the stationary brushes. The brushes are in the great majority of cases of carbon. Finely powdered graphite combined with a binder material is placed in moulds, and under pressure and heat treatment formed into solid units of any desired dimensions. The quality of the material in its finished form, the contact area and contact pressure are important factors in the problem of securing satisfactory commutation. While the commutation process has many phases, it centers on the

change in the flow of current in the armature coils directly connected to the commutator bars in contact with the brushes. As the several bars pass in succession under any given brush the currents must in like order reverse in direction. Hence, during the process of commutation the energy stored magnetically around the armature coil carrying currents to the brush must be removed, and while the bar is still in contact with the brush a like amount of energy must again be stored while forming the magnetic field around the armature coil when the current is formed in the reverse direction. Both the decrease and building up of the magnetic field around the armature coil, unless compensated by counter-flux changes, induce voltages which may reach such magnitude as to cause sparking at the brushes or even start a flashover between the terminals. It should be kept in mind that cutting lines of force that thread the armature, from whatever source, by the relative motion of the conductors and the flux lines generate voltage in the loop short circuited by the brushes. Moreover, the magnetic fields around the armature currents represent energy which must pass over the commutator brush contact during the reversal of current direction.

The commutation cycle for a single coil *a* of an armature is illustrated in Fig. 1 where the current density at the brush-contact surface is assumed to be uniform throughout, producing what is known as "linear commutation," a more or less ideal condition. Each path of the armature is indicated as carrying a current of 30 amp. which combine at the brush to give a total brush current of 60 amp. Arrows together with numerals show the direction and magnitude of currents in all circuits. It will be noted that the armature winding is drawn stationary while the brush has been assumed to move. This was done merely for convenience for representing the progress or relative positions of the armature with respect to the brush by the line XY, which passes through the center of the coil *a* in all cases. In comparing the positions of the brush with the line XY it will be noted that the brush positions advance in successive drawings, a distance of one-fourth of the brush width. Accordingly, to keep uniform current density over the brush surface it is necessary to shift one-fourth of the total brush current from the right of XY to the left of XY for each step. Furthermore, since each segment has been assumed to cover two-thirds of the brush surface, only 40 amp. can pass through any single complete segment surface.

The currents in all circuits can then be easily determined as indicated in Fig. 1. Examination of Fig. 1 shows that the current in coil *a* has been uniformly changed from a maximum of 30 amp. in one direction to a maximum of 30 amp. in the other direction when uniform current density is maintained at the brush surface. It is also shown that the complete reversal is performed in a time equal to that required for the armature to move a distance equal to the brush width only, since at brush position No. 1 commutation is just ready to start while in brush position No. 5 it has just been completed. By increasing the brush width the time of commutation may be increased but this also increases the number of coils short circuited by the brush. Of course, as was stated in the discussion under armature windings, it is necessary to have wider brushes for multiple-type windings in order that continuous contact may be had with all circuits. In order to reduce the number of coils short circuited by the brush it would appear that for simplex windings, such as the one shown in Fig. 1, the brush width should not be greater than the segment width, in which case not more than one coil could possibly be short circuited at a time. Operating experience, however, has proved that so narrow a brush will not produce satisfactory results. Consequently, the brush width is seldom made less than that shown in Fig. 1.

In the practical operation of any dynamo there are several factors which must be taken into account which tend to produce far different results than those indicated in Fig. 1. First, if during the commutation period no voltage were induced in the coil, that is, if the coil were in the true flux-neutral position; second, if the coil had no self- or mutual inductance; and third, if the coil had no resistance, then linear commutation would result due to the uniform shifting of the contact resistance as the segments passed under the brush. For instance (Fig. 1) let the brush positions 3, 4, and 5 be examined. In regard to segment 5 it will be noted that the contact area with the brush is decreasing and that, therefore, the contact resistance between this segment and the brush is increasing in direct proportion. While this procedure is taking place to the right of XY just the opposite, that is, a directly proportional decrease of contact resistance is taking place on the left of XY. Consequently, the coil current in a is being uniformly changed from one direction to the other without producing any sparking at the brushes.

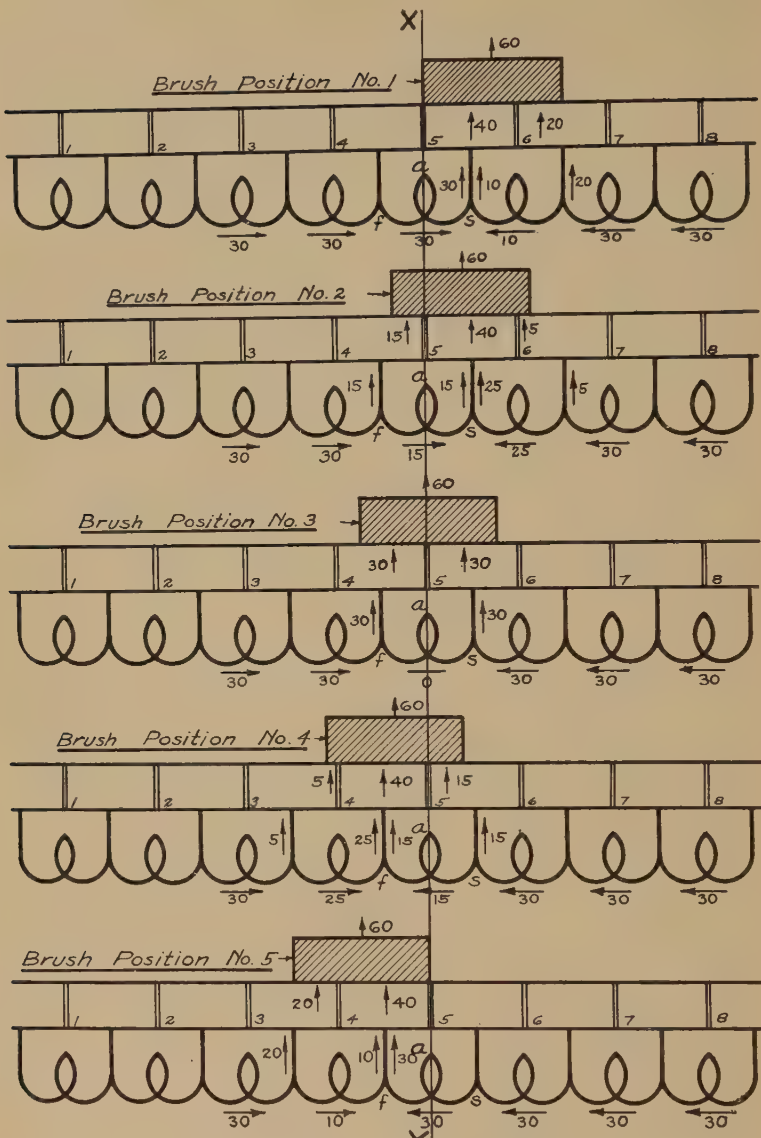


FIG. 1.—Stages of commutation.

The three above-mentioned conditions for linear commutation can rarely, if ever, be obtained in practice. By making the field strength of a machine relatively strong or stiff with respect to the armature, the neutral zone may be fairly well protected from the effects of armature reaction and the resulting voltage may be kept relatively low. It is for this reason that the armature loading of the older machines was limited in proportion to the field strength. Also, if the interpolar space is not too narrow a larger neutral zone may be maintained. This may be accomplished by keeping the pole face span down to about 65 per cent of the pole pitch. The inductance of the coil may be reduced by having only one turn between segments and may be further decreased by the use of short-pitch windings as explained in the discussion under armature windings. While the effect of the coil resistance, as will be shown, is to produce a variation from linear commutation, yet it is hardly ever of such magnitude as to be a major cause for poor commutation.

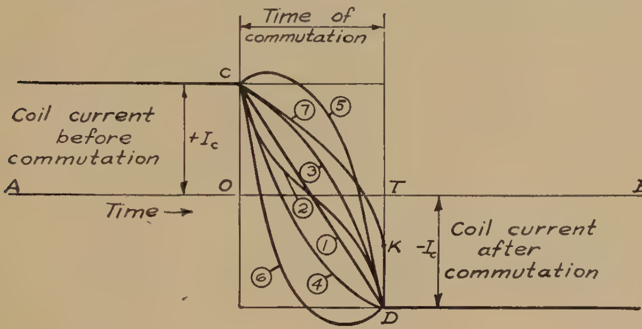
From the foregoing, it should be evident that linear commutation is not the only type of satisfactory commutation or that it cannot be fairly well approximated without the three foregoing conditions being attained. It is plain that if during the process of commutation, a voltage be induced or generated in the coil of such a value as to exactly neutralize the above conditions, linear commutation will be realized. It is further evident that it is practically impossible in practice to produce such properly varying counter-e.m.f's., however, so that only an approximate equalization can be attained by this method. In all cases the brush-contact resistance must be relied on as an important factor in the commutation process. The resulting commutation may not be linear but in general will prove to be quite satisfactory.

The coil resistance has been mentioned above as one of the factors causing non-linear commutation. Usually, however, the coil resistance is considered a distinct advantage when inductance and an induced voltage are present in the coil, because it limits the short-circuit current tending to flow when in contact with the brush. This advantage is far greater than the coexisting disadvantage. Reference is again made to Fig. 1 in order to analyze the effect of the coil resistance when inductance and an induced voltage are not present. It will be noted that the terminals of coil a are indicated by the points s and f. In brush position 2 the current from the left-hand side of the armature has a choice of

two paths in reaching the brush, one being from the point f through segment 4 and the other from the point s through segment 5. It is evident that the added resistance of the coil in one of these two paths will tend to change the division of current from what it would be if the contact resistance alone controlled the division of current. For brush position 2 the current through segment 4 would actually be greater than 15 and the current of coil a correspondingly less. That is, the change in current would take place at a faster rate than for linear commutation. At brush position 3, although there is still a choice of two paths from either side of the armature, no current will be flowing in coil a since the currents exactly balance and the contact resistances are equal. For brush position 4 part of the current from the right-hand side of the armature will flow through segment 4 from the point f; the division of current will be such that more than 15 amp. will flow through segment 5 and a correspondingly less amount through coil a. Again the rate of commutation has been modified, but in this case there is a slowing up instead of the acceleration which occurred during the first half of the commutation period.

In order to compare and show the effects of the different factors during the commutation period a current-time variation of the coil current is shown in Fig. 2. Since for linear commutation the change is uniform, the current variation will follow the straight line marked 1. The effect of the coil resistance is shown by curve 2, which indicates that the reversal of current is at first accelerated and later retarded. Curve 3 shows the effect of inductance and an induced voltage in retarding commutation. Curve 4 shows the effect of a normally desired commutating e.m.f. which would compensate more or less for the effects causing curve 3. Curve 5 shows a case of excessive undercommutation in which the coil current actually increases. This may be caused by the commutating e.m.f. being in the wrong direction. Curve 6 shows a case of overcommutation caused by too large a commutating e.m.f. Ordinarily, a machine may be so adjusted that no sparking will occur at the beginning of commutation, that is, at the leading brush tip. Sparking, however, may still persist or appear at the trailing brush tip. This may easily happen if the final rate of change of coil current is infinite as indicated by curve 7 (Fig. 2). If at the final time of commutation the current has only reached the value of TK, whereas it should have been

TD, then the magnetic field which gives rise to the inductance of the coil would have to be very abruptly formed or readjusted. Since this necessarily requires some time, sparking will occur. Sparking, however, should not be considered as the sole criterion of satisfactory commutation. In curves 5 and 6 (Fig. 2) sparking may, more than likely, not occur at either edge of the brush, yet the reversal of current may be decidedly unsatisfactory due to the localized high-current density in the brush which may cause abnormal heating of both the brush and commutator, which, in turn, seriously affects the contact resistance and condition of the commutator surface.



1. Linear commutation.
2. Effect of coil resistance.
3. Effect of coil inductance.
4. Effect of commutating e.m.f.
5. Under commutation.
6. Overcommutation.
7. Retarded commutation.

Fig. 2.—Short-circuit currents:

In summing up the foregoing discussion it is evident that two basic principles underlie the means and methods that have been developed for securing satisfactory commutation. The first, in time of development, is generally referred to as the *resistance method* and is based on adjustment of the *contact resistance* between the brush and the commutator in combination with the shifting in position of the brushes. The second, or *counter-e.m.f. method*, which is by far the more important, aims to neutralize or compensate for the armature reaction by the use of *interpoles* and *compensating windings*. The contact resistance is used as an auxiliary to further improve the commutation obtained by the counter-e.m.f. method. In good or ideal commutation, the current in the coil itself should decrease at practically a uniform rate and reach zero value at a point corresponding to the middle of the brush.

The current should then increase at a uniform rate and reach its normal value in the opposite direction by the time the short circuit is opened as the coil passes from under the brush. This is the ideal or straight-line commutation and gives a uniform current distribution over the brush surface, as has already been stated in the discussion of Fig. 1.

Unless corrective measures are possible, however, the current in the coil tends to continue in the same direction as before its terminals were short circuited. Moreover, the current produced in the short-circuited coil, as a result of voltage generated by the cutting of the armature flux, will add to the normal or load current before reversal occurs. The resultant current in the coil may therefore actually increase in value as the coil passes under the brush. In this event the conditions are much worse at the moment the coil passes out from under the brush than if no short-circuit current had been present.

The introduction of resistance greatly reduces the short-circuiting current and thereby assists the reversal, but ideal conditions can only be obtained by introducing a counter-e.m.f. into the local short-circuited path to balance the tendency, (that is, the voltage due to self-induction) of the load current to continue in its former direction. It is evident that this counter-e.m.f. must be in the reverse direction to the short-circuit e.m.f. set up by the armature field. Therefore, to gain this end it is necessary to provide a magnetic field opposite in direction to the armature field. In practical designs this is obtained in several ways:

(*a*) By shifting the brushes forward or backward until the coil under commutation comes under an external field (the fringe of the field pole) of right direction and value. This is the usual practice in non-commutating pole machines.

(*b*) A special commutating field of right direction and value may be provided by means of either commutating poles or compensating windings or both.

Since it is not practicable to shift brushes with variations in load, it is evident that the first method, under (*a*), can give only an average condition and that the brushes must be set so as to give ideal commutation at half load. By the second method, under (*b*), having the excitation of the commutating poles and compensating windings in series with the load circuit, sufficiently correct commutating e.m.f's. can be obtained automatically over a

very wide range of operation. It should be noted that whether the reversing field is obtained by shifting the brushes, as under (*a*), or by means of commutating poles or compensating windings, as under (*b*), the contact resistance still is in all cases an important factor. A relatively high *contact resistance* is of very great help in commutation. This is especially true in large capacity machines as the coil resistance is very low. In small dynamos the resistance of the coil itself helps to limit the short-circuit current.

Contact Resistance. Contact Voltage Drop.—*Contact resistance* and *contact drop* relate to the very thin space between the commutator and the brush-contact surfaces. The terms *brush resistance* and *brush drop* frequently used in discussions on commutation do not refer to the brush itself but to the contact resistance and contact drop that develops when an electric current leaves the brush and enters the commutator or flows in the reverse direction. The resistance of the armature coils, the commutator bars, and the brushes themselves are, in most machines, quite small as compared to the contact resistance. The quality of brush material, smoothness of commutator surface, extent of contact area, contact pressure, current density, peripheral speed, temperature of the brush, and commutator surface are factors that enter into the contact-resistance equations and directly affect the commutation.

From the above it is evident that the contact resistance per unit area is not constant. It increases with the peripheral velocity and decreases with the brush pressure. A very important condition is that the contact resistance varies almost *in inverse proportion to the current density*. The contact drop is approximately constant over a considerable range. It is therefore evident that the contact resistance does not have the properties of a true resistance, but has the characteristics of the arc. It seems that the current is transferred across the contact layer by means of myriads of extremely short arcs.

If the voltage drops at the brush contact be plotted as ordinates and the current density as abscissæ (Fig. 3) the resulting curve will be somewhat similar to a saturation curve. The voltage drop starts at zero, increases in practically a straight line, then bends over and becomes nearly parallel to the X axis (Fig. 3).

For good commutation, the commutator must be a true cylinder having a hard glossy surface. The coefficient must be

low and the brushes must be in continuous contact and at constant pressure against the commutator. Any change in brush pressure caused by roughness of the commutator surface or variations from the cylindrical form will cause sparking and give unsatisfactory commutation.

Variations in temperature over the contact surfaces is the chief factor in causing changes in the contact resistance. A rise in temperature decreases the contact resistance. Aside from friction the heat generated over the contact area arises from the RI^2 losses of the load current and the short-circuit current.

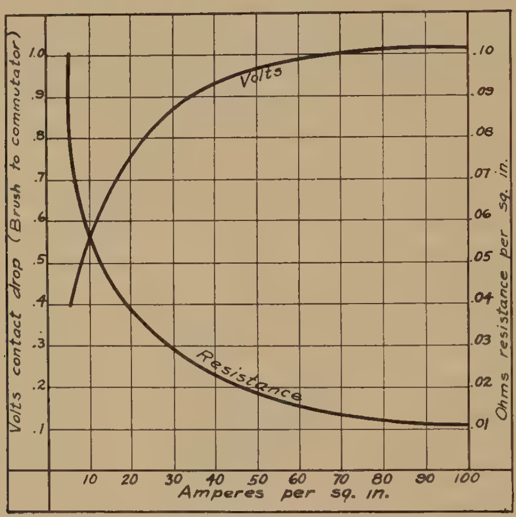


FIG. 3.—Brush resistance and voltage drop.

Hence the temperature rise is interlinked with the current-density factor. The range for the current density in carbon brushes is from 30 to 60 amp. per square inch, while the contact drop varies from 0.3 to 1.4 volts but is usually within 1 to 1.25 volts. Low-carrying capacity is generally coupled with high voltage drop; that is, the voltage drop is a limiting factor for current density.

The brush pressure is also an important factor in the contact resistance equation. In stationary dynamos brush pressures range from 1 to 3 lb. per square inch. For machines that are not stationary, vibration and jarring make a higher brush pressure necessary. In railway motors the range per brush pressure is from 3 to 8 lb. per square inch. The permissible peripheral

speeds of the commutator is largely limited by the friction of the brushes. As brushes have been improved the commutator speeds have increased.

The old type carbon brush would not operate above 4,000 ft. per minute, while with better quality of brushes the upper limit of permissable peripheral speeds has been raised to 7,500 ft. per minute. Higher peripheral speeds are possible with commutators of large diameters but short axially. Good brush contact is more readily maintained at 5,000 ft. per minute with a commutator 50 in. in diameter than with one of 10 in. in diameter.

Sparking.—It is generally assumed that myriads of extremely short arcs provide the essential mechanism for transferring the current over the contact areas between the brushes and commutator. If for any reason all of the conditions for satisfactory commutation are not fully met, so that the voltage at some part of the brush and commutator contact surface exceeds the normal contact drop, *sparking* will result. In fact, sparking is the warning signal indicating failure in securing entirely satisfactory commutation. The direct cause of sparking is excess voltage between the brush and commutator; usually between the trailing tip of the brush and the departing commutator bar or between the entering bar and the leading tip of the brush. The contact drop for satisfactory commutation is about from 1 to 1.25 volts; if between brush tip and adjacent commutator bar the voltage increases, so as to exceed the sparking voltage, sparking will take place.

Sparking Criterion.—The effect of the inductance of the armature coil on the short-circuiting current must be analyzed in greater detail than given in the previous general discussion. Since the armature coil is imbedded in slots in the iron of the armature core, considerable flux will be formed about the conductors. In full-pitch windings, the coils at both the top and bottom of a slot will undergo commutation simultaneously and the inductive flux interlinking the coils will be almost twice as large as in the case of short-pitch windings. There will also be flux lines interlinking the front and back connections, but as these portions of the coil are surrounded by air and not iron clad the inductive effect will be much less than for that portion of the coil lying in the slot. From tests made on a large number of machines, it has been found that the number of flux lines which link 1 in. length of the slot part of a coil for each ampere con-

ductor, in the group of conductors simultaneously short circuited, is approximately 10; while for the end connections the corresponding number of lines are approximately 2. If the dimensions of the coil are known the approximate total inductance of a coil can be readily determined using the above stated quantitative relations.

Having determined the coil inductance, the reactance voltage may be computed. When the current in a coil having an inductance of L henrys is changing at the rate of $\frac{di}{dt}$ amp. per second, an e.m.f. of $L\frac{di}{dt}$ volts is induced in such a direction as to oppose the change of current in accord with Lenz's law

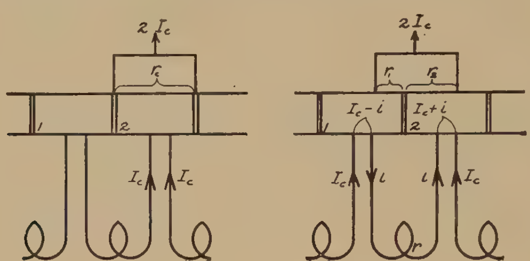


FIG. 4.

By the application of Kirchhoff's laws to the above stated quantitative relations an equation may be written for the voltage drop for the closed circuit that is formed when a coil is being short-circuited.

Let I_c = the current to be commutated.
i = the instantaneous value of the short-circuited coil current.
T = the time of commutation.
t = time measured from the beginning of commutation.
r_c = the total contact resistance of the brush with the commutator.
r = resistance of the short-circuited coil.
L = total inductance of coil.

Let the width of the brush be equal to the segment width as indicated in Fig. 4. As soon as the brush touches segment 1 commutation begins and the brush contact resistance r_c is separated into two parts. Let r_1 be the resistance from segment 1 to

the brush and r_2 the resistance from segment 2 to the brush. At any particular instant the current through r_2 will be $I_c + i$ and that through r_1 will be $I_c - i$. It is also evident that

$$r_1 = r_c\frac{T}{t} \text{ and } r_2 = r_c\frac{T}{T - t} \tag{1}$$

Since the sum of all the voltages around the short-circuited coil must equal zero the following equation may be written:

$$(I_c - i)r_1 = (I_c + i)r_2 + ri + L\frac{di}{dt} \tag{2}$$

or

$$(I_c - i)r_c\frac{T}{t} = (I_c + i)r_c\frac{T}{T - t} + ri + L\frac{di}{dt} \tag{3}$$

A complete general solution of equation (3) is not readily obtained but a solution under the condition existing at the final instant of commutation, which is the one of most importance when considering the conditions required for securing satisfactory commutation, can be obtained quite readily.

Rewriting equation (3),

$$L\frac{di}{dt} = -ri + r_c(I_c - i)\frac{T}{t} - r_c(I_c + i)\frac{T}{T - t} \tag{4}$$

When $t = T$ and $i = -I_c$, which are the conditions at the final instant of commutation,

$$L\frac{di}{dt} = rI_c + 2I_cr_c - r_cT\frac{0}{0} \tag{5}$$

The last term of equation (5) may be evaluated by differentiating, with respect to the independent variable t, both numerator and denominator of the expression from which it was obtained.

Differential of

$$\frac{I_c + i}{T - t} = \frac{\frac{di}{dt}}{-1} = -\frac{di}{dt} \tag{6}$$

Equation (5) then becomes

$$L\frac{di}{dt} = rI_c + 2I_cr_c + r_cT\frac{di}{dt} \tag{7}$$

Hence,

$$\frac{di}{dt} = \frac{I_c(r + 2r_c)}{L - r_cT} \tag{8}$$

That is, the rate of change of current at the final instant of commutation is given by equation (8).

If $L = r_cT$, di/dt the rate of change, will be infinite. Sparking will naturally occur as has already been shown for curve 7 in Fig. 2.

If $L > r_cT$, di/dt the rate of change as indicated by the slope of the line is positive, indicating that the current has been reversed to a greater extent than necessary, and quite likely will cause sparking as illustrated by curve 6 in Fig. 2.

If $L < r_cT$, di/dt will be negative and commutation may be sparkless and also entirely satisfactory, depending on how far the relation differs from the first condition above, that is, where $L = r_cT$.

From the above it is readily seen that the first two conditions between L and r_cT must be avoided and that the relation $L < r_cT$ is *the criterion of sparkless commutation.* Whether the commutation actually is sparkless or not depends on the magnitude of the difference between L and r_cT.

While the above criterion of sparkless commutation shows the limits in value of the inductance of the coil, a further relation may be obtained which determines the limits of the *average reactance voltage* of the coil. If both terms of the sparking criterion $L < r_cT$ be multiplied by $2I_c/T$ the following relation is obtained:

$$\frac{2I_c}{T}L < 2I_cr_c \tag{9}$$

Since $2I_c/T$ is the average rate of change of current of a coil during the commutation period, $\frac{2I_c}{T}L$ becomes the average reactance voltage. It is also evident that $2I_cr_c$ is the drop in voltage at the brush contact. Equation (9) therefore shows that the condition for sparkless commutation is that the average reactance voltage must be less than the brush contact drop. It is not to be inferred that equations (8) and (9) are identical indications of sparkless commutation because equation (9) has for its argument the *average* reactance voltage which of course is based on the entire time of commutation, while equation (8) relates only to the end of commutation which for sparkless commutation is obviously of prime importance.

In the foregoing discussion the effect of the generated voltage in the coil by the cutting of the fringing flux of the adjacent field poles has not been mentioned. If the factors causing this voltage are known a term may be inserted in equation (2) and the final rate of change of current found by the same method as used

above. There are various other causes that tend to exceed the sparking voltage that need not be enumerated here and form the minor limits in the design and operation of direct-current machines. For machines with commutating poles or compensating windings a corresponding counter-e.m.f. is produced which largely neutralizes the armature reaction. For non-commutating pole machines the effect of the armature flux can only be met by shifting the brushes with variations in load.

Modern direct-current machines are, however, expected to commutate without sparking from no load to 25 per cent overload without shifting the brushes. This is accomplished by placing the brushes in the position giving ideal commutation for about 65 per cent full load; that is, approximately half way between the position for no load and 25 per cent overload. At no load the armature field would be zero and the voltage produced by cutting the fringe of the main field must not exceed the sparking voltage. At 25 per cent overload the armature reactance voltage would be greater than the counter-e.m.f. generated in the short-circuited coil by the main field. This excess must also not exceed the sparking voltage if the machine shall have satisfactory commutation over the whole range of load. The above relations are illustrated in Fig. 5 for a certain machine operating successively under three different conditions; first, as a shunt generator; next, as a flat-compounded generator; and, finally, as an overcompounded generator. Regardless of which type of generator is being considered the reactance voltage will be the same for all of them when operating at the same load current. This is indicated in Fig. 5 by all the reactance voltage curves having the same slope. The voltage generated in the coil undergoing commutation due to its cutting lines of force from the fringing field, however, is not the same for the three types of generators. In the shunt machine the coil voltage due to cutting lines of the fringing flux decreases considerably from no load to full load because the main field flux is weakened and shifted by the armature reaction. For the flat compounded machine, although the armature reaction is just as strong as before, the main field has been strengthened and consequently the fringing flux is practically constant, giving a nearly constant generated coil voltage. For the overcompounded machine the main field may be so greatly strengthened that the coil voltage increases. The difference between the reactance voltage and generated coil voltage will always

be the voltage that tends to produce sparking. Since at no load the reactance voltage is zero, the brushes should not be shifted farther than to the point at which the coil generated voltage equals the sparking value as indicated in Fig. 5.

From the foregoing it is evident that an overcompounded generator would give satisfactory commutation over a wider range of load than a flat-compounded generator and that a flat-

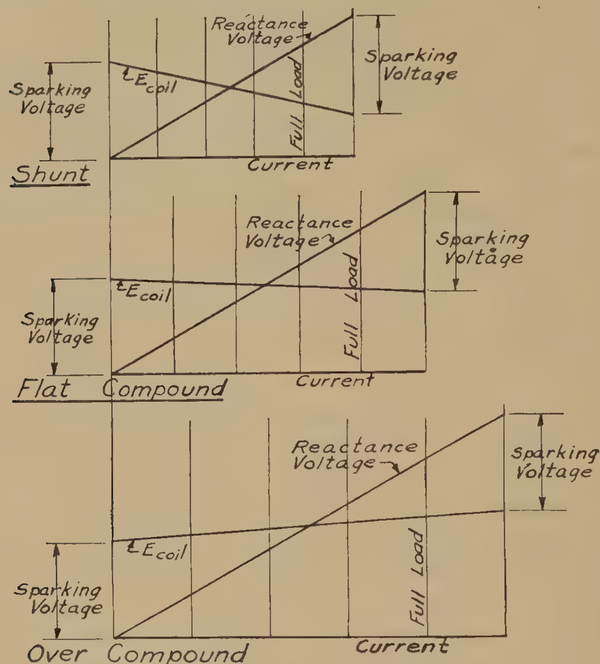


Fig. 5.

compounded machine would give greater range of load than a shunt machine.

Another important factor is the value of the contact resistance, especially as it relates to current density at the brush tips. If the contact resistance becomes too low the currents and, hence, the resistance voltage in the short-circuited coil may exceed the sparking voltage and thus cause sparking at either one or the other of the brush tips.

Sparking is often due to irregularity in the commutator surface as a loose bar or high mica between the bars. To eliminate the latter obstacle the mica is undercut, leaving a shallow grove

between the bars so that the brush can come in contact only with the copper bars.

The immediate result of sparking is pitting or pocketing and blackening of the commutator. The effect is cumulative in that the roughening of the commutator further increases the sparking. Moreover, the sparking generates heat which raises the temperature of the brush contact thereby lowering the contact resistance. This, in turn, increases the sparking.

Flashing.—One of the limiting factors in the design of commutating machinery is *flashing*. This may be of several kinds, but in essence it is a short circuit over some part of the commutator surface. A flash may originate between two adjacent bars at some point between the brush arms and then quickly develop into a general flashover. Or, there may be excessive sparking or an initial arc formed by the edge of the brush which may be carried around the armature, thus increasing in volume until it becomes a flashover to the opposite terminal or some other part of the machine. Flashes originate from a variety of causes, some of which may be normally present in the machine, while others may be of an accidental nature. The carrier of the flashover current is in all cases vaporized conducting material. True flashing is associated with vaporization, so that in many cases the initial cause of flashing may be determined by finding the source of vaporization.

If the heat developed by or in the vapor of a small arc between bars or at the brush tip is sufficient to produce more conducting vapor, the initial arc will quickly increase into a flashover.

The causes for flashing are many, but the more common may be grouped as follows:

(*a*) Ring fire or arcs between adjacent commutator bars.

(*b*) Sparking at the brushes.

(*c*) Excessive overloads or short circuits.

(*d*) Sudden break and make of circuit under load.

If small particles of conducting material become lodged between commutator bars, currents will flow which heat the particles to incandescence; and as the glowing points are on the rapidly rotating armature they will give the appearance of streaks of light or ring fire. The glowing particle may develop sufficient heat to produce conducting vapors which may become the origin of flashover. Severe sparking at the brushes may cause vaporizations of the copper or other gaseous conducting material which

would be carried along the rapidly rotating armature and lead to a flashover.

Excessive overloads may produce voltages so high per armature coil or between commutator bars that the resulting heavy currents develop conducting vapor under the brushes. These vapors will naturally be carried forward by the rotating commutator and thus cause flashing. In non-commutating pole machines with the brushes shifted under the fringe of the main field, as an aid to commutation, flashing sometimes results when such heavy loads are interrupted. If the rupture is very abrupt the sudden collapse of the magnetic field around the armature coil causes a rise in voltage which may be sufficient to start a flashover. This is particularly likely to happen in cases where the flashing limits are about reached before the circuit was interrupted. When a circuit breaker opens under heavy overload, or on a short circuit, flashing is liable to follow, although in some cases the short circuit may cause the flashover before the circuit breaker opens.

If a short circuit across the terminals occurs on a direct-current generator, either with or without external resistance a heavy current rush will result in accord with Ohm's law. The current rush is of short duration as the armature reaction will demagnetize the main field. For a short circuit without external resistance the current may reach very large values probably 25 to 40 times full-load current. Under such abnormal conditions any direct-current generator will have severe flashing at the brushes.

In direct-current railway motors flashing at the commutator is not a rare occurence. One common cause of flashing, particularly at high speed, is jolting of the brushes away from the commutator, usually due to rough track. An arc is formed between the brush and the commutator, which is carried around to the other terminal over the commutator surface. Another source is the break in the motor current when passing over a gap or dead section in the trolley circuit. Similarly, a break in the rail return circuit will cause a sudden "break and make" in the current which may cause flashover.

It is usually assumed that commutation conditions for a dynamo are practically the same whether operated on a generator or a motor. However, flashing conditions are different in an important respect. In the generator the field distortion produced

by the armature reaction crowds the flux into the trailing tip of the brush; that is, away from the leading brush tip. In motors the reverse is true in that the heaviest flux density and hence the highest voltage between armature bars comes in the leading brush tip. As the vapor, which is essential for flashing, is carried by the commutator in the direction of rotation, it is evident in this particular that conditions for flashing are different for motor or generator operation. Flashing is always undesirable as in many cases it may cause serious damage to the machines. This is particularly true in large machines or where power can be concentrated in the flashover.

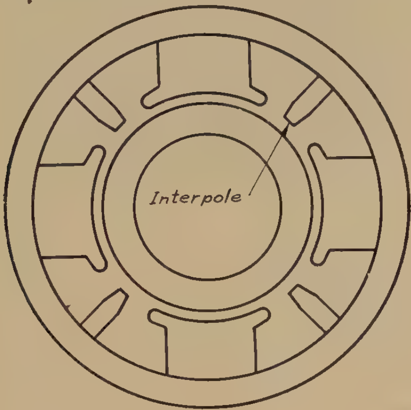


FIG. 6.—Location of interpoles.

Commutating Poles.—The commutating poles are located in the magnetically neutral planes midway between the main field poles, as shown in Fig. 6. On account of their location *commutating poles* are also called *interpoles.*

The purpose of the commutating poles is to neutralize or counteract the armature field or armature reaction. The field excitation of the commutating poles is provided by a series winding so that whatever changes occur in the armature field a change, as nearly similar as possible will take place in the commutating pole field. A commutating field coil is shown in Fig. 7, and an interpole with winding in Fig. 8.

The field structure of an interpole motor with commutating poles and series connections is shown in Fig. 9. In order to aid commutation the local flux produced by the commutating pole must be in opposite direction to the interpolar flux produced

by the currents in the armature winding. To accomplish this the magnetomotive force of the interpole winding must obviously be greater than the magnetomotive force of the armature windings in the commutating zone. An armature coil, cutting the commutating pole flux generates an e.m.f. proportional to the flux, the speed and the number of conductors in series. This voltage is in opposition to the e.m.f. in the short-circuited coil induced by the slot and end connection.

FIG. 7.—Commutating field coil. (*General Electric Company.*)

For ideal commutation these e.m.f's. should be equal and opposite. In perfect commutation the current in the short-circuited coil decreases at about a uniform rate to zero and then increases at the same rate to normal value in the opposite direction by the time the coil leaves the brush. Under these conditions there are no local currents in the short-circuited coil.

FIG. 8.—Commutating pole core and field coil. (*General Electric Company.*)

These conditions can be approximated only in so far as the e.m.f. obtained from the interpole at all times balances the armature e.m.f's. in the short-circuited coil.

The actual effect of the interpole is shown in Fig. 10. If the machine has no compensating winding the flux will be distorted under load such that with the brush in the geometrical neutral position, that is, half-way between the poles, considerable voltage

would be generated in the coil by the flux *a-b*. In order to counteract the inductive effect of the coil, the brushes must be shifted beyond the new flux neutral point *c* to a point *e*, where

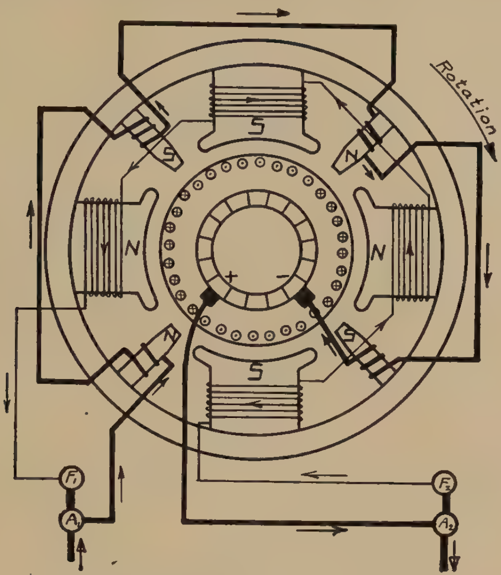


FIG. 9.—Circuit diagram of interpole dynamo.

the necessary commutating flux exists. Point *e*, Fig. 10, is a very unsatisfactory location of the brush for good commutation, however, not alone for the fact that the flux will radically

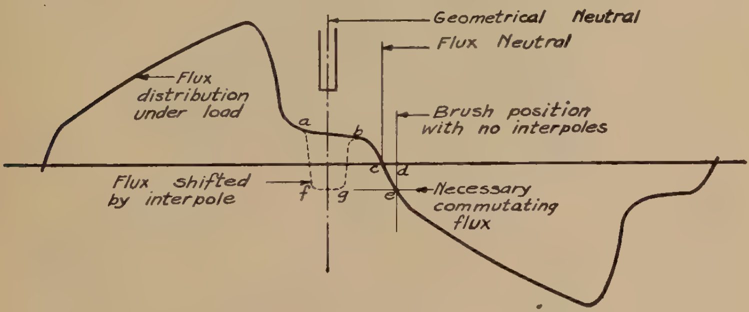


FIG. 10.—Effect of commutating pole.

change for variations in load but also because of the steep slope of the flux-density curve in that sparking may occur at both edges of the brush, although at the middle of the brush commuta-

tion may be perfect. The logical position of the interpole is therefore at some point where the flux is practically uniform, such as *ab*. If the interpole strength is sufficient to make the resultant flux negative, as illustrated by *f-g* in Fig. 10, satisfactory commutation should be obtained.

If taken separately the e.m.f's. produced in the short-circuited coil by the interpole and armature fluxes may be quite large, but since the two are in opposition only the difference at any instant need be considered. In practice it is not possible to obtain exact equality between the interpole and armature e.m.f's. This difference must be taken care of by the contact resistance of the brushes in order to secure satisfactory commutation.

In the design of commutating poles the e.m.f's. due to armature fluxes are determined first and then the interpole flux is made of such magnitude that it will generate an equal or slightly larger e.m.f. than would be generated by the armature flux alone.

The brush setting in relation to the interpoles is of importance because the interpoles are fixed in position and the maximum point of the armature magnetomotive force is determined by the location of the brushes. Any shifting, forward or backward, of the brushes, will cause a change in the flux direction and will cause improper commutation in some of the armature coils.

Since the function of the interpole is merely to introduce a voltage in the coil undergoing commutation, it is immaterial whether this be accomplished at only one or both sides of the coil. If the voltage is to be generated in both sides, there will be as many interpoles as main poles. It is evident, however, that essentially good results could be obtained if alternate interpoles were omitted if those remaining are made twice as strong. In a wave-wound multipolar machine having but two brushes a coil undergoing commutation consists of as many conductors as there are interpolar spaces each one of which is passing through its own neutral interpolar region. A single commutating pole, if large enough, may thus be placed between any two main poles and prove sufficient for the entire machine. In practice, however, it is usually impossible to design a single pole of sufficient strength. Two or more interpoles are consequently used.

Saturation of the interpole magnetic circuit, to some extent due to high leakage of the interpole flux, greatly modifies the useful flux during heavy overloads or short circuits, and leads, under abnormal conditions, to excessive sparking or flashing.

If, however, the interpole is combined with a compensating winding in the main poles, the leakage will be greatly reduced and the machine will carry much heavier overloads without flashing or excessive sparking.

In motors having interpoles, the location of the brushes is especially important for other reasons than the prevention of sparking.

It is evident that shifting of the brushes causes the interpole flux to move in part or even in full away from the coil undergoing commutation, and to induce an e.m.f. in the active voltage generating conductors of the armature. Depending on the direction in which the brushes are shifted from the correct neutral position, the interpole flux may be made to either add or subtract from the main field flux. As a result the machine takes on the characteristics of either a series or differentially compound motor tending to either slow down or speed up and even run away when load is placed on the motor.

Compensating Windings.—The compensating winding is placed in grooves or slots in the main-field pole faces. A pole core showing the slots for the compensating windings is shown in Fig. 11.

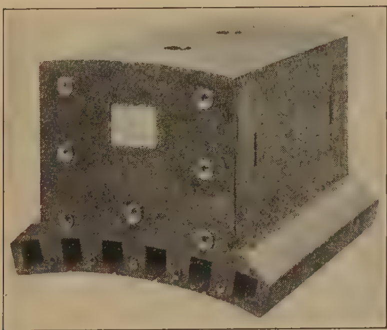


Fig. 11.—Compensating winding slots in core of field pole. (*General Electric Company.*)

The compensating winding is connected in series to the armature circuit and so arranged that the current in each conductor of the winding flows in opposite direction to that of the current in the adjacent armature conductor. In Fig. 12 is shown the field structure of a 14-pole direct-current mill motor having thereon the main fields, interpoles, and compensating windings. A section of the field structure in Fig. 12 is shown in larger scale

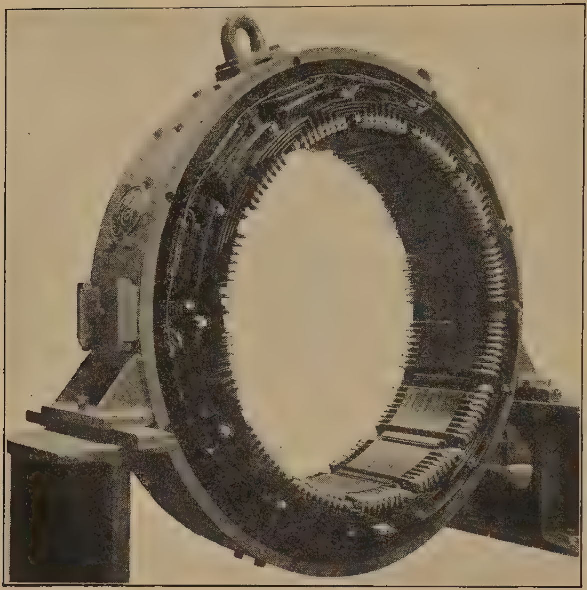


Fig. 12.—Motor field structure. (*General Electric Company.*)

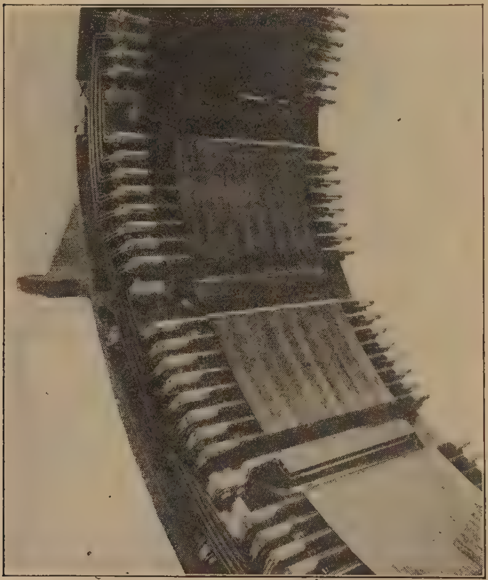


Fig. 13.—Section of field structure. (*General Electric Company.*)

in Fig. 13. It should be noted that the compensating winding is connected in series with the interpole excitation. In effect the compensating winding supplements the interpoles in neutralizing or compensating the armature reaction and especially in preventing field distortion.

The main purpose of the compensating winding is to prevent distortion which is not accomplished by the interpole magnetomotive force. Under heavy overloads, short circuits or any sudden change of load current, the e.m.f. produced in the compensating winding will be approximately proportional to the armature reaction. Moreover, the compensating winding prevents excessive flux leakage during heavy overloads or short circuits.

PROBLEMS

1. Sketch a four-pole machine with interpoles similar to Fig. 6. Indicate the direction of rotation and show the proper sequence of polarity of the poles (north or south) to give correct commutation in a generator. Do likewise for a motor.

2. If an interpole shunt motor is reversed in rotation by reversing the shunt-field leads, will it be necessary to change the connections of the interpole windings in order to get correct commutation?

3. (*a*) If the interpoles of a shunt generator are correctly connected for a given direction of rotation, will they be correct for rotation in the opposite direction without changing interpole or shunt-field leads? Explain by means of a sketch of a gramme-ring winding in a machine having four field poles and interpoles. Assume a given direction for the residual magnetism.

(*b*) Having decided whether the original connections are correct or not, and having made any changes deemed necessary, will the polarity of the machine be changed when run in the opposite direction?

(*c*) How could an opposite polarity from that of (*b*) be obtained? Explain each part briefly. Use sketches where necessary.

4. A shunt generator having interpoles runs in the clockwise direction when operating as a generator. Must any interpole connections be changed in order that this machine may be used as a motor operating in either direction or with either polarity at its terminals?

5. If the machine of Problem 2 were a compound motor would any changes be necessary in the series-field connections?

6. If the machine of Problem 3 were a compound generator would any changes be necessary in the series-field connections? Explain briefly for both (*a*) and (*c*).

7. If the machine of Problem 4 were a compound generator would any changes be necessary in the series-field connections? Explain briefly.

8. From the results of the questions in the above problems what conclusions may be made about the interpole connections of motors and generators?

CHAPTER XVIII

LOSSES, EFFICIENCY, HEATING, RATING[1]

The transformation of mechanical energy into electrical energy by the generator and the reverse process in the motor are always accompanied by the conversion of part of the energy received by the machine into heat. As the heat serves no useful purpose but must be radiated or otherwise removed from the several parts of the machine in order that the temperatures shall not become too high, this wasted portion of the energy is generally referred to as the *losses*. Hence, in every dynamo, whether used as a generator or a motor, the *power input*, that is, the rate of receiving energy, is greater than the *power output*, the rate the machine delivers energy.

The losses represent energy transformed into heat, and hence under operation the temperature in the several parts of the machine will rise above the surrounding air until the rate of heat dissipation (radiation, conduction, convection) becomes equal to the rate of heat generation. The rise in temperature due to the losses is a very important factor in limiting the capacity of any given dynamo; that is, in many respects the permissible rise in temperature determines the limits within which the machine must be operated. It is evident that the relative magnitude of the losses is of importance, not only on account of the value of the wasted energy but more particularly as regards the con-

[1] The tabulated data and rules in this chapter are from the December, 1925 issue of the A.I.E.E. Standards. The *A.I.E.E. Standards* adopted by the American Institute of Electrical Engineers, the *N.E.M.A. Apparatus Standards* sponsored by the National Electrical Manufacturers Association, and the *A.E.S.C. Standards* approved by the American Engineering Standards Committee form the basis for standard practice in electrical engineering in the United States. The rules and regulations of the International Electrical Commission (I.E.C.), with which the standards of the A.I.E.E., N.E.M.A., and A.E.S.C. are in full accord, form the foundations for organized standard practice in all parts of the world. The reader is advised to consult the latest editions of the above publications for detailed information and the precise wording of the rules and standards that govern the ratings and tests of electrical machinery.

sequent rise in the temperature as a limiting factor in the rating and operation of the machine.

Losses.—In direct-current machines the converting of mechanical or electrical energy into heat is accomplished in a number of ways in the several parts of the machine; that is, the losses come from several sources and accordingly may be separated into groups, such as *copper losses, iron losses, friction and windage losses, stray-load losses,* etc.

The RI^2 *losses* include the armature, shunt field, series field, compensating winding, and interpole *resistance losses,* together with the brush *contact loss.* Rheostat losses are included with the field-winding loss when they are present, whether the machine is self- or separately excited.

The *no-load core losses* are found chiefly in the armature iron and are produced by hysteresis and eddy currents. These losses are due to the reversal of magnetic flux in the armature iron as it moves across a pair of poles. Since a full cycle is completed for each pair of north and south poles, the direction of flux is alternating at a frequency depending on the armature speed and the number of poles in the machine.

In Chap. XI formulæ were derived for both eddy current and hysteresis loss in terms of frequency and flux density. It is, however, well-nigh impossible to apply these equations to rotating armatures because of the large variation of flux density throughout the cross-section of the core. These losses, however, can be obtained with a high degree of accuracy by test.

The *windage and friction losses* are made up of the bearing and brush friction of the machine and the air resistance to turning of the armature. These losses are independent of the load and in all except very high speed machines are quite small. The bearings and windage losses are very difficult to separate and only their combined value can be obtained by test.

The *stray-load losses* are several in number; namely, eddy-current loss in the armature conductors, short-circuit loss of coils during commutation, increased-core loss due to flux distortion and tooth-frequency losses due to flux distortion. The *eddy-current loss* in the armature conductors may be produced under load conditions by two causes. First, each conductor when carrying current sets up a magnetic flux which is reversed when commutation takes place. This collapse and building up of flux will induce eddy-current losses in near by conductors.

Second, armature reaction will cause flux distortion over the pole face and thereby produce much higher flux densities in some of the teeth than others. Higher tooth densities will cause greater slot leakage and the pulsation of the slot leakage flux through the copper conductors will produce additional eddy current losses. The *increased core losses due to flux distortion* are caused by the maximum flux density being increased by armature reaction, although the average flux density may remain the same. Under no-load conditions the pole face of a machine will have a considerable variation in flux density for every given point on its surface, because of the changing reluctance of the magnetic circuit caused by the slots in the armature. A *hysteresis- and eddy-current loss* is thus produced at tooth frequency in the pole face. Due to armature reaction during load the magnitude of the pulsations is changed, which causes an actual increase in the core losses.

The foregoing losses may also be classified on the basis of the degree of accuracy with which they can be computed or measured, as shown in Table XIII.

TABLE XIII.—CLASSIFICATION OF LOSSES

Accurately measurable	Approximately measurable or determinable	Indeterminable
No-load core losses, including eddy-current losses in conductors at no-load.	Brush friction loss	Iron loss due to flux distortion
Load RI^2 losses in windings. No-load RI^2 losses in windings.	Brush-contact loss	Eddy-current losses in conductors due to transverse fluxes occasioned by the load currents
	Losses due to windage and to bearing friction	Eddy-current losses in conductors due to tooth saturation resulting from distortion of the main flux
		Tooth-frequency losses due to flux distortion under load
		Short-circuit loss of commutation

The losses in direct-current dynamos should be measured, calculated or conveniently taken as specified in the following rules and regulations of the A.I.E.E. Standards:

5-361. RI^2 Losses.—The RI^2 losses shall be based upon the current and the measured resistance, corrected to 75°C.

5-362. Bearing Friction and Windage. (*a*) *General.*—Drive the machine from an independent motor, the output of which shall be suitably determined. The machine under test shall have its brushes removed and shall not be excited. This output represents the bearing friction and windage of the machine under test.

(*b*) *Engine-type Generators.*—In the case of engine-type generators, the windage and bearing friction loss is ordinarily very small, amounting to a fraction of 1 per cent of the output. This loss shall be neglected owing to its small value and the difficulty of measuring it.

5-363. Brush Friction of Commutator and Collector Rings.—(*a*) Drive the machine from an independent motor, the output of which shall be suitably determined. The brushes shall be in contact with the commutator, but the machine shall not be excited. The difference between the output obtained in the test in paragraph 5-362 and this output shall be taken as the brush friction. The surfaces of the commutator and brushes should be smooth and polished from running when this test is made.

(*b*) Experience has shown that wide variations are obtained in tests of brush friction made at the factory before the commutator and brushes have received the smooth surfaces that come after continued operation. Conventional values of brush friction, representing average values of many tests shall be used, as follows:

	Watts per square inch of brush contact surface per 1,000 feet per minute peripheral speed
Carbon and graphite brushes	8.0 watts
Metal graphite brushes	5.0 watts

In the event that these conventional values are questioned in any case, the brush friction shall be measured as in (*a*) above.

Where the actual values of the brush-friction loss for a given kind of brush for a certain application have been accurately determined these values may be used in similar cases.

5-364. Core Losses.—Drive the machine from an independent motor, the output of which shall be suitably determined. The brushes shall be in contact with the commutator and the machine shall be excited, so as to produce at the terminals a voltage corresponding to the calculated internal voltage for the load under consideration. The difference between the output obtained by this test and that obtained by test under paragraph 5-363 (*a*) shall be taken as the core loss.

It is also recognized practise to determine the core loss by driving the machine with only one brush on each stud in contact with the commutator

and taking the difference of the losses of the machine unexcited and excited as above.

5-365. Brush-contact Loss.—A total drop (of positive and negative brushes) of 2 volts shall be assumed as the standard drop in determining brush-contact loss for carbon and graphite brushes with pigtails attached. A total drop of three volts shall be assumed where pigtails are not attached. One-quarter of 1 volt for each collector ring shall be used at all loads in calculating the brush-contact loss for metal-graphite brushes.

5-366. Stray-load Losses.—The stray-load losses include the items in the column of Table I headed "Indeterminable." For calculating the conventional efficiencies of direct-current generators and motors, stray-load losses shall be taken as 1 per cent of the output, except that for motors of 200 h.p. at 575 r.p.m. and smaller, no value has yet been assigned, and for the present stray-load losses shall be omitted.

5-367. Miscellaneous Losses. (*a*) *Field-rheostat Losses.*—All losses due to field rheostats, either series or multiple connected, shall be included in the determination of the efficiency, even when the machine is separately excited.

(*b*) *Ventilating Blower.*—When a separately driven blower supplies air to a machine set the power required to drive it shall be charged against the complete unit, but when one or more separately driven blowers supply air through a single duct to two or more machines the power required to drive the blower or blowers shall be charged against the plant or station and not against the machine set.

(*c*) *Other Auxiliary Apparatus.*—Auxiliary apparatus, such as a separate exciter for a generator or motor, shall have its losses charged against the plant of which the generator and exciter are a part, and not against the generator. An exception should be noted in the case of turbo-generator sets with direct-connected exciters, in which case the losses in the exciter shall be charged against the generator. The actual energy of excitation and the field-rheostat losses, if any, shall be charged against the generator.

5-368. Other Recognized Methods of Determining Losses. *Running Light Method.*—The machine is run at no load as a motor and the input measured. This input, after deducting any series RI^2 losses, represents the sum of the losses, which shall be regarded as constant, discussed in paragraphs 5-362, 5-363, and 5-364. The losses at any load may be obtained by adding to these constant losses the RL^2 losses based upon the current and measured resistance, the brush-contact losses, the stray losses, and the miscellaneous losses.

It will be noted that the methods for determining the several losses in direct-current machines as given in the foregoing rules call for a calibrated driving motor. If such a machine is not available the losses are then found as described in Rule 5-368 but by this method they cannot be separated into their component parts as readily as by using a calibrated motor. The *running-light* data are obtained with the *same induced voltage* acting in the machine as under load conditions and with the same

speed. An example will serve to show how both of these relations are obtained. A 25-kw., 250-volt generator is given a running-light test and the following data obtained:

Armature resistance	= 0.082 ohms.
Full-load armature current, neglecting field current	= 100 Amp.
No-load armature current	= 6 Amp.
Full-load speed	= 960 r.p.m.
Generated e.m.f. (approximate) at full load	= 250 + 100(0.082) = 258.2 volts.
Impressed terminal voltage running light (same as generator voltage in generator at full load)	= 258.2 + 6(0.082) = 258.7 volts.

With 258.7 volts impressed on the machine the field current is then varied until the speed becomes 960 r.p.m. This value of field current is the normal field current of the machine from which the field loss can be computed. The armature input at no load minus the armature copper loss gives the *stray-power loss* (core losses, windage and friction). These losses remain practically constant at all loads in constant-potential constant-speed machines.

In addition to the already mentioned methods of separating the losses of a machine there is another known as the *retardation method* which gives fairly accurate results on large machines. This method is as follows: The machine to be tested is started as a motor and the excitation is adjusted to normal value. The armature circuit is then broken and the field current is kept constant. Readings of speed are taken at definite intervals of time as the armature slows down. From these data a speed-time curve may be plotted as illustrated by the lower curve in Fig. 1. The rate of deceleration of the armature, as the machine slows down, is at every point along the curve directly proportional to the force required to keep the armature turning at that speed. If the force to maintain rotation drops to one-half then the deceleration has also decreased to one-half and the slope of the speed-time curve is one-half the initial slope. The stray-power losses (core losses, windage and friction) can thus

be found at any speed, since these are the only losses which produce deceleration under the given conditions.

$$\text{Stray power loss} = kn\frac{dn}{dt} \tag{4}$$

In order to find the value of the constant k, the stray-power loss must be computed from a running-light test at some convenient point on the curve. Usually more than one test is run and the average value obtained for k is used in equation (4).

To separate the losses, speed-time deceleration curves similar to the above are obtained with both armature and field circuits broken. In this case the deceleration will be caused by friction and windage. The difference in the two cases gives the core

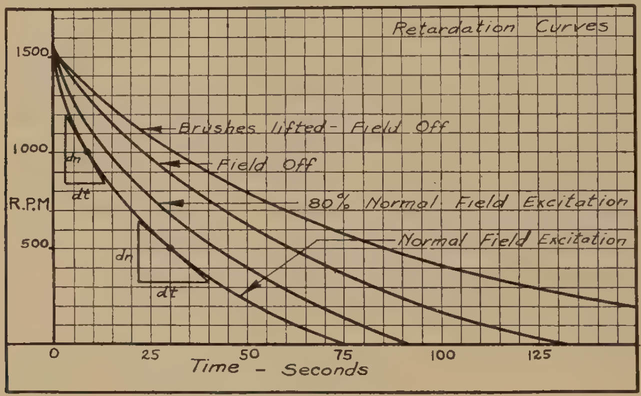


FIG. 1.—Retardation curves.

losses. A third deceleration curve, for which the brushes are lifted, in combination with the above curve, makes it possible to compute the bearing friction and windage. It is well to note that the constant k in the above equation remains the same for all the curves, since the deceleration is being produced on the same rotating armature, the moment of inertia of which stays constant in the three cases.

Efficiency.—The efficiency of a direct-current commutating machine is defined as the ratio of the useful power output to the total power input.

$$\eta = \text{efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{input} - \text{losses}}{\text{input}} \tag{5}$$

Efficiency is usually expressed in the percentage of the input instead of the fractional ratio as represented by equation (5).

In most cases the losses are small in comparison to the input or output, and therefore it is preferable to measure the losses and either the input or the output instead of computing the efficiency from input and output data. The losses themselves are composed of several factors, some of which are small in comparison to the rest and some of which cannot be readily obtained from test data. For this reason two methods for obtaining efficiency are in general use; namely, *directly measured* and *conventional efficiency*.

(1) *Directly measured efficiency* is obtained from simultaneous measurements of input and output or by the accurate determination of all the component losses in combination with measurements of either input or output. In obtaining the directly measured efficiency of a machine the input and output should be measured by one of the following methods:

(*a*) Input and output are measured simultaneously at the terminals of the machine by means of electrical measuring instruments.

(*b*) The mechanical power is measured by means of brake or dynamometer, the electrical power by means of electrical measuring instruments.

(*c*) The mechanical power is measured by means of a calibrated auxiliary machine, the electrical power by means of electrical measuring instruments.

Mechanical power delivered by the machine should be measured at the pully, gear, or coupling on the motor shaft, in order to exclude the loss of power in belt or gear friction. The efficiency should be measured at the rated voltage and speed. Readings are usually taken at ¼, ½, ¾ full, and 1¼ rated load, and as nearly as possible at the final temperature that obtains after operating for the required time in accord with the specified rating.

The directly measured efficiency of large machines is very often obtained by the *pump back* or Kapp's opposition method of testing, which requires two practically identical machines, one or both of which are to be tested. One of the machines is used as a motor to drive the other as a generator. The output of the generator is fed back to the motor terminals where it combines with power from an outside supply circuit to furnish the necessary motor current. The current required by the motor from the power-supply circuit (in addition to the generator current)

is comparatively small, as it merely supplies the energy dissipated by the losses of the two machines. The advantage of the pump-back method is that comparatively very little energy is required. The machines are actually under load conditions and the losses can be ascertained quite accurately. The main disadvantage is that two, essentially duplicate, machines are required. It is generally assumed that the stray-power losses in the two machines are in direct proportion to their generated or back electromotive forces. This method of dividing the stray-power losses is not, however, strictly correct.

(2) *Conventional efficiency* is obtained from the component losses; most of which are accurately determinable and the remainder of which are assigned conventional values; or, all of the losses may be determined by conventional methods of computation. The efficiency obtained in this way is the ratio of the output to the sum of the output and the losses, or of the input minus the losses to the input. In obtaining the conventional efficiency of constant-voltage, constant-speed machines the rated voltage and speed are used; for adjustable-speed motors, the base speed. The efficiency of all apparatus, at all loads, is generally based on a reference temperature of 75°C. In computing conventional efficiencies the following losses are considered:

(*a*) RI^2 losses in armature and field windings;
(*b*) Bearing friction and windage losses;
(*c*) Brush friction loss;
(*d*) Core loss;
(*e*) Brush-contact loss;
(*f*) Stray-load loss (generally omitted);
(*g*) Miscellaneous losses.

The efficiency that has been discussed in the preceeding paragraphs is called the *over-all efficiency* as distinguished from the *mechanical* and *conversion efficiencies.* The generator receives mechanical power and delivers electrical power, but the conversion is not complete, as part of the energy is consumed by the mechanical losses, namely, core losses, friction and windage. The core loss, although in the electric form before it is changed into heat, is classified as a mechanical loss because of its initial brake action on the armature. After subtracting the above losses from the mechanical input the remainder will be the actual electrical power converted from which the efficiency of conversion

can be computed. For generators the above relations are expressed by equations (6), (7) and (8).

$$\text{Electrical power developed} = \text{mechanical input} - \text{friction, windage and core losses} \tag{6}$$

$$\text{Efficiency of conversion} = \frac{\text{electrical power developed}}{\text{mechanical power input}} \tag{7}$$

$$= \frac{\text{output} + RI^2 \text{ losses}}{\text{mechanical power input}}$$

$$= \frac{\text{output} + RI^2 \text{ losses}}{\text{output} + \text{all losses}}$$

$$\text{Electrical efficiency} = \frac{\text{electrical power output}}{\text{electrical power converted}} \tag{8}$$

$$= \frac{\text{electrical power output}}{\text{electrical power output} + RI^2 \text{ losses}}$$

For motors the corresponding efficiencies are given by equations (9) and (10).

$$\text{Efficiency of conversion} = \frac{\text{mechanical power developed}}{\text{electrical power input}} \tag{9}$$

$$= \frac{\text{electrical power input} - RI^2 \text{ losses}}{\text{electrical power input}}$$

$$= \frac{\text{mechanical power output} + \text{stray power losses}}{\text{electrical power input}}$$

$$\text{Mechanical efficiency} = \frac{\text{mechanical power output}}{\text{mechanical power developed}} \tag{10}$$

$$= \frac{\text{mechanical power output}}{\text{mechanical power output} + \text{stray power loss}}$$

Maximum Efficiency.—For the purpose of computing conventional losses in constant-potential constant-speed machines the losses are divided into two groups:

(*a*) Losses that are essentially constant at all loads;

(*b*) Losses that vary with the load.

It is evident by inspection all but the RI^2 losses in the armature, the series field, the commutating pole and the compensating windings are essentially constant. This includes the RI^2 losses in the shunt-field winding. The copper losses in the armature and series circuits vary as the square of the current and hence as the square of the load in constant potential machines.

Let P = the load or output,
P_c = the constant losses
kP^2 = the variable losses where k is a constant

Then the efficiency

$$\eta = \frac{P}{P + P_c + kP^2} \tag{11}$$

Evidently the maximum efficiency for variable load may be obtained from equation (11) by equating the first derivative, with respect to the variable load, to zero.

$$\frac{d\eta}{dP} = \frac{P + P_c + kP^2 - P(1 + 2kP)}{(P + P_c + kP^2)^2} = 0 \tag{12}$$

Hence for maximum efficiency,

$$P_c = kP \tag{13}$$

Substituting in equation (11) the maximum efficiency is expressed by equation (14)

$$\eta_{\max} = \frac{P}{P + 2P_c} \tag{14}$$

In the above derivation of maximum efficiency all the variable losses have been assumed to vary as the square of the power. There are, however, losses which would be more correctly represented if assumed to vary directly as the power. For instance, the brush loss of a machine could be represented by kP, because the brush drop tends to remain fairly constant, regardless of the load current. If this additional term kP be included in the expression for efficiency from which the maximum efficiency is derived it will be found that the maximum efficiency occurs where $P = kP$ which is identical with the result obtained when all of the variable losses were assumed to vary with the square of the power. The curves illustrated in Fig. 2 show the constant losses, variable losses, and efficiency curve with the maximum value for the load at which the constant and variable losses are equal.

In series motors the losses cannot be grouped as constant and variable in the manner described above. The flux and armature speed, and hence the magnitude and frequency of the currents in the armature iron vary with the load. Because of the varying permeability of the iron for the range of magnetic flux densities from no load to full load, neither the eddy-current loss, represented by: $k_c(\text{speed})^2B^2$, nor the hysteresis loss, expressed by: k_h $(\text{speed})B^{1.6}$, would be directly proportional to the power or to

the power squared. If the permeability of the iron core were fairly constant then the eddy-current loss would be very nearly

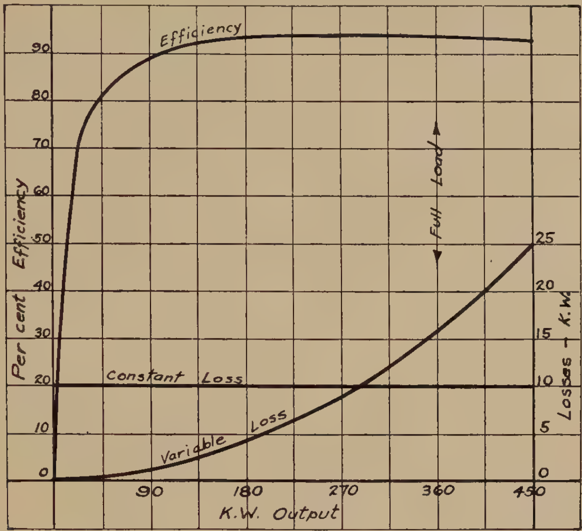


FIG. 2.—Efficiency load curve. Direct-current generator.

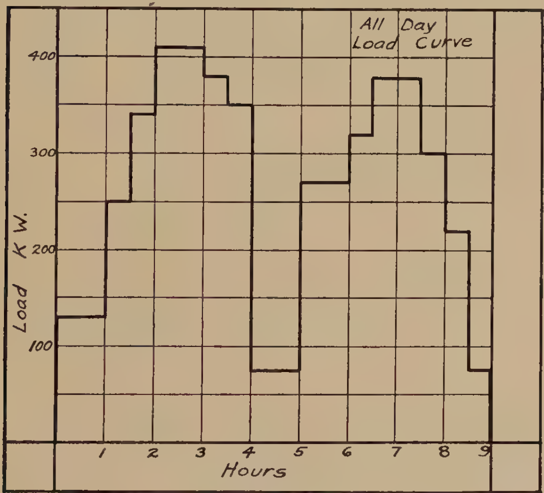


FIG. 3.—Typical all-day load curve.

constant for all loads, while the hysteresis loss would increase slightly. It should also be noted that the windage and friction loss which is fairly constant in most constant potential direct-

current machines, varies over a wide range in the series motor because of the large variation in armature speed. From the above it is evident that under operating conditions with both speed and load variable, as in electric railway service, the maximum efficiency of the series motor becomes a complex function that cannot be expressed in the form of a simple equation.

Under practical operating conditions the load varies and as the charges for electrical power are based primarily on the energy consumed over a given period, that is in kilowatt-hours, it is generally important to know not merely the efficiency for any given load but also the ratio of the net *energy output* to the total *energy input* for a given period. If given for the time of a working day this ratio is known as the *all-day efficiency of the machine*. A typical all-day load curve is shown in Fig. 3.

Determination of Losses in Series Motors.—For determining the stray losses of shunt machines only one set of readings usually under running light (no outside load) conditions, need be taken, as the speed varies so little from no load to full load that the stray losses may be considered as constant for all loads. For the series motor, however, both the speed and the armature flux vary between limits, widely apart so that a series of observed data must be obtained under varying speed as well as varying armature-current conditions. From the data thus obtained curves may be plotted showing the core loss for any speed and any armature current and also the windage and friction losses for any speed within the range of the motor tested.

During the test the series-field current is supplied from an outside source. That is, the motor is separately excited in order that the field current may be varied independently of the current flowing in the armature; usually accomplished by means of a lamp bank resistor. The armature current is likewise obtained from an independent source, preferably from a generator, the voltage of which can be varied as desired. The circuit connections for the test are shown in Fig. 4.

For a series of values of the field current under running light (no outside load) conditions the corresponding readings of armature voltage and current and field current are recorded. The speed is kept at the desired constant value by adjusting the voltage impressed on the armature. For each running light set of readings the armature copper loss is subtracted from the armature input leaving the stray-power losses consisting of core losses,

windage and friction. These values of stray-power losses are plotted as shown in Fig. 5 against the corresponding values of field current. By running tests at several constant values of

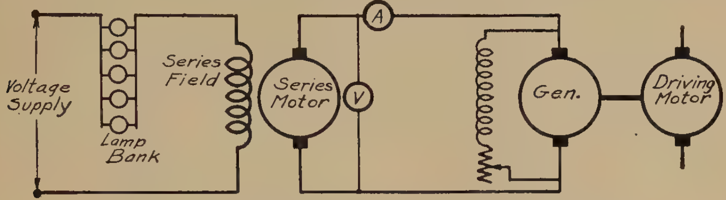


FIG. 4.—Circuit diagram for test of series motor.

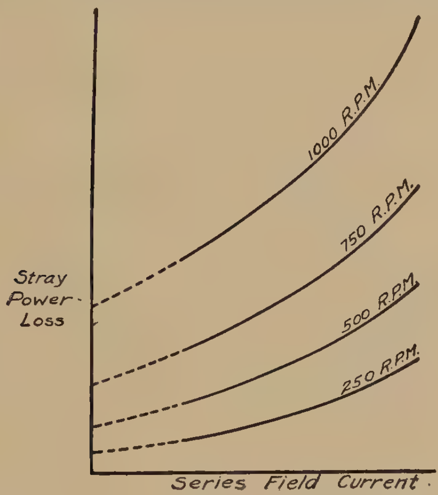


FIG. 5.—Stray-power loss curves at constant speeds.

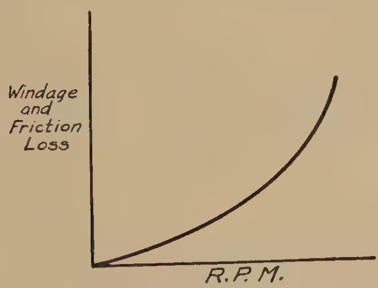


FIG. 6.—Windage and friction loss of series generator.

speed a corresponding number of curves may be plotted as in Fig. 5. If the curves of Fig. 5 be extended to the zero axis of ordinates as indicated by the dotted lines, the intercepts on the

vertical axis will indicate the windage and friction losses corresponding to the several speeds since the field current is zero and consequently the core loss would be reduced to zero.

The friction and windage loss curve in relation to speed may then be plotted as in Fig. 6 from the intercepts obtained in Fig. 5.

If from the ordinates of each of the curves in Fig. 5, the windage and friction losses be subtracted, curves may be plotted showing

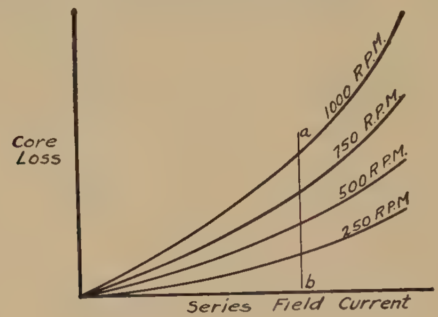


Fig. 7.—Core losses at constant speeds.

the core loss alone, varying for different values of field current for the several constant speeds as shown in Fig. 7.

It is often more desirable to have the core loss plotted against r.p.m. at constant values of field current. These curves may be obtained from Fig. 7 by drawing vertical lines such as *ab* at any

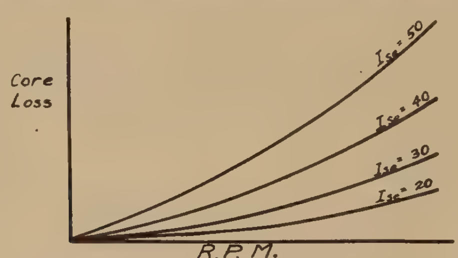


Fig. 8.—Core losses at constant current.

desired value of field current and then plotting the intercepts against r.p.m. as in Fig. 8.

It is evident that from Figs. 7 and 8 the core loss may be obtained for any speed and for any field current, which for the series motor is the same as the armature or line current. The application of the losses as plotted in Figs. 6, 7 and 8 is considerably more complicated in computing the efficiencies of series

motors than in the similar operation for shunt motors. In shunt motors the copper loss alone varies with different assumed values of armature or load current, the stray losses being considered constant due to nearly constant speed. In series motors, an assumption of armature current necessitates the finding of the speed corresponding to the given current, if the motor is operated on constant voltage mains. The correct speed can be approximated by the following method: From the running-light readings for any given speed, which were obtained for computing the stray-power loss curves of Fig. 5, values of $\phi Z'$ may be computed corresponding to the different values of field current. The values

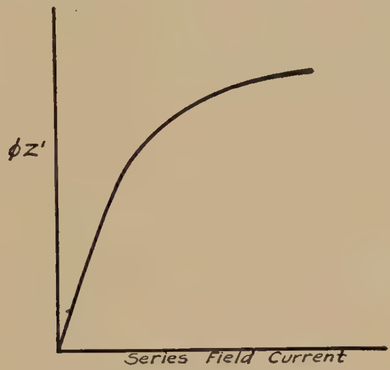


FIG. 9.—$\phi Z'$ saturation curve.

of $\phi Z'$ are computed from the speed equation (15) in which, all the factors for the speed are known except $\phi Z'$

$$n = \frac{V - R_a I_a}{\phi Z'} \tag{15}$$

It is immaterial which speed is chosen in computing $\phi Z'$, since its value for any particular machine depends only on the field current. Having calculated $\phi Z'$ for several values of field current a curve may be plotted as in Fig. 9.

For any value of armature or load current the corresponding $\phi Z'$ may be obtained from Fig. 9. By substituting values for $\phi Z'$ in equation (16) the corresponding speed may be computed.

$$n = \frac{V - (R_a + R_{se})I_a}{\phi Z'} \tag{16}$$

Having determined both the speed and field current the correct core, windage, and friction losses may be obtained from Figs. 6,

7, and 8. The copper losses $(R_a + R_{se})I_a^2$, may be computed and the over-all efficiency of the motor calculated.

The foregoing method of obtaining the efficiency by losses of a series motor neglects the effect on speed caused by armature reaction. This error, however, cannot be unduly large and the calculated efficiency values should be approximately correct.

Temperature Limits.—Heat is generated in all electrical machines by the energy losses incidental to their operations.

TABLE XIV.—CLASSIFICATION OF INSULATING MATERIALS

Class	Description of material
O..............	Class O insulation consists of cotton, silk, paper, and similar organic materials when neither impregnated nor immersed in oil
A..............	Class A insulation consists of cotton, silk, paper, and similar organic materials when impregnated or immersed in oil; also enamel as applied to conductors. (An insulation is considered to be "impregnated" when a suitable substance replaces the air between the insulated conductors. The impregnating substance, in order to be considered suitable, must have good insulating properties; must entirely cover the fibers and render them adherent to each other and to the conductor; must not produce interstices within itself as a consequence of evaporation of the solvent or through any other cause; must not flow during the operation of the machine at full working load or at temperature limit specified; must not unduly deteriorate under prolonged action of heat.)
B..............	Class B insulation consists of inorganic materials such as mica and asbestos in built-up form combined with binding substances. If Class A material is used in small quantities in conjunction for structural purposes only, the combined material may be considered as Class B provided the electrical and mechanical properties of the insulated winding are not impaired by the application of the temperature permitted for Class B material. (The word "impair" is here used in the sense of causing any change which could disqualify the insulating material for continuous service.)
C..............	Class C insulation consists of inorganic materials such as pure mica, porcelain, quartz, etc.

Necessarily, the temperatures in the various parts of a generator or motor will rise until the rate of heat dissipation (radiation conduction, and convection) becomes equal to the rate of heat generated. The temperature limits on which the rating of the machine is based are largely determined by the physical properties of the respective insulating materials. Insulating materials are classified as shown in Table XIV as a basis for establishing temperature for rating purposes.

TABLE XV.—METHODS OF TEMPERATURE DETERMINATION

Method	Description of method
Thermometer	This method consists in the determination of the temperature, by mercury or alcohol thermometers, by resistance thermometers, or by thermocouples, any of these instruments being applied to the hottest part of the machine accessible to mercury or alcohol thermometers
Resistance	This method consists in the determination of temperature by comparison of the resistance of a winding at the temperature to be determined with the resistance at a known temperature
Embedded detector	This method consists in the determination of the temperature by thermocouples or resistance temperature detectors, built into the machine as specified in the section of the Standards dealing with the specific kind of machine

Three methods for temperature determination, as defined in Table XV, are used in commercial practice; namely, the *thermometer* method, the *resistance* method, and the *embedded-detector* method. The permissible temperature limits for the several kinds of insulating materials are based on laboratory tests and from experience gained in the operation of electrical machinery. It is evident that it would be very desirable to register the temperature of the *hottest spot* in a machine, but this can seldom be done in practice. The temperature observed at a very specific point may not be the highest in the machine under the given conditions. Hence a distinction is made in permissible temperature limits based on the *hottest spot*, or on the *observable* temperatures. The specified difference by which the observable

temperature is generally assumed to be lower than the hottest spot depends on the method in measuring the temperature, as follows:

Thermometer method	15°C.
Resistance method	10°C.
Embedded-device method	5°C.

In Table XVI are given the generally used *limiting observable temperatures* and the corresponding hottest-spot temperature for direct-current generators and motors (for railway motors see p. 397).

TABLE XVI.—LIMITING TEMPERATURES

	Class O material, degrees, Centigrade	Class A material, degrees, Centigrade	Class B material, degrees, Centigrade
Limiting hottest-spot temperature	90	105	125
Limiting observable temperatures			
(*a*) Thermometer method	75	90	110
(*b*) Resistance method	80	95	115
(*c*) Embedded-detector method	85	100	120

Rating tests on electrical machines are, however, generally based on observable *temperature rises* instead of the limiting observable temperatures. As the surrounding air seldom exceeds 40°C. the *limiting observable temperature rises* are in general obtained by subtracting 40°C. from the limiting observable temperatures given in Table XVI.

Rating.—The rating of a machine or apparatus is an arbitrary designation of an operating limit. In direct-current machines the rating specifies the output and such other characteristics as speed, voltage, and current as may be assigned to it by the manufacturer. The *rating of generators* is expressed in *kilowatts* available at the terminals of the machine at a specified speed and voltage. The *rating of motors* is expressed in kilowatts or *horsepower* available at the shaft at specified speed and voltage.

Several kinds of rating are recognized in commercial practice; namely, *continuous* rating, *short-time* rating, *nominal* rating, and ratings for *intermittent*, *periodic*, and *varying duty*.

Continuous rating specifies the load the machine can carry continuously without causing temperatures or other established limitations to be exceeded.

Short-time rating defines the load that can be carried for the time specified in the rating, the machine starting cold, without causing the temperature and other specified limitations to be exceeded. Standard periods for short-time rating are 5, 10, 15, 30, 60, and 120 min.

The nominal rating of a generator specifies the constant load which, having been carried without causing further measurable increase in temperature rise, may be increased 50 per cent for 2 hours without causing the established limitations for nominally rated machines to be exceeded.

The rating for *intermittent*, *periodic*, and *varying* duty may be continuous, short-time, or nominal, for which the thermal effects are as nearly as possible those of the actual service. For continuous and short-time rating, the temperature rise (thermometer method) in the various parts of the machine must not exceed the rating given in Table XVII.

TABLE XVII

Item	Type of enclosure	Limiting temperature rise, degree Centigrade		
		Class O insulation	Class A insulation	Class B insulation
1. Armature windings, wire field windings and all windings other than 2.........	All types except totally enclosed	35	50	70
	Totally enclosed	40	55	75
2. Single layer field windings with exposed uninsulated surfaces and bare copper windings	All types except totally enclosed	45	60	80
	Totally enclosed	45	60	80
3. Cores and mechanical parts in contact with or adjacent to insulation	All types except totally enclosed	35	50	70
	Totally enclosed	40	55	75
4. Commutators and collector rings	All types except totally enclosed	50	65	85
	Totally enclosed	50	65	85
5. Miscellaneous parts (such as brush holders, brushes, pole tips, etc.,) other than those whose temperatures affect the temperature of the insulating material may attain such temperatures as will not be injurious.				

The corresponding temperature rise limits (thermometer method) for nominal rating are shown in Table XVIII.

Table XVIII

Item	Limiting temperature rise	
	Class A insulation	Class B insulation
1. Armature windings, wire field windings and all windings other than 2	55	75
2. Single layer field windings with exposed uninsulated surfaces and bare copper windings	65	85
3. Cores and mechanical parts in contact with or adjacent to insulation	55	75
4. Commutators and collector rings	65	85
5. Miscellaneous parts (such as brush-holders, brushes, pole tips, etc.,) other than those whose temperatures affect the temperature of the insulating material may attain such temperatures as will not be injurious.		

It is generally assumed that the temperature rise is the same for all cooling air temperatures between the limits of 10 and 40°C. Specifications for the location of thermometers and other details for the application of the above rules for testing or determining the rating of direct-current machines may to best advantage be obtained from the A.I.E.E. or other published standards referred to on the first page of this chapter.

Rating of Railway Motors.—In the A.I.E.E. standards a separate section is devoted to railway motors, as the service requirements differ from those of the stationary power motor. The basis for the rating of railway motors is essentially the same as for stationary motors, but due to the intermittent nature of the load, the limiting temperature rise is somewhat greater, as shown in Table XIX. The ratings recognized for railway motors are:

(*a*) One-hour rating;
(*b*) Continuous rating of ventilated motors;
(*c*) Continuous rating of enclosed motors.

A railway motor is totally enclosed if the covering prevents circulation of air between the inside and the outside of the core, but not sufficiently to be termed air tight. In a ventilated motor the external cooling air is circulated through the machine.

TABLE XIX.—LIMITING TEMPERATURE RISE FOR RAILWAY MOTORS

Item	Type of enclosure	Method of temperature determination to be employed	Limiting temperature rise degrees Centigrade			
			One-hour rating		Continuous rating	
			Class A insulation	Class B insulation	Class A insulation	Class B insulation
1. Armature and field winding	Ventilated	Resistance	100	120	85	105
		Thermometer	80	95	65	80
	Totally[1] enclosed	Resistance	110	130	95	115
		Thermometer	90	105	75	90
2. Cores and mechanical parts in contact with or adjacent to insulation	Ventilated	Thermometer	80	95	65	80
	Totally[1] enclosed	Thermometer	90	105	75	90
3. Commutators..........	Ventilated	Thermometer	95	110	80	95
	Totally[1] enclosed	Thermometer	105	120	90	105
4. Miscellaneous parts (such as brush holders, brushes, pole tips, etc.) other than those whose location is such that they may injuriously affect the adjacent insulation may attain such temperatures as will not be injurious in any other respect.						

[1] The temperature rises of totally enclosed motors are taken as 10°C. higher than the ventilated motors since the cooling on stand test will be inferior to that obtained in service.

The *one-hour rating* of a railway motor is defined as the output measured in horsepower which the motor can carry for 1 hr. on stand test, starting cold, at its rated voltage without exceeding the temperature limits in Table XIX.

The *continuous rating of ventilated motors* is the output at the motor shaft measured in horsepower or kilowatts which the motor can carry for an unlimited period on stand test at its rated voltage, with the ventilating system as in service, without exceeding the temperature limits given in Table XIX. Direct-current motors may also be given a continuous rating in amperes at full-, three-fourths, and half-rated voltage. The *continuous rating of totally inclosed motors* is given in amperes at three-fourths and half-rated voltage.

Commercial machinery is subjected to other tests or limiting conditions, of which the *dielectric test*, the *insulation resistance*, and the regulation (Chap. XV) are the most important.

Dielectric Test.—The standard test voltage for all direct-current machines, except as otherwise specified, is on alternating voltage whose effective (virtual) value is twice the rated voltage of the machine, plus 1,000 volts. For railway motors the test

voltage is twice the rated voltage of the motor, plus 2,000 volts, using alternating currents of commercial frequency. The test voltage is applied continuously for a period of 60 sec. Machines for use on circuits of 25 volts or lower, as on low-voltage battery circuits, automobiles, etc., are tested with 500 volts.

Commercial dielectric tests should be made on the completely assembled machine, and not with individual parts; and necessarily the machine should be in good condition. High-voltage tests to determine whether or not the specifications are fulfilled are admissible on new machines only. For a description of the methods for measuring the voltage for high-voltage tests see A.I.E.E. Standards, Sec. 4, "Standards for the Measurement of Voltage in Dielectric Tests."

Insulation Resistance.—The insulation resistance of electric machinery is of doubtful significance as compared with the dielectric strength. It is subject to wide variations with temperature, humidity, and cleanliness of the parts of the machine. When insulation resistance falls below prescribed values it can, in most cases of good design and where no defects exist, be brought up to the required standard by cleaning and drying the machine. The insulation resistance, therefore, may provide a useful indication as to whether or not the machine is in suitable condition for application of the dielectric test but should not be considered as an independent requirement.

The insulation resistance in meghoms of a machine at operating temperature should not be less than the quotient of the rated voltage divided by the rating in kilowatts plus 1,000. The test is made with all circuits of equal voltage above ground connected together by applying a direct current voltage of 500 volts.

For authoritative statements of rules and regulations governing the rating of direct-current machines, reference is made to Sec. 5 and 11 of the A.I.E.E. Standards.

PROBLEMS

1. A 100 h.p. 200-volt, 800-r.p.m. motor takes an armature current of 360 amp. at full load. The armature resistance (not including the brushes) is 0.036 ohms. If at no load this machine takes an armature current of 19.6 amp., what voltage should be impressed at no load (running-light test) in order to approximate full-load conditions?

Find the stray-power loss of this machine.

If the normal field current of this machine is 15.2 amp. find the efficiency when I_a = 100, 200, and 370 amp.

2. A 75-kw. 110-volt generator, when running light as a motor, takes an armature current of 15.7 amp. and a field current of 20 amp. The armature resistance is 0.021 ohms. (not including the brushes). What voltage should be impressed for the running-light test in order to approximate full-load conditions?

Find the stray-power loss.

Find efficiency when I_a = 200, 400, 600, 700 amp.

3. Two similar 10-kw. 220-volt generators are connected together both mechanically and electrically in order to determine their stray losses by the Kapp opposition method. The output of one of the machines acting as a generator is fed back into the armature of the other machine which is acting as a motor driving the generator, the machine acting as a motor receives additional current from the 220-volt supply mains. The amount of power drawn from the mains is that required to compensate for the losses in both machines. The two shunt-field windings obtain current from the same 220-volt supply mains by separate circuits. The generator-field current is 2.1 amp. and the motor-field current is 1.36 amp. When the generator is delivering its rated armature current of 47.6 amp. the line supplies 7.4 amp. to the armature of the motor. If the armature resistance is 0.222 in each machine, *including* the brushes, find the stray-power loss of each machine. What is the efficiency of each machine under the above conditions?

4. A 20-kw., 250-volt, long shunt, compound generator has a stray-power loss of 856 watts. The armature resistance is 0.094 ohms (not including the brushes), the shunt-field resistance 106 ohms, and the series-field resistance 0.036 ohms. Find the conventional efficiency, efficiency of conversion, and electrical efficiency when this generator delivers its rated full load at rated voltage?

5. A 35-h.p. 125-volt, short-shunt compound motor has a stray-power loss of 1,430 watts. The armature resistance is 0.052 ohms (not including the brushes), the shunt-field resistance 62 ohms and the series-field resistance 0.021 ohms. When the motor takes 230 amp. from 125-volt mains find the conventional efficiency, the efficiency of conversion and the mechanical efficiency.

6. If the generator having the efficiency curve plotted in Fig. 2 is used to supply the load in Fig. 3, find the all-day efficiency of the generator operating for 9 hr.

7. The armature and field resistance of a 500-volt generator were measured after the machine had been standing idle in a room at a temperature of 72°F. Using a voltage of 480 volts the field current is 5.2 amp. With 106 amp. passing through the armature, a voltage drop of 18.6 volts is noted between marked commutator segments. After running the machine for 2 hr. the above readings are repeated. 480 volts across the field produce 4.8 amp. field current. With 102 amp. passing through the armature, a voltage drop of 21.2 volts is noted between the same commutator segments. What is the temperature rise of both the field and armature windings in degrees, centigrade?

CHAPTER XIX

ELECTROLYTIC CONDUCTION. BATTERIES. ELECTROLYSIS

Chemical action in electrolytic cells or batteries is a very important source of electric energy. A cell consists of two plates of conducting material, one electrically positive and the other negative, surrounded by an electrolytic fluid in a container. The chemical action of the *electrolyte* on the two plates generates voltage which causes a current to flow if the two plates are connected by an external circuit. The word *battery* was originally applied to a group of cells coupled together but is sometimes used either for a single cell or for several cells connected together so as to form a unit group source of electromotive force. The projecting terminals of the plates are the poles or electrodes to which the external circuit is connected. The terminals are called "positive" (cathode) and "negative" (anode) and are marked + and − in such a manner that in the *external part of the circuit* the current flows from the positive to the negative electrode, while *inside the cell* the current flows from the anode to the cathode.

Inside the cell the current is transmitted by electrolytic conduction through the electrolyte between the two electrodes. In order that a solution shall be an electrolyte; that is, capable of electrolytic conduction, part of the molecules of the chemical compound in solution must dissociate into ions carrying positive and negative charges of electricity. Thus in an aqueous solution of copper sulfate some of the $CuSO_4$ molecules are dissociated into Cu and SO_4 ions; the Cu ion carrying a positive charge of electricity and the SO_4 ion a negative charge, quantitatively equal to the positive charge on the Cu ion.

The direction of the current flowing in the cell is stated to be the same as the direction of migration of the positive ion, or opposite to the movement of the negative ions. Thus in a Zn-copper-sulfate-Cu element, as in the Daniell cell, the direction of the current in the electrolyte is from the zinc to the copper.

The zinc electrode is called the *anode* and the copper the *cathode* on the basis of current flow inside the cell. On the basis of current flow in the outside circuit the copper electrode is the *positive terminal* and the zinc electrode the *negative terminal.*

When a current flows through the electrolyte, either as a result of chemical reactions originating in the cell or because of voltage impressed on the electrodes from some outside source, the electricity is carried as charges on the migrating positive and negative ions, which move in opposite directions through the electrolyte. The chemical reactions take place at the surface of the two electrodes where the charged ions combine to form chemical compounds.

The basic quantitative relations between chemical reactions and electrolytic conduction of electric currents are expressed by two laws of electrolysis established by Faraday in 1833.

Faraday's first law states that *if nothing but the desired reaction occurs at the anode and at the cathode as the result of the passage of electricity, the quantities of material changed at the anode or cathode depend only on the quantity of electricity passing.* That is, these quantities depend only on the product of the current and the time. A current of I amp. for t sec. will cause the same quantity of chemical change as I/n amp. for nt sec. where n may have any numerical value.

Faraday's second law states that *the quantity of gas set free or metal deposited is proportional to the equivalent weight of the gas or metal, and that* 96,540 *coulombs deposit or set free* 1 *g. equivalent of the metal or gas.*

The 96,540 *coulombs per gram equivalent* is the fundamental unit for all electrolytic processes and is called *one faraday.* The symbol is $+F$ for positive monovalent ions and $-F$ for negative monovalent ions. A bivalent ion has $2F$ charges per gram molecule.

The *equivalent weight* or *gram equivalent* is defined as the quotient of the atomic weight of the given material divided by the valency. Silver is monovalent and has an atomic weight of 107.93. Hence 96,540 coulombs, or 1 *faraday*, would deposit 107.93 g. of silver. Copper is bivalent and its atomic weight is 63.6. Hence equivalent weight of copper $= \frac{63.6}{2} = 31.8$ g. That is, 96,540 coulombs, or 1 faraday, would deposit 31.8 g. of copper.

The *electrochemical equivalent* of any element is defined as the atomic weight divided by the product of 96,540 and its valency. Therefore, the electrochemical equivalent of copper = $\frac{63.6}{2 \cdot 96{,}540} = 0.000329$ g. That is, 1 amp. for 1 sec. will deposit 0.000329 g. of copper, or 1 amp.-hr. will deposit 1.186 g. of copper. The number 96,540 (1 *faraday*), representing the coulombs required per monovalent gram-ion, is based on the atomic weight of silver (0.001118 g. per coulomb) adopted as standard by the International Electrical Congress in 1893 as part of the definition of the ampere.

$\frac{107.93}{0.001118} = 96{,}538.5$ coulombs for the monovalent gram-ion of silver, which is very nearly 96,540, the figure generally used. From Faraday's second law, one amp.-hr. of electricity will deposit 4.025 g. of silver.

The *e.m.f. of a cell* is the total voltage generated, that is, the open-circuit voltage between the terminals. If the external circuit is closed and a current flowing, the terminal voltage is less than the e.m.f. of the cell, due to voltage drop inside the cell. The voltage loss in the cell is caused by the internal resistance drop and by the effects of polarization.

Polarization in a cell relates to secondary reactions largely caused by changes in concentration of the electrolyte and by the accumulation of gases on the surface of the plates. Thus, a current flowing in a simple *zinc-dilute sulfuric acid-copper* cell will cause hydrogen to form on the copper electrode until the surface is covered by a layer of hydrogen gas. This in effect changes the copper electrode to a hydrogen electrode and causes a reduction of the terminal voltage. In order that the original e.m.f. of the cell may be continuously available the polarization process is neutralized by the introduction of a *depolizer* in the cell. In commercial practice depolarizers are of two general types:

(*a*) A salt of the cathode metal is placed in the electrolyte so that when the current flows the positive ions in solution are deposited on the cathode (positive electrode), thus merely adding metal of the same kind to the cathode or positive electrode. In the *Daniell* and *Gravity* cells, copper sulphate surrounds the cathode or positive copper plate. The current flowing in the cell deposits copper on the copper plate and thus polarization is prevented.

(*b*) The cathode (positive electrode) is surrounded by a compound rich in oxygen which is capable of furnishing negative ions to the solution as its metal constituents possess different valencies. In the Leclanché cell and in dry cells, manganese dioxide surrounds the positive carbon electrode, the hydrogen combining with the oxygen to form water, while the manganese dioxide (MnO_2) reduces to manganese peroxide (Mn_2O_3).

The *internal resistance* relates largely to the electrolyte as the resistance of the electrodes themselves is very small. It varies with the temperature and solution density and other factors,

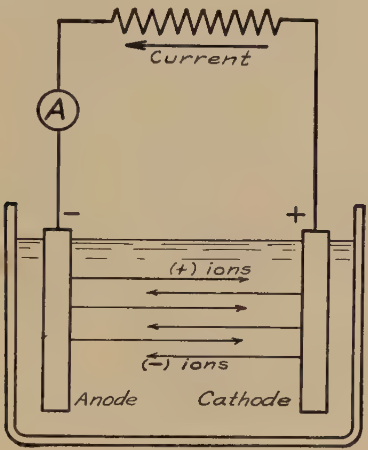


FIG. 1.—Battery circuit.

but follows the general law of resistance in electrical conductors: that is, *varies directly as the length of path* and *inversely as its cross-section.* Hence, to keep the internal resistance small, the distance between the plates should be short and the cross-sectional area large.

The voltage relations and the application of Ohm's law to the battery circuit in Fig. 1 is given by equation (1).

In Fig. 1 and equation (1) let:

E = e.m.f. of cell, total voltage generated or open-circuit voltage.
E_t = terminal voltage, circuit closed, current of I amp. flowing.
E_p = counter-voltage due to polarization.
R_n = internal resistance of circuit.
R_x = external resistance of circuit.

$$E - E_p - R_n I = E_t = R_x I \qquad (1)$$

For batteries in which the polarization is negligible ($E_p = 0$ in equation (1)) the internal resistance of the cell is expressed by equation (4).

$$E = (R_n + R_x)I \qquad (2)$$

$$E_t = E - R_n I = E \frac{R_x}{R_n + R_x} \qquad (3)$$

$$R_n = \frac{E}{I} - R_x = \left(\frac{E}{E_t} - 1\right) R_x \qquad (4)$$

Classification.—Batteries are grouped as primary and secondary on the basis of whether or not the chemical processes are reversible. Both groups are subdivided with respect to the chemical reactions involved and the materials used in their construction. A skeleton outline of a classification of primary batteries is shown in Table XX and a corresponding outline in Table XXI for secondary or storage batteries.

Primary Batteries.—In nearly all primary batteries the negative electrode or *anode* consists of a metallic zinc rod, plate, cylinder or such other form as may be desirable. Chemical

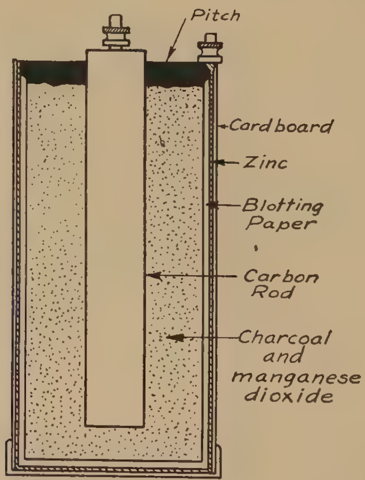


FIG. 2.—Cross-section of dry cell.

action changing the metallic zinc to zinc sulfate, zinc chloride, or other zinc compounds with a corresponding change in the composition of the electrolyte is the cause and source of the electric energy produced by the battery. The zinc is the fuel consumed in the process of generating electric energy. In practical oper-

ation the process is not reversible. When all the zinc has been changed from the metallic state into chemical compounds the generation of electric energy ceases and the battery is worn out. If the same container is to be used further the cell must be replenished with new chemicals.

TABLE XX.—PRIMARY BATTERIES

	Negative electrode anode	Electrolyte	Positive electrode cathode	Name of cell
I. Wet	1. Zinc......	zinc sulfate, copper sulfate solution $Zn + ZnSO_4 + CuSo_4 \rightarrow 2ZnSO_4 + Cu +$ electric energy	copper	Daniel, Gravity
	2. Zinc......	sulfuric and nitric acid solution $Zn + H_2SO_4 + 2HNO_3 \rightarrow ZnSO_4 + 2H_2O + 2NO_2$ electric energy	carbon	Bunsen, Grove
	3. Zinc......	chromic acid or potassium or sodium bichromite with sulfuric acid solution $3Zn + 2CrO_3 + 6H_2SO_4 \rightarrow Cr_2(SO_4)_3 + 3ZnSO_4 + 6H_2O +$ electric energy	carbon	Grenét, Fuller
	4. Zinc......	salammoniac solution and manganese dioxide $Zn + 2NH_4Cl + 2MnO_2 \rightarrow ZnCl_2 + 2NH_3 + H_2O + Mn_2O_3 +$ electric energy	carbon	Leclanché
	5. Zinc......	caustic soda or caustic potash solution $Zn + 2NaOH + CuO \rightarrow NaZnO_2 + H_2O + Cu +$ electric energy $Zn + 2KOH + CuO \rightarrow KZnO_2 + H_2O + Cu +$ electric energy	copper oxide copper	Edison, Lalande
		Standard cells		
	6. Zinc......	mercuric sulfate $Zn + Hg_2SO_4 \rightarrow ZnSO_4 + 2Hg +$ electric energy	mercury	Clark
	7. Cadmium.	mercuric sulfate $Ca + Hg_2SO_4 \rightarrow CaSO_4 + 2Hg +$ electric energy	mercury	Weston
II. Dry	1. Zinc......	salammoniac solution and manganese dioxide $Zn + 2NH_4Cl + 2MnO_2 \rightarrow ZnCl_2 + 2NH_3 + H_2O + Mn_2O_3 +$ electric energy	carbon	Practically all types

In dry cells the zinc electrode is usually in the form of a hollow cylinder and serves as the container for the electrolyte, depolarizer and positive electrode, usually of carbon. A paper lining or a layer of pulp or plaster of Paris on the inside of the zinc container is saturated with the electrolyte, an aqueous solution of sal ammoniac and zinc chloride. The lining also serves to keep the zinc from coming in contact with the manganese dioxide and crushed carbon which are packed around the inside or posi-

tive electrode. The sal ammoniac solution is held by capillary attraction in the paper lining and in the interstices of the porous carbon and the manganese dioxide, but serves the same purpose as the fluid electrolyte in the wet cells. The top of the cell is sealed with asphalt or some other material in order to prevent, or at least greatly retard, the evaporation of the water in the electrolyte. In some cases the electrolyte contains small quantities of gelatine, glycerine, zinc chloride, etc., as an aid in retarding evaporation or to reduce local action on open circuit.

Amalgamation of Zinc Electrodes.—Commercial zinc used for battery electrodes contains particles of carbon, iron, and other impurities which are electrically positive with respect to metallic zinc. The particles of carbon, iron, etc. that are on the surface of the positive electrode and therefore come in contact with the electrolyte form with the zinc miniature cells producing local currents that waste both the zinc and the active material of the electrolyte. As this local action is in progress continuously, it is evident that it must be reduced to a minimum, otherwise, the life of the battery will be greatly shortened. If the electrodes were made of chemically pure zinc, there would be no local action, but this would greatly increase the cost of the cell. Practically, the same chemical advantage is gained by coating the zinc electrode with a thin layer of mercury. The mercury dissolves some of the zinc, forming an amalgam, thus letting pure zinc ions come in contact with the electrolyte, while at the same time the mercury covers the particles of carbon, iron, etc. and thereby prevents local action.

Two simple processes of zinc amalgamation are in use:

(*a*) The zinc is first cleaned with dilute sulfuric acid and then the mercury is rubbed over the zinc surface with a cloth swab.

(*b*) A small quantity of mercury, about 3 per cent, is mixed with the molten zinc just before it is cast into rods or plates.

Rating of Primary Batteries.—Three forms of rating are used for primary cells, depending on the purpose for which the battery will be used. The *ampere-hour rating* is based on the amount of material available in a single charge. Zinc, which is generally used for the positive electrode, is bivalent and its atomic weight is 65.37. From Faraday's second law, the electro-chemical equivalent of zinc is therefore:

$$\frac{\text{atomic weight}}{\text{valency} \cdot 96{,}540} = \frac{65.4}{2 \cdot 96{,}540} = 0.000338 \text{ g.}$$

Hence, 1 amp.-hr. requires 1.219 g. of zinc, or 1 g. of zinc will produce 0.82 amp.-hr. of current.

Primary cells are sometimes *rated in terms of the current* which can be obtained for a short time when the cell is short circuited through an ammeter. In certain cases, especially when polarization enters as an important factor, the rating may also be expressed in terms of the current that can be maintained for considerable length of time.

Rating by service tests, depending on the type of service to be rendered, as for telephone work, gas-engine ignition, flash-lamp service, etc., are also used in practice. These tests are imperical and the detail specifications may be found in electrical engineering handbooks.

Efficiency of Primary Batteries.—The *efficiency of primary cells* may be stated either in terms of ampere-hours or in watt-hours. The *ampere-hour efficiency* is the ratio between the numbers of ampere-hours obtainable in the external circuit and the number theoretically available. The *watt-hour efficiency* is the ratio of the product of the obtainable ampere-hours and the terminal voltage to the product of the theoretically available ampere-hours multiplied by the total e.m.f. of the battery. The watt-hour efficiency may also be stated as the ratio of the electric energy in the external circuit to the expended chemical energy.

The internal losses are due to several factors:

(*a*) Local currents in or on the surface of the electrodes.

(*b*) Local currents due to inequalities in the electrolyte.

(*c*) Irreversible secondary actions, as the escape of gaseous products, etc.

(*d*) RI^2t heat losses in the cell.

The actual efficiency of primary cells is, in general, low but as only small amounts of energy are involved, the efficiency is usually not a controlling factor. Primary cells, and especially dry cells, are adapted for such purposes as require very small currents continuously or limited amounts of electric energy intermittently. The more common applications are gas-engine ignition, pocket-flash lamps, telephone work, and the operation of relay controls on a great variety of mechanisms.

Storage Batteries.—The basic difference between primary and secondary or storage batteries is that in the latter the electrochemical processes are reversible. When the storage battery

has delivered electric energy until exhausted it can be recharged by supplying electric energy from some outside source. Moreover, the process of charging and discharging can be repeated over and over again. That is, the secondary cell is merely a device or contrivance by which electric energy first can be transformed and stored as chemical energy, and, second, the stored chemical energy can later be automatically reconverted into electric energy.

The battery *discharges* when the chemically stored energy is delivered as electric energy to the outside circuit. *Charging* is the process of restoring the active materials in the battery by passing through it a direct current in the opposite direction to that of discharge.

The active materials of the battery are substances that react chemically to produce electric energy during discharge, and conversely store energy chemically when the battery is charged. The chemical action in the battery requires a large surface of the active material in contact with the electrolyte. In order to provide support for the finely divided active materials and paths of low resistance for the currents the electrodes are constructed on a frame structure called the *grid.* The *positive plate* consists of a grid and the active material from which the current flows to the external circuit when the battery is discharging. The *negative plate* consists of a grid and active material to which the current flows from the external circuit when the battery is discharging. The *polarity* of the battery is based on the electrical conditions that determine the direction of current flow. By common usage the current in the outside circuit during discharge of the lead battery is said to flow from the peroxide of lead in the positive plate and to the sponge lead in the negative plate. In the Edison battery the current, in the outside circuit, flows from the nickel peroxide in the positive plate and to the iron of the negative plate.

On the basis of the chemical reactions involved only two kinds of storage batteries are of marked commercial importance,

(*a*) The lead or acid battery.

(*b*) The Edison, nickel-iron or alkaline battery.

A third type, the Hubbell cell, is used to a limited extent in miner's lamps.

The active materials in the lead cell are lead peroxide, sulfuric-acid solution, and sponge lead. A large variety of lead cells carry-

ing different trade names are in commercial use. The differences lie in mechanical design of the grids and in the methods used in the manufacture of the plates. All types of lead-storage batteries are based on the same chemical process shown in Table XXI.

The active materials in the Edison cell are nickel oxide, caustic-potash solution, iron oxide, and iron. The Hubbell cell has the same electrolyte and positive electrode material as the Edison battery but cadmium is used in place of iron for the anode or negative electrode

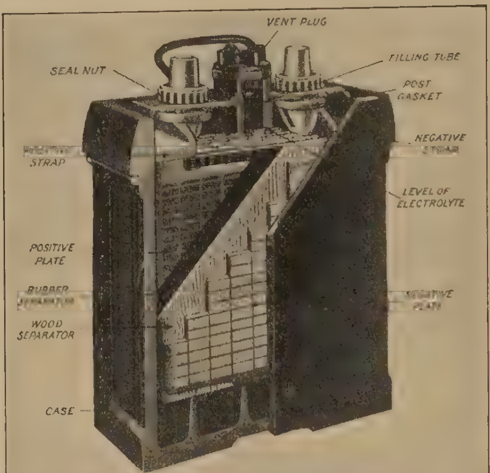
TABLE XXI.—SECONDARY OR STORAGE BATTERIES

	Negative electrode anode	Electrolyte	Positive electrode cathode
(*a*) Lead cell	Sponge lead Reaction at anode (negative plate) $Pb + H_2SO_4 \rightleftarrows PbSO_4 + 2H +$ electric energy Reaction at cathode (positive plate) $PbO_2 + H_2SO_4 \rightleftarrows PbSO_4 + H_2O + O +$ electric energy Combined reaction in cell $Pb + PbO_2 + 2H_2SO_4 \underset{\longleftarrow}{\overset{\text{discharge}}{\longrightarrow}} \text{charge } 2PbSO_4 + 2H_2O +$ electric energy From left to right gives reaction during discharge From right to left gives reaction when charging	caustic potash solution	lead peroxide
(*b*) Edison cell	Iron, an iron oxide $Fe + N_2O_31.2H_2O + 1.8H_2O \underset{\longleftarrow}{\overset{\text{discharge}}{\longrightarrow}} \text{charge } Fe(OH)_2 + 2Ni(OH)_2 +$ electric energy From left to right gives reaction during discharge From right to left gives reaction when charging	caustic potash	an oxide of nickel, nickel hydrate

On the basis of type of service rendered, storage batteries are conveniently divided into two groups—stationary batteries and portable batteries. Stationary batteries include load-regulating batteries, line batteries, peak-load batteries, oil-switch and exciter reserve batteries, stand-by batteries, telephone-exchange batteries and farm batteries. Portable batteries include starting and lighting batteries, vehicle batteries, car-lighting batteries, elec-

tric-locomotive batteries, mine-lamp batteries and ignition batteries.

The Lead Battery.—In the construction of lead storage batteries two general methods are used for forming the plates—the Planté and Fauré or pasted plate processes. In both cases the desired end is to secure as large a surface as possible of the active material and at the same time sufficient structural strength coupled with low resistance in the cell.



Sectional View of Battery, Showing Construction Details

Fig. 3.—Sectional view of portable lead storage battery. (*Electric Storage Battery Co.*)

In the Planté process the surface of the metallic lead is greatly increased by rolling or spinning and then by oxidizing the surface layer into a spongy mass by the passage of electricity and by application of nitric or acetic acid.

In the Fauré process the plate structure is cast in the form of lead grids. A small amount of antimony is added to the molten lead in order to increase the structural strength of the grid. A paste of red or gray oxide of lead is then pressed into the interstices of the grid. This paste material becomes firmly attached to the grid by subsequent action of electric currents.

Plates formed by either the Planté or Fauré processes are assembled into positive and negative groups, which properly spaced, are fused to connecting bars so that when assembled and placed in a container the positive and negative plates alter-

nate in position. Insulating separators of perforated hard-rubber strips or treated wood prevent the adjacent plates from coming in contact with each other.

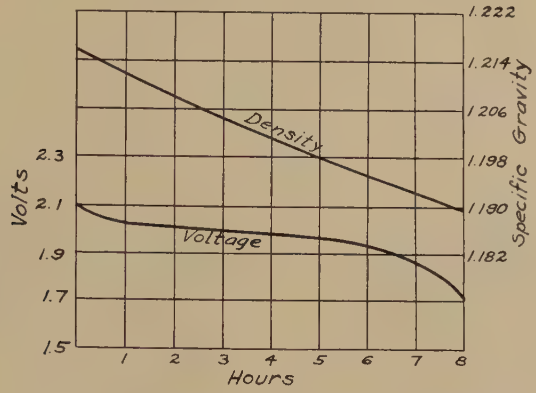


FIG. 4.—Specific gravity of solution. Charge curve.

The electrolyte of the lead storage battery is an aqueous solution of sulfuric acid, having a density or specific gravity of about 1.2 (Baumé) when first placed in the cell. The chemical

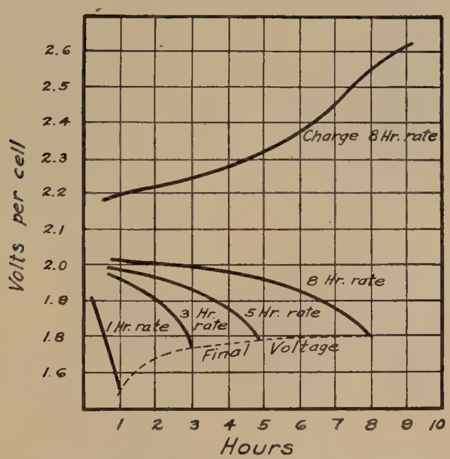


FIG. 5.—Charge and discharge curves of lead storage batteries.

reactions as shown in Table II modify the sulfuric-acid content in solution so that the specific gravity of the electrolyte is highest, 1.22, when the cell is fully charged and decreases to 1.176 when discharged.

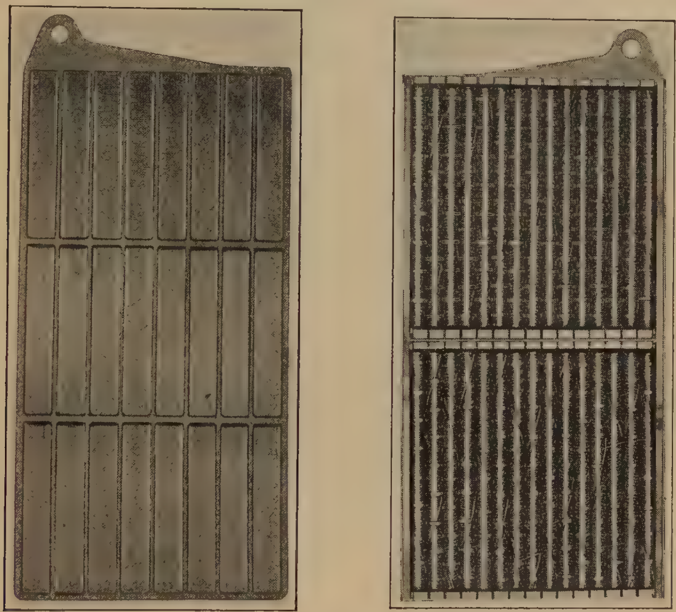
In the operation of storage batteries the charging and discharging rates must be considered. The *charging rate* is the current expressed in amperes at which the battery should be charged. Two methods are used for charging: (*a*) The *constant-current* charge in which the current is maintained at a constant value throughout; (*b*) The *constant-potential* charge in which the voltage at the terminals of the battery is held at a constant value. A *modified constant-potential system* is one in which a fixed resistance is inserted in the charging circuit in order to limit the initial current. Near the end of a charge the rate is reduced to a smaller value known as the *finishing rate* to prevent excessive gassing. In lead batteries it is sometimes necessary to complete the reduction of the lead sulphate by an *equalizing charge*. The equalizing charge is ordinarily at the finishing rate or less.

In order that lead batteries in wet storage shall have the least possible deterioration a very low charge, about 1 per cent of the finishing rate and known as the *trickle charge*, is applied continuously.

The Edison Battery.—The nickel-iron-alkaline cell was invented by Edison in 1901. The action of charging and discharging produces a transfer of oxygen between the plates. The negative plate or anode consists of iron and iron oxide, while the positive plate or cathods is of nickel, nickel oxide, and nickel hydrate. Structurally, the negative plate is a grid of rolled steel in which the many small rectangular perforated pockets are filled with finely powdered iron oxide (FeO). A number of the unit pockets are combined into groups forming a flat thin negative plate as shown in Fig. 6 (*a*). The grid and pockets are made of nickel plated steel.

The positive plate is formed from cylindrical tubes containing the active material. The tube is made from a thin, perforated, nickel-plated steel ribbon wound spirally into the form of a cylinder and then banded by nickel-plated steel rings. The tubes are filled with powdered nickel hydrate ($Ni(OH)$), which changes to an oxide of nickel during the forming treatment after the cell has been assembled. To reduce the internal resistance of the cell the nickel hydrate is alternated with layers of nickel flake when tempered into the tubes. A number of tubes are mounted on a nickel-plated steel frame or grid forming a flat rectangular plate, as shown in Fig. 6 (*b*).

The positive and negative plates are assembled into groups mechanically and electrically united by means of nickel-plated steel rods, nuts and washers. The groups are assembled into complete elements or cells by intermeshing the plates and placed in a nickel-plated steel container as shown in Fig. 7. The negative group always has one more plate than the positive, and hence both outside plates are negative. Insulation between the alternate positive and negative plates is provided by hard-rubber



(*a*) Negative plate. (*b*) Positive plate.

Fig. 6.—Edison cell. (*Edison Storage Battery Co.*)

pins. Rubber sheets insulate the outside negative plates from the sides of the container; and hard-rubber frames insulate the other sides and bottom of the container. Soft-rubber bushings and hard-rubber screwcaps, or gland caps, insulate the pole pieces where they project through the cover.

The electrolyte is a dilute solution (about 12 per cent) of potassium hydroxide (KOH). A small amount of lithium hydroxide increases the capacity and life of the cell. When new, the specific gravity of the electrolyte is 1.25. The specific gravity decreases slowly when the battery is in service. Since the electrolyte is merely a carrier of the electric current and does not

change in composition when the battery is charging or discharging, a comparatively small amount of liquid is required. The capacity of the nickel-iron-alkaline cell depends on the number

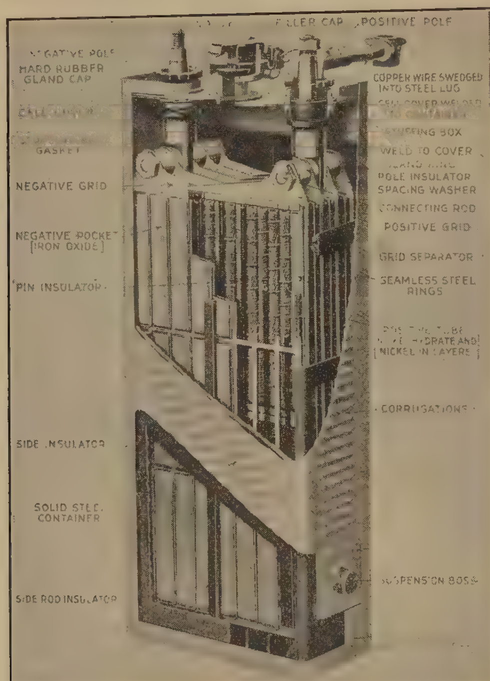


FIG. 7.—Sectional view of Edison cell. (*Edison Storage Battery Co.*)

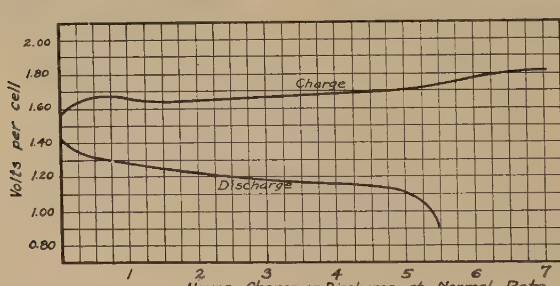


FIG. 8.—Edison battery. Charge, discharge, ampere-hour curves.

and type of its positive plates. A typical charge and discharge ampere-hour curve for Edison batteries is shown in Fig. 8. The average voltage of the Edison cell is 1.20 volts when discharged at normal rate to a low limit of 1.0 volt per cell.

Accidental conditions as short circuiting or charging in reverse direction or complete discharge do not injure the Edison battery. Vibration and jolting seldom cause injury. Prolonged overheating above 115°F. and prolonged operation with electrolyte level below plate tops will cause permanent injury.

Rating of Storage Batteries.—In discussing ratings and efficiencies of storage batteries the definitions of the technical terms used must be clearly kept in mind.

The *ampere-hour capacity* is the number of ampere-hours which can be delivered by a cell or battery under specified conditions as to temperature, rate of discharge, and voltage. The capacities of storage batteries for different classes of service are not directly comparable. The capacity depends on the discharge rate, reference temperature of the electrolyte, specific gravity of the electrolyte and the final voltage allowed. The *discharge rate* is the current expressed in amperes at which the battery shall be discharged. The *reference temperature* has been standardized at 25°C. The *specific gravity* is the ratio of the weight of a given volume of the liquid electrolyte to the weight of the same volume of water at a specified temperature. In storage-battery work the specific gravity is usually measured by a hydrometer. Upon reaching the prescribed *final voltage* the discharge is considered complete. The final voltage is usually chosen so that the useful capacity of the cell is realized.

The *rating* of storage batteries is based on the magnitude of the current that can be maintained continuously over a stated period of time, usually 5 or 8 h. Specifically the rating should state:

(*a*) Ampere-hour capacity.
(*b*) Discharge rate.
(*c*) Reference temperature of discharge, 25°C.
(*d*) Specific gravity of electrolyte at 25°C.
(*e*) Final voltage limit per cell on discharge.

In tests for rating storage batteries the ambient temperature, or room temperature, should be from 5 to 8°C. below the temperature of the electrolyte at the beginning of the discharge and kept constant throughout the discharge.

Efficiencies of Storage Batteries.—Due to the difference in service rendered three forms of efficiencies are used in connection with storage batteries: *ampere-hour efficiency*, *watt-hour efficiency* and *volt efficiency*.

Ampere-hour efficiency is defined as the ratio of the ampere-hours output of a cell or battery under specified conditions of temperature, rate of discharge and final voltage to the ampere-hours of input required for complete charge. The *watt-hour efficiency* is the ratio of the watt-hours of useful output to the watt-hours of input expended in charging. The so-called *volt efficiency* is the ratio of the average voltage of the cell or battery on discharging to the average voltage on charge.

Efficiency is the ratio of useful output to the required input. From an electrochemical standpoint the ampere-hour efficiency is desired, as this represents the ratio of the quantity of electricity delivered by the battery to the quantity required for a complete recharge, expressed in ampere-hours.

The *watt-hour efficiency* relates to the energy involved and represents the ratio of the energy delivered by the battery to the energy required in charging, expressed in watt-hours measured at the terminals of the battery. To obtain the watt-hour efficiency of the installation the measurements are made at the busses. The watt-hour efficiency of a battery is sometimes taken as the product of the ampere-hour efficiency and the volt-efficiency.

The watt-hour or energy efficiency of lead storage batteries is from 70 to 92 per cent, depending on load conditions. Under approximately similar conditions the efficiency of the Edison battery is approximately 60 per cent. Variation in temperature greatly affects the efficiency. Chemical action becomes more rapid as the temperature increases, while the resistance of the electrolyte decreases. For temperatures in excess of 38°C. the life of the battery is shortened. At low temperatures the ampere-hour capacity in both the lead and Edison batteries is greatly reduced.

Uses of Storage Batteries.—The uses of storage batteries are many and varied but may be grouped under the following general divisions:

(*a*) In direct-current power systems to insure continuity of operation, to provide regulation and to secure operating economy.

(*b*) For starting automobiles and for ignition purposes.

(*c*) To reduce load fluctuations in street-railway systems.

(*d*) In the lighting of railway trains.

(*e*) For propulsion of electric trucks, automobiles, and other vehicles.

(*f*) In radio receiving sets and broadcasting stations.

(*g*) In telephone centrals.

In direct-current power systems, particularly in connection with Edison three-wire distribution networks in metropolitan areas, storage batteries are used to insure continuity of service, to improve regulation and for carrying part of the peak load. For these purposes the storage batteries are "floating on the line" and located in substations near important load centers or in the central stations. For industrial plants where even momentary interruptions in the supply of power would cause serious

FIG. 9.—Central station standby storage battery. (*Electric Storage Battery Co.*)

damage, storage batteries are kept floating on the busbars. In small isolated power plants, as on farms or in villages, storage batteries are used to furnish the power required during hours of light load as it would be more expensive to operate generators continuously.

Most automobiles are equipped with storage batteries for starting and ignition purposes. Likewise a large number of radio sets use storage batteries as a source of plate supply and filament heating.

Electric trucks propelled by storage batteries are extensively used for short-haul purposes or when stops are frequent, as in factories, on railway stations, freight depots and docks. The

use of storage-battery locomotives in large mines and manufacturing plants or tenders for trains hauled by trolley locomotives is rapidly increasing. Storage batteries are indispensable in the navy, particularly in battleships and submarines. Storage batteries are used in train lighting, generally in connection with either axle-driven generators or with a prime-mover generator unit in the baggage car.

The Edison nickel-iron storage battery is more rugged and stands much more abuse than the lead cell but its energy efficiency is considerably lower. The nature of the service to be rendered determines whether the lead or the nickel-iron type of storage battery should be used. The Edison battery is adapted for trucks, tractors, locomotives, battery-propelled railway cars, railway-car lighting, mine lamps, railway signals, police and fire-alarm systems, etc. The types manufactured at present are not suitable for use with automobile starting motors which require as high as 20 times normal discharge current.

Electrolysis.—Electrolysis is a general term for the electro-separation or decomposition of chemical compound by electric currents in passing through electrolytic cells. The refining of copper and other metals, electroplating, electrotyping, and similar electrolytic processes are examples of electrolysis. An electrolytic cell consists of an anode through which the current enters the electrolyte which forms the conducting medium and a cathode by which the current leaves the cell. If the anode consists of a metal which can combine with the iron set free by the electrolytic process then the anode will be decomposed or corroded.

In grounded electric distribution systems only a part of the return current will flow through the metallic conductor provided for this purpose. The remaining part, that is, the *stray currents* will flow through the parallel conducting paths in the earth or adjacent pipes or mains of water, gas or heating systems. This is particularly the case in electric street railways, using the track rails as return conductors, the positive generator terminal being connected to the trolley and the track rails to the negative terminal of the generator. If all of the current after passing through the motors in the street car returned to the generator in the station through the rails no damage would be done. But these rails are in contact with the ground and hence a part of the returning current flows through the earth in paths more or

less parallel to the tracks and reenters the metallic rail circuit at points in the neighborhood of, or at, the power station.

The conductivity of the earth depends on the moisture content and the amount of salts left in solution. Moisture in the soil makes the earth a good electrolytic conductor. If iron pipes or mains of nearby gas or water systems are located more or less parallel to the street-railway tracks the stray currents are greatly increased as the currents will divide in direct proportion to the conductivity of the rails and the parallel paths through the earth to the power station.

In Fig. 10 let G represent the generator in the power station with the positive terminal connected to the trolley AB. At a distance ND from the station on the car tracks is a street car C. Parallel to the tracks is a water main MQ. The earth is moist

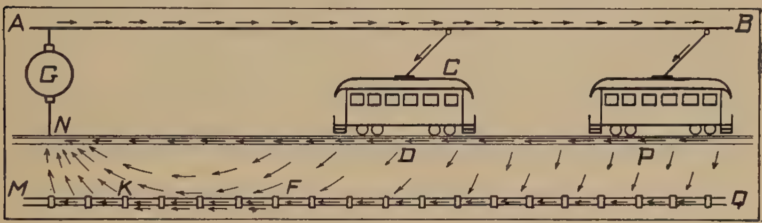


FIG. 10.—Illustrating electrolysis.

and forms a good electrolyte. Stray currents leave the soil from D to P and enter the water main over the distance FQ while the stray currents leave the pipe in the area marked KM and return to the generator at N. The corrosion in the water mains due to electrolysis occurs in the area KM where the current leaves the iron main, the anode of the electrolytic conduction. Corrosion also on the right side of each pipe junction, because the resistance through the joint is greater than in the pipe and hence part of the stray current in a pipe section shunts the joint by passing around through the soil and reentering the next section of pipe on the left side of the junction. The greatest corrosion occurs where the stray current leaving the water main is at its greatest density, generally near to the power station.

Underground structures or lead cable sheaths gas and water-service pipes may also be damaged and completely destroyed by electrolysis produced by stray currents passing from one metallic system to another, although the transfer point may be far away from return feeder connections. Stray currents may

cause corrosion of lead around cables if water in the conduits or the damp conduits themselves provide conducting paths from the lead cable sheaths. Because of the high electrochemical equivalent of lead and the thin walls of the lead sheath, lead-covered cables are more quickly destroyed by electrolysis than iron pipes.

The effects of electrolysis on cast-iron pipes differ to a marked degree from corresponding action on wrought-iron and steel pipes. In the first place the amount of the corrosion is generally much greater in the wrought-iron and steel pipes. This is due to the higher resistance of the cast-iron pipes of the same dimension as the wrought-iron or steel pipes as well as to the much higher resistance of the lead joints of the cast-iron pipe as compared to the bolted coupling joints of the wrought-iron or steel pipes. Because of the much higher resistance in the circuit the stray currents are greatly reduced in the cast-iron pipes with a corresponding reduction in the corrosion produced by electrolysis. Hence the cast-iron pipe is preferable—although the rate of corrosion by electrolysis for a given current passing from an iron pipe to the soil is practically the same for cast iron, wrought iron and steel. Moreover, the appearance of the corroded parts shows a marked difference. The outside appearance of a cast-iron pipe corroded by electrolysis gives little evidence of the changes that have taken place as the iron oxides formed are held in place by the graphite. The corroded material has the consistency of hard graphite but possesses very little mechanical strength. A physical examination with a test hammer is often necessary to determine the extent of damage done by electrolysis. For wrought-iron and steel pipes the effect produced by electrolysis is more apparent. The action is more localized and at points where the current leaves the pipe, pits are formed as the oxide of iron formed by electrolysis is carried away and becomes diffused in the soil.

If stray currents exist in a district having water or gas mains or iron or lead pipes for any purpose more or less imbedded in moist earth, electrolysis of the iron or lead will result. The only way to prevent electrolysis is to completely insulate the return circuit so that no stray current will be formed. This is gained by street-car systems by the use of the double trolley or with insulated third and fourth rails. The principal objection to the underground double-trolley system is the very high cost. The

overhead double-trolley system has a higher first cost compared to the single-trolley plan and also complications in the operation especially at crossings, on account of having two overhead trolley wires.

Many methods have been used with considerable success for minimizing electrolysis from street-railway stray currents. The more important are the *drainage method* and the *insulated-return-feeder system.*

The stray currents cause corrosion on leaving the metallic conductor to enter the moist soil in contact with the pipes or cables. The drainage method consists in providing metallic conductors in which the stray current returns to the rails or to the negative busbar in the generating station. Protection for lead sheaths or cables is obtained by bonding the cable sheath to the return circuit, or to the station negative busbars. The drainage method may, however, merely transfer the difficulty to some other part of the system. For the bonding necessarily reduces the resistance in the stray-current circuits and hence the stray currents may be greatly increased. Thus the corrosion at the pipe joints may be greatly increased and may more than offset the gain made in reducing the corrosion in the section the stray currents leave the pipe on returning to the power-station busbars.

By means of insulated return feeders the distribution of ground potential may be regulated so as to minimize the corrosion produced by stray currents. To determine the locations, connection points as well as size and number of insulated return feeders, extensive current and potential surveys of the district served must be made and the system of feeders so arranged that at no point will the voltage gradient exceed permissible values. In some systems negative boosters are connected to the negative return feeders in such a manner that the e.m.f. of the boosters is towards the current station, thus in effect compensating for the resistance drop in the return feeder. With properly bonded tracks and an adequate insulated return feeder system it is possible to reduce the stray currents in underground piping and cable systems to any desired value and to a large extent almost eliminate electrolysis by stray currents as a factor in the corrosion of underground pipes or cables.

In reinforced concrete, electrolysis may prove a serious menace. Stray electric currents flowing from the iron-reinforcing bar into

the moist concrete in which the bar is imbedded will cause oxidation of the iron by electrolysis. Since the oxides of iron thus formed occupy more space than the original iron, great stresses may be produced in the concrete that will crack or crumble the concrete. Extensive investigations on electrolysis have been made by the U. S. Bureau of Standards, the results of which have been published in a series of technologic papers and which also give comprehensive reviews of many phases of this important field of electrolytic conduction.

PROBLEMS

1. A current of 680 amp. flows for 24 hr. through a solution of copper sulphate in a plant for refining copper. Find the amount of copper deposited.

2. Zinc has an atomic weight of 65.37 and a valence of 2. If a cell maintains a current of 0.26 amp. for 3 months, what weight of zinc is consumed in the cell?

3. Four plating vats are connected in series. The plating materials are silver, copper, nickel, and zinc, respectively. The densities of these elements are, respectively, 10.6, 8.89, 8.80, and 7.19 as compared to water. Nickel has an atomic weight of 58.68 and a valence of 2. Find the thickness of plating produced in each vat if the four articles of the same shape being plated simultaneously by a current of 50 amp. for 4 hr. are each 3 sq. ft. in area.

4. Iron has an atomic weight of 55.85 and a valence of 2. An iron water pipe carries a stray trolley-system current of 10 amp. How long will it take to remove by electrolysis 10 lb. of pipe material at the point where the current leaves the pipe?

5. A 10-volt battery has an internal resistance of 0.06 ohms. What is the maximum power that may be obtained in an external circuit from this battery? When delivering maximum power what is the efficiency of the battery?

6. A battery in a fully charged condition discharges at a 75-amp. rate for 7.6 hr. at an average terminal voltage of 112 volts. The battery is then charged up to its original condition by a current of 87 amp. flowing for 7.2 hr. at an average voltage of 128 volts. Find the ampere-hour and watt-hour efficiency of the battery under the above cycle of operation.

7. A 60-cell battery in a fully charged condition is fully discharged by a current of 93 amp. flowing for 8 hr. The battery is then charged to the original condition by a current of 100 amp. flowing for 8 hr. and 20 min. If the voltage per cell varies both during charge and discharge, as indicated in the curves of Fig. 5, determine the ampere-hour and watt-hour efficiency over the above cycle.

CHAPTER XX

TRANSMISSION AND DISTRIBUTION

In the design, construction, and operation of electric-power systems three general divisions are recognized: *generation*, *transmission*, and *distribution;* each of which has many well-defined subdivisions.

Generation relates to the machinery and appliances in, or directly connected with, the power plant or generating station. *Transmission*, in the restricted technical sense, pertains to comparatively long-transmission lines, usually carrying current at high voltage, between the generating station and substations and includes protective and control apparatus directly related to the high-tension lines. *Distribution* covers the low-tension network, with accessory appliances, delivering power to individual consumers. In large systems the distribution division begins at the substations and ends at the terminals to which the machines, lamps or appliances, that consume the power, are attached. In systems and for parts of systems in which the distance from the generating station to the load is comparatively short the transmission division proper is omitted and the distribution network begins directly at the power plant.

For each of the three divisions, specific but different conditions must be met that impose limitations in the design and operation of the system. Necessarily the basic requirements in the design of any power system are those of safety and economy. The aim and purpose of the engineer is to deliver to the customer the required power at the lowest possible cost. To meet these fundamental requirements very many factors must be taken into consideration. Some of these factors apply only to special cases or are of special importance in a particular power system. Most of the factors, however, apply to all cases.

Through the efforts of many industrial organizations, government agencies and individuals, rules and regulations relating to *safety requirements* that experience and extended investigations have proved necessary and desirable in electric power transmission and distribution systems have been compiled into what

are known as the *National Electrical Code,* sometime referred to as the *Underwriters' Rules,* and the *National Electrical Safety Code.* The original National Electrical Code was drawn in 1897. It is revised every 2 years in order that it may be kept in accord with current developments and advances in the electrical art. The code relates particularly to the distribution division and has proved of great economic importance in relation to fire hazard and fire insurance. On the basis of safety the permissible terminal voltages of the distribution network have been standardized. Thus the terminal voltage for incandescent lamps and household appliances is from 110 to 120 volts. Other standard distribution voltages for supplying power to motors of various sizes and under specified safety restrictions are 220, 440, and 550 volts. In direct-current electric railways 600, 750, and 1,500 volts at the motor brushes are generally used. Insulation, protective devices and construction regulations are to a large extent standardized and detail requirements specified in the National Electrical Code. The National Electrical Safety Code, of more recent origin, does the same service to industry with regard to life hazard as is done by the National Electrical Code in regard to fire hazard. With the very general use of electricity in all industrial fields, in homes, in transportation and communication it is evident that well-defined standards guarding the fire and life hazards are essential.

Direct-Current Power Transmission.—Of the many factors that enter into the problem of securing the most economical transmission and distribution of electric power, the RI^2 losses in the conductors are of first importance. The loss of power when transmitting electric energy over metallic conductors, usually copper wires or cables, varies directly as the product of the resistance and the square of the current.

$$\text{Line loss, } P \alpha RI^2 \tag{1}$$

The total power delivered to the load is proportional to the product of the current and the load voltage. Hence for any given amount of power delivered the current would vary inversely as the voltage. Therefore, the losses as given in equation (1) may, for any given amount of power transmitted, also be expressed as proportional to the quotient of the line resistance R and the square of the load voltage E as in equation (2)

$$P \alpha \frac{R}{E^2} \tag{2}$$

In Chap. II it was shown that the resistance varies as the product of the resistivity ρ at the given temperature, and the distance or length of conductor l divided by the conductor cross-sectional area A.

$$R = \rho\frac{l}{A} \tag{3}$$

Since the conductors are generally wires of uniform cross-section, the weight of copper ${}_cM$ varies as the product of the distance of transmission, and the cross-section of the conductors.

$$\text{Weight of copper } {}_cM \propto lA \tag{4}$$

Therefore from equations (2), (3), and (4) considering the resistivity ρ constant, the power loss varies for a given amount of power delivered, as shown in equation (5).

$$P \propto \frac{l^2}{{}_cME^2} \tag{5}$$

For a given distance of transmission l and for a given quantity of power delivered to the load, the relation of power loss, weight of copper, and load voltage is given by equation (6).

$$P \propto \frac{1}{{}_cME^2} \text{ or } {}_cM \propto \frac{1}{PE^2} \tag{6}$$

That is when transmitting power over any given distance:

(*a*) For a given weight of copper, *initial investment*, the power loss in transmission and distribution varies inversely as the square of the voltage;

(*b*) For a given power loss, *operating expense*, the weight of the copper required varies inversely as the square of the voltage. In either case, therefore, it is desirable, from the economic point of view, to use the highest permissible voltages in the several parts or sections of electric transmission and distribution systems.

The relations expressed by equation (6) are of far-reaching importance. The permissible voltage for the several divisions in the power transmission system varies widely. Comparatively low voltage at the generator terminals, high tension on the long transmission line, moderately high pressure on the main sections of the distribution network, with low voltage at the load terminals must be provided. Hence, for economical generation, transmission and retail delivery of electric energy it is essential that the voltage be raised at the generator terminals, kept high on the transmission line, lowered to moderate values at the substations for the primary distribution system and again

lowered to safe terminal voltages for the service wires connected to the customers' lamps, motors, and other electric appliances. This raising and lowering of the voltage is accomplished automatically in a highly satisfactory and economical manner by means of transformers when using alternating currents. The transformer is wellnigh ideal for this purpose. It has low first cost, no moving parts, requires little space and care, operates at high efficiency and can be built so as to give practically any desired voltage ratio. For direct currents similar changes in voltage require rotating machinery, which has much higher first cost, requires more attention and has lower operating efficiency than the transformer. Moreover, range of voltages is much smaller while the space and operating expenses are greater. The superiority of the transformer over the rotating machinery for raising or lowering the voltage is so pronounced that almost all transmission of electric energy is accomplished by using alternating currents. Direct currents are not used for long-distance transmission except on a few power lines in Southern Europe of comparatively small capacity. Direct current is used for the transmission of power over short distances, that is, for the *distribution* of electric power if conditions imposed by the nature of the service make the installation as a whole desirable and economical, although the line losses are greater than would be required for transmitting an equal amount of power by alternating currents.

Thus, it is found advantageous to use direct currents for electric railways, high-speed elevators, variable-speed motors, arc welding, electrolytic processes, arc lamps, etc. and in combination with large storage batteries for insuring greater reliability of power supply in metropolitan areas. See Chap. XXII.

The Thury System.—Outside of a few installations using the *Thury system* in Europe, direct currents are not used for long distance transmission of electric energy. The Thury system of power transmission has been in successful operation in several plants in Italy and France. In principle this system is similar to the series lighting systems. The basic plan of the Thury system is to provide a constant-current power supply, letting the voltage vary with changes in the load. The generators as well as the motors are connected in series as indicated in Fig. 1.

Each generator is mounted on an insulated platform and connected to its prime mover by an insulated coupling. The current

is maintained constant in value by automatic devices which control the prime-mover speed, shift the brushes, and shunt the field. Large adjustments are provided by starting up or shutting down of generators. Generators are shut down or taken out of service by short circuiting the fields. The motors receiving the power are in construction similar to the generators. Motors are regulated to give constant speed by centrifugal governors which move the brushes and operate the field rheostats. The Thury system will not serve for general distribution of power but merely for

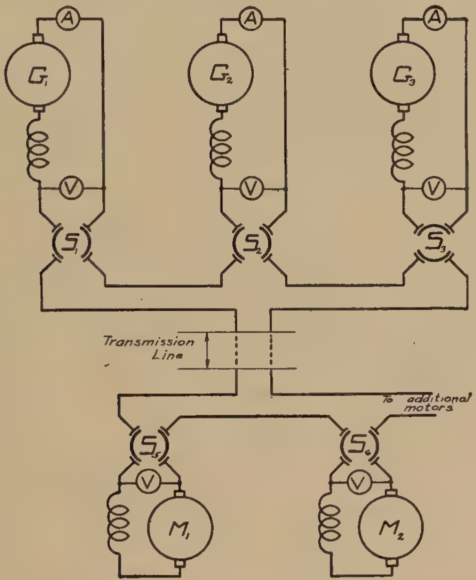


FIG. 1.—Circuit diagram. Thury system of power transmission.

power transmission from generating station to substations. In the substation the energy is transformed by means of motor-generator sets and then delivered to the customer over constant-potential distribution systems.

In comparison to constant-potential long-distance transmission, the more important advantages claimed for the Thury system are:

(*a*) Unity power factor at all loads,
(*b*) Higher effective voltages for the same insulation,
(*c*) Negligibly small dielectric losses,
(*d*) Two wires only to be insulated,

(*e*) A number of stations can be operated in series and a station can be connected to the line at any point,

(*f*) Switching arrangements very simple.

(*g*) Desirable in connection with variable load hydraulic stations,

(*h*) Adapted for industrial loads requiring constant torque.

The following disadvantages of the Thury System should be noted:

(*a*) Heavy insulation between generators and motors to ground. Insulated floors and insulating couplings,

(*b*) Generator units of comparatively small capacity,

(*c*) Line loss constant at all loads,

(*d*) Less effective for constant head hydraulic stations,

(*e*) Lack of overload torque characteristics in motors.

Since the generators and motors are all connected in series, the voltage between the machines and the ground becomes very high, although the voltage between the windings and the frame in any one of the machines does not exceed 5,000 volts. Therefore each machine must be insulated from the ground for a much higher voltage than the windings from the frame of the machine. Necessarily each machine must in all cases be connected through an insulating coupling to a prime mover for the generator or the load for the motors.

In America the constant-potential multiple system has become so firmly established and offers so many advantages that it is not likely that in the future any constant-current series system will be used for long-distance transmission of power. All types of constant-current generators that were generally used for supplying power for series arc-lamp lighting circuits, twenty or more years ago, are now obsolete. Magnetite arc lamps, connected in series, still in use for street lighting, are in most cases provided with constant direct current by mercury arc rectifiers. The distances involved, however, are comparatively short and the service rendered brings these circuits properly into the distribution division.

Direct-current Distribution Systems.—The primary division in classifying distribution systems, both for direct and alternating currents, is based on the method of making circuit connections. The load supplied by an electric circuit generally consists of several parts or units as a number of lamps or motors

or electric appliances. These load units can be connected either in series or in multiple. Hence the basic divisions are:

(*a*) Series circuits in which the same current flows through all the load units. Therefore series circuits require constant-current supply so that for changes in load the voltage is varied. Series circuits are sometimes called *constant-current systems*.

(*b*) Multiple or parallel circuits in which the several load units are connected in parallel, each taking a part of the total current. Multiple systems properly operate on constant voltage power supply, while the current varies with the load. Parallel circuits are generally referred to as *constant-potential systems*.

(*c*) Combinations of series and parallel circuits. In the multiple or constant potential system a few of the load units are in some cases connected in series, while the rest are connected in parallel between the mains, in which constant voltage is maintained.

Direct-current Series Circuits; Constant-current Systems.— The use of series distribution systems is limited by safety requirements, as the voltage to ground increases with the number of units connected in series. Hence series circuits cannot be used in residences or where people can come in contact with the wires or the appliances. In street lighting the series circuit can be used to advantage for three reasons:

(*a*) The conductors can be placed out of reach on poles, thus making high voltage to ground permissible.

(*b*) The several load circuit units, lamps, require the same amount of current and hence adapted to a constant-current supply.

(*c*) The low-voltage rating of each lamp, the long distance between the lamps and the extent of the area to be served would make a constant-potential distribution system very expensive as the amount of copper required would be very much greater than for the series circuit.

Probably the most extensive use of series direct-current circuits for the distribution of power is in connection with the magnetite arc-lamps for street lighting. These circuits carry 6.6 amp. and each lamp requires 70 volts for normal operation. The number of lamps in a single circuit varies greatly, depending on the area to be lighted. If a circuit has 80 lamps the constant current at 6.6 amp. passes through all the lamps and the total voltage of the circuit would be 5,600 volts. Any change in the

number of lamps in circuit will require a corresponding change in the total voltage without change in the strength of the current. In the case that the filament of any single lamp burns out the current is interrupted and all of the lights go out momentarily. In the base of each lamp are two fusible metal surfaces attached to the ends of the filament and insulated from each other by a thin piece of insulating material. If the filament fails the current of the whole system becomes zero and the total line voltage will be concentrated at this particular lamp. The resistance between the discs is punctured and an arc is formed between the fusible discs in the lamp, causing them to melt together and thereby close the circuit. The voltage required for the lighting circuit will be slightly reduced by an amount equal to the voltage

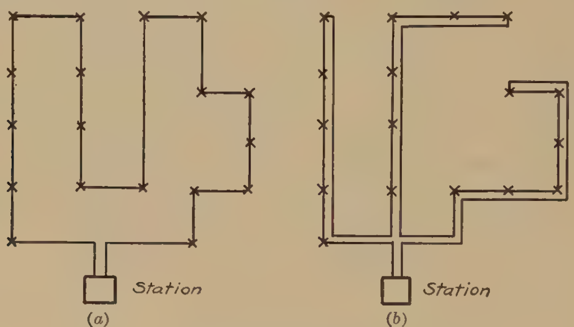


FIG. 2.—Arc-lamp street-lighting circuits.

required by the one lamp, but this is automatically taken care of by the regulating device in the power supply, which is so adjusted as to maintain constant current instead of constant voltage.

There are two standard methods for arranging the wires in street-lighting circuits as shown in Fig. 2 and known as the open-loop and the parallel-loop systems. In the open-loop system the shortest route is sought without reference to the separation of the outgoing and returning sections of the conductor. This system requires a minimum of copper. In the parallel-loop system the outgoing and return parts of the conductor are kept near to each other. More copper is required, but inductive disturbances are greatly reduced and testing for faults is facilitated. Other important applications of series circuits are found in telegraphs, fire-alarm systems, train lighting and the windings of the field poles of generators and motors.

Direct-current Multiple Circuits or Constant-potential Systems.—By far the greater part of direct-current power distribution, on the kilowatt-hour basis, consists of multiple circuits, that is, over constant-potential systems. The metropolitan areas of large cities are in most cases serviced both by direct- and alternating-current networks, in order to insure continuity of service. Direct-current, constant-potential systems having large storage batteries floating on the mains are considered the most reliable for insuring continuity of service. Direct currents are also preferable for elevator service, variable-speed motors, and various other industrial loads found in metropolitan areas.

In the distribution of power by direct currents several factors must be given consideration, in addition to the limitations

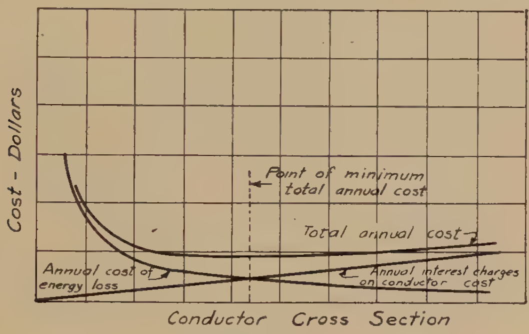


FIG. 3.—Graph illustrating Kelvin's law.

on the permissible voltage that may be used. The more important conditions that must be met are:

1. The wires must be large enough to carry the required current within safe heating limits so as to avoid fire hazards. The allowable carrying capacities of copper wires when used under ordinary conditions are given in Table XXII.

2. The wires must be of sufficient mechanical strength. For pole-line construction No. 8 A.W.G. is the smallest size of wire used.

3. The arrangement of the circuits and the distribution of copper must be such that the voltage drop will keep within specified limits for all changes in load. For incandescent lamps the permissible voltage variation for most economical operation is very small; within two volts above and two volts below the rated voltage. For motors and household appliances a wider

TABLE XXII.—COPPER CONDUCTORS

A.W.G.	Area circular mils	Diameter, mils		Per 1,000 ft.		Carrying capacity		
		Solid	Strand	Pounds bare	Ohms at 20°C.	Rubber covered	Other insulations	Underground cable
	2,000,000	......	1,631	6,200	0.0052	1,050	1,670	1,085
	1,500,000	......	1,412	4,631	0.0069	850	1,360	895
	1,000,000	......	1,152	5,070	0.0104	650	1,000	695
	900,000	......	1,093	2,762	0.0115	600	920	650
	800,000	......	1,031	2,453	0.0129	550	840	607
	700,000	......	464	2,145	0.0148	500	760	558
	600,000	......	893	1,837	0.0173	450	680	505
	500,000	......	815	1,530	0.0207	390	590	450
	400,000	......	728	1,223	0.0259	330	500	390
	300,000	......	630	917	0.0345	270	400	323
	250,000	......	575	764	0.0414	240	356	292
0000	211,600	460.00	528	640.5	0.0489	210	312	260
000	167,805	409.64	470	508	0.0617	177	262	225
00	133,079	364.80	418	402.8	0.0778	150	220	195
0	105,593	324.95	373	319.5	0.0981	127	185	168
1	83,690	289.30	328	253.3	0.1237	107	156	146
2	66,370	257.63	292	200.9	0.156	90	131	125
3	52,640	229.42	260	159.3	0.197	76	110	108
4	41,740	204.31	232	126.4	0.248	65	92	91
5	33,100	181.94	....	100.2	0.313	54	77	76
6	26,250	162.02	....	79.46	0.394	46	65	64
7	20,820	144.28	....	63.02	0.497	......	.....	53
8	16,510	128.49	....	49.98	0.627	33	46	45
9	13,090	114.43	....	39.63	0.791	......	.....	38
10	10,380	101.89	....	31.43	0.997	24	32	33
12	6,530	80.81	....	19.77	1.59	17	23	24
14	4,107	64.08	....	12.43	2.52	12	16	18
16	2,583	50.82	....	7.82	4.01	6	8	
18	1,624	40.30	....	4.92	6.37	3	5	
20	1,022	31.96	....	3.09	10.14			
22	642.5	25.4	....	1.95	16.12			
24	404.0	20.1	....	1.22	25.63			
26	254.1	15.9	....	0.769	40.75			
28	159.8	12.6	....	0.484	64.79			
30	100.5	10.0	....	0.304	103.0			
32	63.2	7.95	....	0.19	161.8			
34	39.8	6.31	....	0.120	260.5			
36	25.0	5.00	....	0.076	414.2			
38	15.7	3.97	....	0.048	658.5			
40	9.9	3.15	....	0.030	1,047.0			

range is permissible but in most cases the incandescent lamps become the controlling factor.

4. After the conditions in (1), (2), and (3) are met the design becomes a problem in economics. Two opposing factors must be

considered. The larger the conductor the greater the initial cost but the less electric energy will be dissipated as heat due to the RI^2 losses. For greatest economy the size of conductor used should be such that the interest on the investment plus the cost of the energy will be a minimum. This requirement will be met if the annual interest on the investment is equal to the annual cost of the electric energy transformed into heat by the RI^2 losses in the conductors. This statement is in essence Kelvin's law for economical transmission and distribution of electric energy. A graphical illustration of Kelvin's law is shown in Fig. 3.

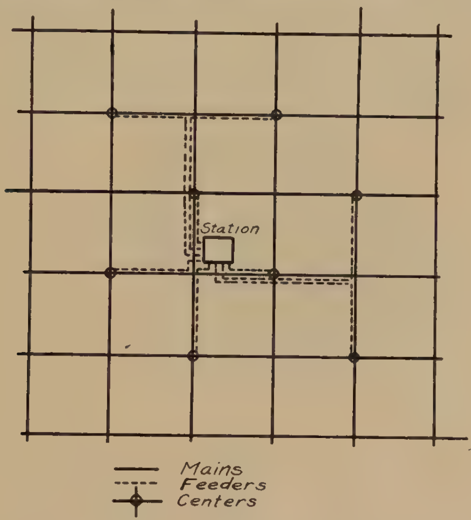


FIG. 4.—Feeders and mains in a direct-current distribution network.

Feeders and Mains.—In constant-potential direct-current systems the conductors delivering power to the service wires leading to the individual load units are called "the mains." In metropolitan areas the mains form an underground network as illustrated by the full lines in Fig. 4. This network receives energy at several points which are connected by means of separate pairs of heavier conductors, called "feeders," to the direct-current busbars in the central station or substation, as indicated by the broken lines in Fig. 4. The centers at which the feeders should be connected to the mains must be determined on the basis of load distribution so as to require a minimum of copper to keep the voltage drops at all points of the distribution network within prescribed limits. Pressure wires are brought back from

each feeder-main center to a station voltmeter in order to enable the station operator to adjust the station-feeder voltage so as to keep the potential of each center at its proper value. Three pilot or pressure wires are generally used in the Edison three-wire system, one connected to the neutral and one to each of the outer wires of the feeding center. The voltage of each feeder at the generating station can be varied independently of the other feeder and the voltage drop in a feeder can be large or small as the distribution of the load on the mains may require, as no loads are supplied at intermediate points. Good practice permits voltage drops as high as 10 per cent in feeders but not to exceed 3 per cent in the mains. The load units are distributed along the mains in a more or less uniform manner but a feeder merely connects the station busbars to one feeding center on the mains.

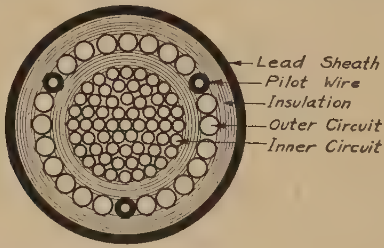


Fig. 5.—Cross-section of cable. Edison three-wire distribution system.

The service lines connect the mains to convenient central points of the customers' load, while the wiring on the customers' premises vary with the character and size of the load.

A number of wiring arrangements are shown in Figs. 5 and 6. In each case a single pair of conductors lead from the supply source. Which form to use depends on the load distribution. The plan selected should give as uniform voltage drop as possible under the given load requirements.

The uniform copper cross-section (Fig. 6 (a)) is applicable for light loads only. For heavier loads the varying cross-section (Fig. 6 (b)) permits a more advantageous use of the copper. Manifestly, if the load is uniformly distributed the conductors should be tapered or diminish in cross-section at each load point. In practice the tapered conductors are not available and to change conductor size at every load point would greatly increase the cost of installation. A change of cross-section in steps that conform with commercial wire sizes used in comparatively few

sections is found more economical. The antiparallel (Fig. 6 (*c*)) the open loop (Fig. 6 (*d*)) and the closed loop (Fig. 6 (*e*)) are advantageous under load distributions forming more or less definite groups.

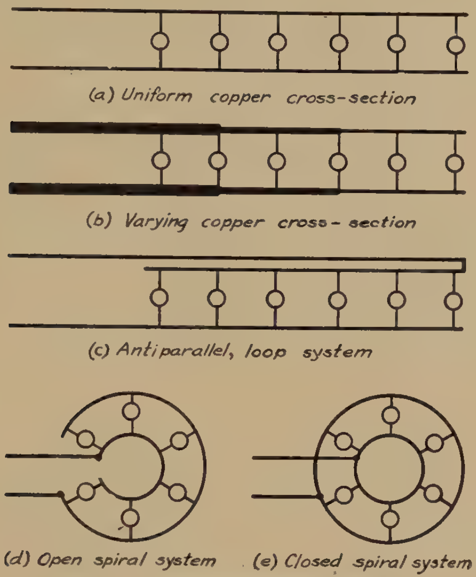


FIG. 6.—Distribution load circuits.

By connecting pairs of incandescent lamps in series as shown in Fig. 7 the voltage between the mains is doubled and hence only one-fourth as much copper would be required for the same voltage drop. This series-parallel arrangement is, however,

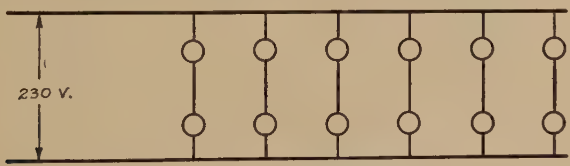


FIG. 7.—Two-wire, series-parallel system.

undesirable and seldom used as it brings double the voltages to the customers' premises and necessitates the turning off or on of two lights at a time.

The Edison Three-wire System.—All the advantages of doubling the voltage are found and at the same time all the

disadvantages of the series-parallel system in Fig. 7 are eliminated by the *Edison three-wire system,* illustrated in Fig. 8. The Edison three-wire system consists of two outside conductors between which pairs of lamps are connected in series, together with a thin wire connected to the midway points between the lamps in the several pairs. From the neutral to either of the

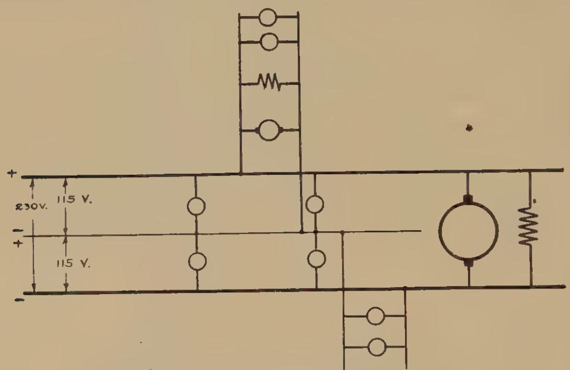


Fig. 8.—Edison three-wire distribution.

outside wires the voltage is approximately 115 volts, and therefore 230 volts between the two outside wires Incandescent lamps, motors, heaters, etc. (115 volts) are connected between the neutral and either outside wire while 230-volt motors and appliances are serviced by the two outside conductors. When

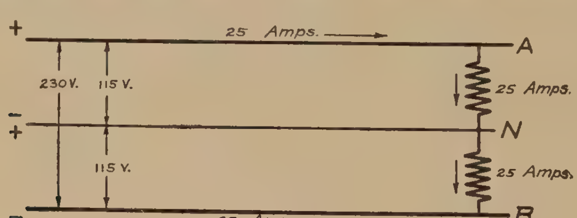


Fig. 9.—Edison three-wire balanced load.

the load is balanced as in Fig. 9 no current flows in the nuetral wire. From Fig. 9 it is evident that only one-fourth as much copper is required in the two outside mains for the same load as would be required in a two-wire 115-volt system. If it be assumed that the neutral wire is of the same cross-section as either of the two outside wires the copper required by the Edison three-wire distribution system will be $\frac{3}{8}$ or $37\frac{1}{2}$ per cent of that

of the corresponding two-wire 115-volt systems. Hence a saving of 62½ per cent of the copper is accomplished by using the Edison three-wire system in place of the two-wire system, although the terminal voltage at each incandescent lamp remains at 115 volts.

If the load is unbalanced as illustrated by Fig. 10 (*a*) and Fig. 10 (*b*) part of the current will flow in the neutral but the voltage at each lamp terminal will remain approximately at 115 volts.

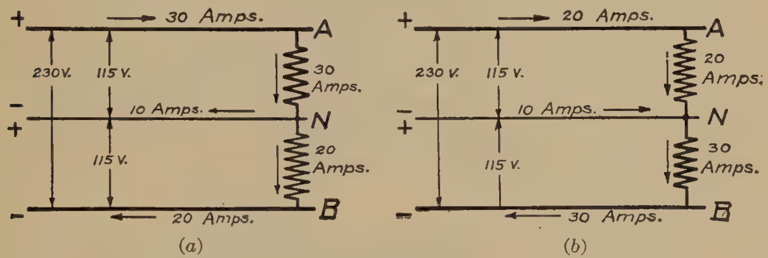


FIG. 10.—Edison three-wire systems, unbalanced loads.

The neutral carries the difference of the currents in the two outside wires. In commercial systems the load is connected so as to secure as nearly balanced load as possible. Hence the greater part of the energy required by the load is distributed at

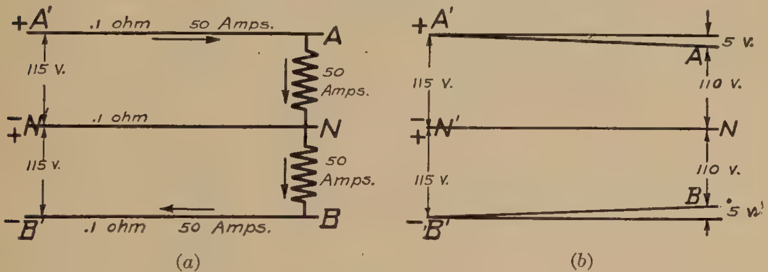


FIG. 11.—Edison three-wire system. Balanced terminal voltages due to balanced loads.

230 volts and only the minor unbalanced part is carried at the less economical lower voltage.

The neutral wire is generally grounded so that on the customers' premises the voltage from either of the outside wires to ground cannot exceed the voltage to neutral.

If the load in the two sides of the Edison three-wire system is balanced the voltage in the two sides will also be balanced as the

drop in the two outside wires necessarily must be equal and as no current flows in the neutral the voltage to neutral must be the same for both sides. This is shown graphically in Fig. 11.

If the loads on the two sides are unbalanced the terminal voltages will also be unbalanced, as shown in Fig. 12. Variations in voltage at load terminals is undesirable and the range must

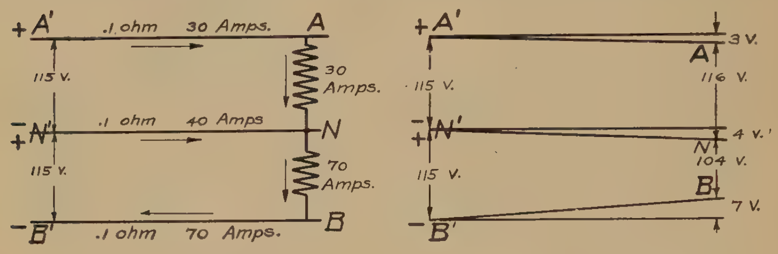


FIG. 12.—Edison three-wire system. Unbalanced terminal voltages due to unbalanced loads.

be kept within narrow limits. In well-regulated large power systems the currents flowing in the neutral does not under normal operation exceed 10 per cent of the total current flowing in the outside conductors.

It is important to keep the neutral wire return circuit closed at all times; for, if the neutral is open, unbalanced load conditions

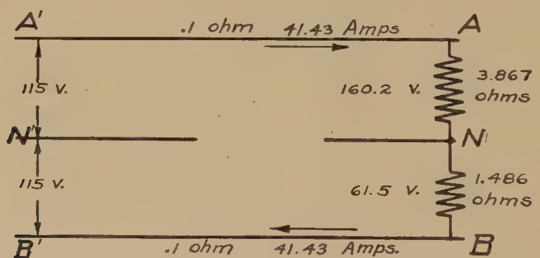


FIG. 13.—Effect of opening neutral of a three-wire system.

may cause excessive unbalancing of the voltages in the two sides of the system.

Let it be required to find the voltages across the two loads AN and BN of Fig. 12 when the neutral is opened as indicated in Fig. 13, assuming that the resistances remain constant. The

resistances AN and BN are readily computed from Fig. 12 before the neutral is opened.

$$R_{AN} = {}^{116}\!/\!_{30} = 3.867$$
$$R_{NB} = {}^{104}\!/\!_{70} = 1.486$$

After opening the neutral the two-load resistances form a straight-series circuit with the outside circuit conductors across the full 230 volts. The current flowing is accordingly.

$$I = \frac{230}{3.867 + 1.486 + .2} = 41.43 \text{ amps.}$$

The voltages across the points AN and NB are,

$$E_{AN} = IR_{AN} = (41.43)(3.867) = 160.2 \text{ volts.}$$
$$E_{NB} = IR_{NB} = (41.43)(1.486) = 61.5 \text{ volts.}$$

It was assumed that the resistances remained the same after opening the neutral as they were before. In the case of incandescent lamps this would not be strictly true. For tungsten lamps the change in resistance as the lamp voltage changes would tend to increase the unbalance. For carbon lamps the unbalance would be decreased by the change in resistance.

Power Supply for Edison Three-wire Distribution System.—From the discussion in the preceding paragraph it is evident that the central-station equipment furnishing power to the Edison three-wire system must provide a terminal for the neutral wire located so as to bring the current flowing in the neutral into the generating mechanism at the voltage point midway between the positive and negative terminals. This may be accomplished in several ways. Of the arrangements that have been invented to meet the requirements of the Edison three-wire system the following have proved to be of commercial importance:

1. Two generators connected in series.
2. One generator and storage battery.
3. One generator and balancer set.
4. Three-wire generators.

In the early development of the Edison three-wire system the power supply was obtained by connecting two generators in series with the neutral wire tapped into their common junction as shown in Fig. 14.

The use of two generators in series is not an economical method as the first cost is higher and the operating efficiency lower than for a single machine that will give the same output. The power supply requirements of the Edison three-wire system can be met by one generator in connection with a storage battery floating on the line as shown diagrammatically in Fig. 15. The neutral

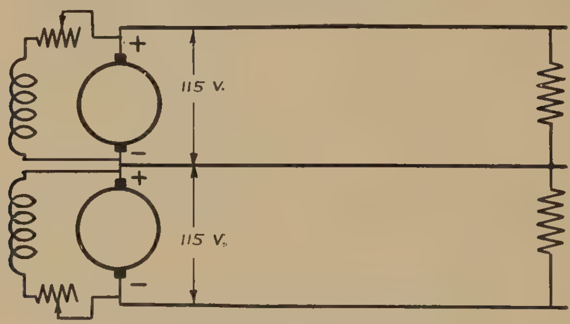


Fig. 14.—Two generators in series supplying power to Edison three-wire distribution system.

wire is connected to the middle point of the battery. By inspection of the circuits in Fig. 15 it is evident that when the load is unbalanced the half of the battery which is on the heavily loaded side of the three-wire system will discharge while the half on the light-load side will be charged.

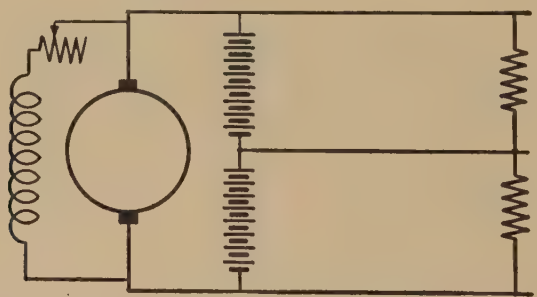


Fig. 15.—One generator with storage battery as power supply for Edison three-wire distribution system.

A more common method for gaining the desired neutral connection is the use of a balancer set in connection with the direct-current generator (Fig. 16). A balancer set consists of a motor and a generator whose armatures are coupled together mechanically. The desired action of the balancer set is that the machine

on the heavily loaded side of the three-wire system shall operate as a generator and supply the required additional power while the machine on the side having the lighter load operates as a motor and thereby counteracts the unbalanced load condition. To gain this end, three arrangements of the field excitation circuits in the balance set may be used:

(*a*) Shunt-field windings with connections as shown in Fig. 16.

(*b*) By using shunt-field winding with connections crossed as shown by the circuit diagram in Fig. 17.

(*c*) By the compound-field windings as indicated in Fig. 18.

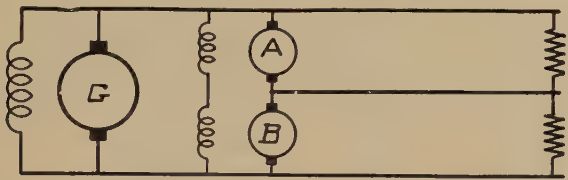


FIG. 16.—One generator with balancer set as power supply for Edison three-wire distribution system.

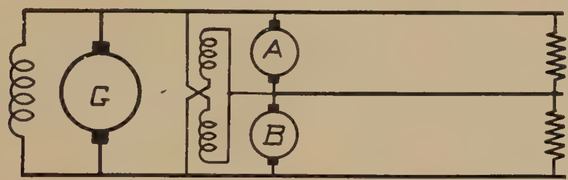


FIG. 17.—One generator with balancer set having shunt-field connections crossed as power supply for Edison three-wire distribution system.

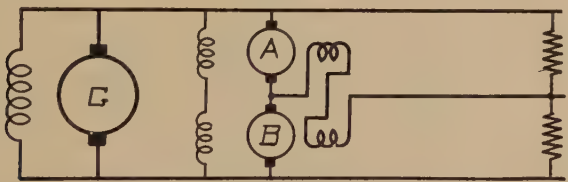


FIG. 18.—One generator with compound-wound balancer set as power supply for Edison three-wire distribution systems.

Consider the conditions for motor and generator action in the balancer set in Fig. 17. If the load is balanced no current flows in the neutral and there is no generator action in the balancer set. A slight motor action prevails in both machines sufficient to overcome the no-load losses of the set. Let the load be unbalanced with the lighter side on machine *B*. The terminal voltage of the armature in *B* is therefore greater than for *A*, and

since the field connections are crossed the field excitation of B, and hence the armature counter-e.m.f. is less than in machine A. Consequently machine B will operate as a motor and machine A as a generator thus counteracting the unbalanced condition of the load. If the unbalancing should change so that the machine B is in the heavily loaded side the relative terminal and induced voltages in the two machines will be reversed and as a consequence machine B will operate as a generator and machine A as a motor. The action of the balancer set again compensates for the unbalanced load condition. Hence the balancer set in Fig. 16 automatically counteracts or compensates for unbalanced load conditions, whether one side or the other has the heavier load. The connections shown in Fig. 15 give similar effects.

More nearly perfect compensation will be produced by balancer sets having compound-field windings connected as shown in Fig. 18. The compensating action is automatic. In the machine operating as a generator the compounding is cumulative and in the machine operating as a motor the fields become differentially compounded. If properly designed and adjusted the neutral current will automatically produce the desired variations in field excitation so that there will be very little change in the terminal voltage of either machine in the balancer set.

Three-wire Generators.—Storage batteries and balancer sets used as auxiliary appliances to provide a terminal connection for the neutral wire are objectionable in that they add to the first cost, require additional station floor space and increase the operating expense. These undesirable features are obviated by using a three-wire generator, invented by Dobrowolsky, for which the circuit diagram in its original form is shown in Fig. 19. The neutral terminal is obtained by connecting a coil of wire FGH, wound on an iron core, to points on the armature winding 180 electrical degrees apart. The coil of wire is tapped at the middle point G for connection to the terminal of the neutral wire in the Edison three-wire system. As the armature rotates the voltage between the points F and H goes through a cycle from zero to a maximum in one direction, back to zero and then a maximum in the reverse direction and back to zero. This voltage cycle causes an alternating current to flow in the coil but the potential at the middle point G is for all positions of the armature midway between the potential of the two brushes. That is, G is the electrical neutral point and serves as the terminal

connection for the neutral wire. In three-wire generators of this type the coil *FGH* with its iron core may be mounted inside the armature. The middle point *G* is connected to a slip-ring terminal for connection to the neutral wire of the Edison three-wire system as shown in Fig. 20

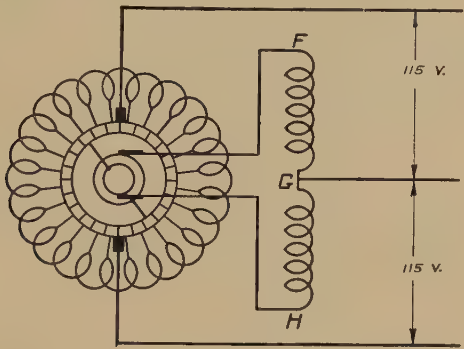


FIG. 19.—Three-wire generator. (*Dobrowolsky.*)

In Fig. 20 two coils are used with their center points connected together by the slip ring. This method tends to stabilize or maintain the neutral point more closely than is possible by the use of a single coil. When the coil is constructed inside of the machine it is usually placed under the armature core or may

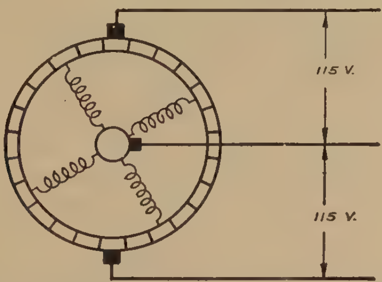


FIG. 20.—Three-wire generator. Double set of coils tapped with armature windings and having a common neutral connection.

actually be wound in the slots alongside of the armature conductors. In either case only one slip ring is necessary as shown by Fig. 20. However, if coils are to be placed externally of the machine as many rings must be used as the number of tapped points.

It is interesting to note that the current that flows in the neutral wire of the Dobrowolsky type of three-wire generator has no

effect on the flux density in the core of the balance coil, since the resulting ampere-turns of one-half of the coil are exactly balanced by those of the other half.

In compound-wound three-wire generators the series-field windings must be in two equal parts. Half of the series-field turns must be connected to each of the outer wires as shown in Fig. 21. If two or more compound-wound three-wire generators are to operate in parallel, two equalizer connections must be provided. The circuit diagrams and connections are shown in Fig. 21.

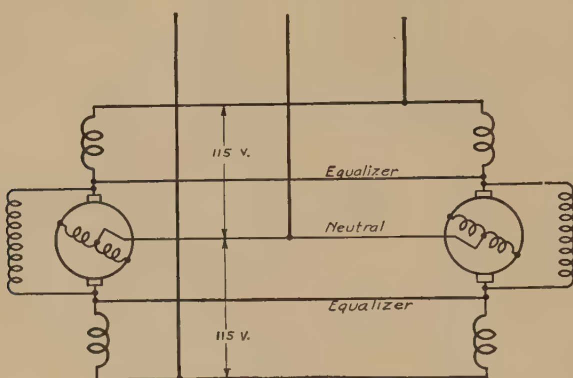


Fig. 21.—Two compound-wound three-wire generators in parallel.

Distribution for Electric Street Railways.—The form of the distribution system for street railways depends on many factors, as the area served, length of individual lines, land contours, density of the traffic, daily and seasonal variations in load, etc.

The central-station generators are compound wound with the series field on the negative, that is, grounded side. The positive terminal is connected to the trolley and feeders through a circuit breaker. In large systems the distribution feeders delivering power to the trolley wires may consist of many sectionalized groups that extend over the various car lines, forming a more or less interconnected network. In essence the distribution system consists of the trolley wire, either alone or in combination with one or more feeders connected as shown diagrammatically in Figs. 22, 23, 24, and 25.

The simplest form, used on short lines having light traffic, shown in Fig. 22 consists of merely the trolley wire. For somewhat heavier traffic and longer lines the ladder system shown in

Fig. 23, consisting of a single feeder connected the trolley wire at several points may prove the most desirable.

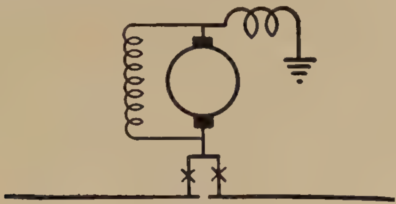


FIG. 22.—Railway trolley circuit.

For still heavier traffic and longer lines multiple feeders connected as shown in Fig. 24 may be necessary. The sectionalizing of the trolley as shown in Fig. 25 serve to localize trouble. A

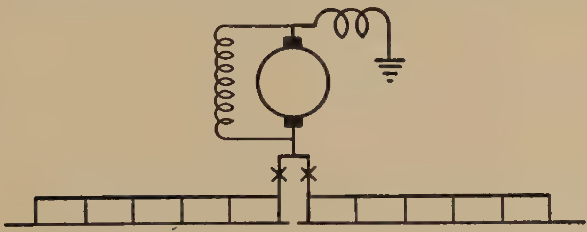


FIG. 23.—Ladder feeder system.

short circuit on one section will not shut down the whole system. Sectionalized systems, however, require more copper in the feeders. As the several car lines forming a street-car system in

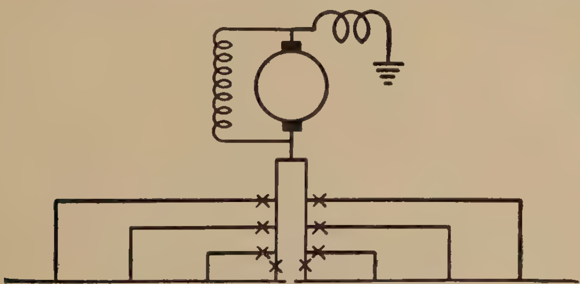


FIG. 24.—Multiple feeder system.

a large city differ greatly in length and in the character of the service required, each set of feeders must be operated more or less as distinct units. In order to hold the operating voltage on the longer lines at a desirable value it is in most cases eco-

nomical to install auxiliary apparatus as boosters and storage batteries to compensate for the resistance drop in the feeders and trolley wire comprising the distribution system.

In long trolley lines it would require a very large amount of feeder copper to give good regulation at the end of the line. It is sometimes more economical to install a storage battery and keep it floating on the trolley at the end of the line as shown in

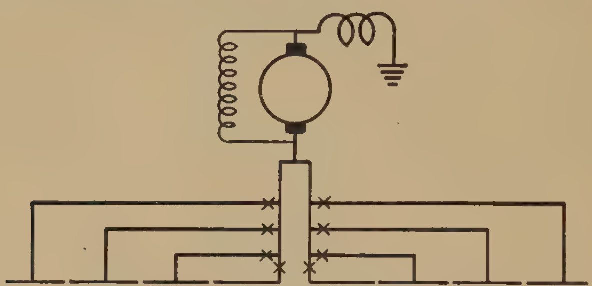


FIG. 25.—Sectionalized trolley-feeder system.

Fig. 26. The battery acts somewhat as a huge balance wheel on the generator. The battery carries a part of the peak loads by delivering energy that has been stored during preceding light-load periods. Thus, the action of the battery greatly improves the voltage regulation and relieves the generator from excessive variations in the load.

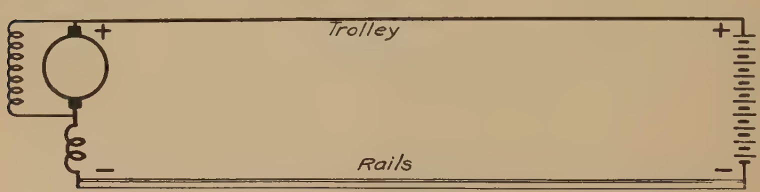


FIG. 26.—Battery at end of line.

Boosters.—A booster is a dynamo whose armature is connected in series with the main supply circuit in order that its generated e.m.f. may add to or subtract from, depending on the direction of generation, the central-station voltage impressed on the circuit. The e.m.f. generated by the booster is only a small percentage of the central-station voltage but the armature conductors must be heavy enough to carry the full-load current. Power for operating the booster is generally obtained by means of a direct-connected

motor receiving current from the central-station constant-potential mains. Boosters may be either series or compound wound and the voltage generated varies in magnitude and may be unidirectional or automatically reversible. The purpose of the booster is to regulate the voltage of the feeder or busbar section of the system to which it is connected. Thus if the central-station voltage supply is constant the booster would automatically compensate for the resistance drop in the feeder under changing load conditions and thereby keep constant voltage at the load. Boosters are frequently used in battery-control circuits to compensate for variations in the battery voltage. A large number of booster-circuit connections have been developed to serve the widely varying requirements of electric circuits or isolated systems with special load characteristics. Only three typical circuit connections will be described.

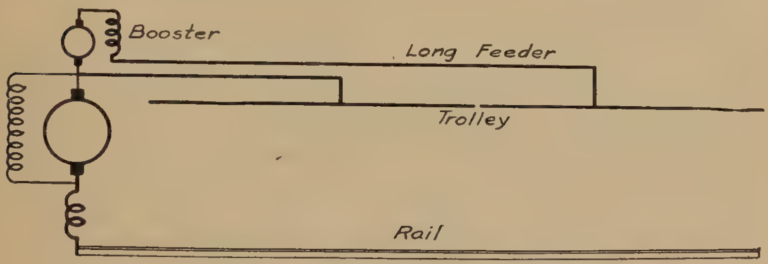


Fig. 27.—Series booster connections.

Series Booster.—The series booster in a feeder circuit for compensation of the resistance drop in the line as shown in Fig. 27 is a simple arrangement. Since the voltage drop in the line is proportional to the current the voltage generated by the booster should likewise be in proportion to the current. That is, the booster should have a straight-line external characteristic.

The voltage characteristics of the series booster are sufficiently close, for practical purposes, to the desired straight line passing through the origin if the field-flux density is well within the saturation point; that is, below the bend in the magnetization curve.

The effect produced by the booster is in the same direction as would result from an increase in the conductor copper but may be made sufficiently large to completely compensate for the ohmic drop in the circuit. A special application of the series booster

to reduce electrolysis from stray current is referred to in Chap. XIX, and illustrated by Fig. 28.

Shunt Booster and Battery.—The shunt booster is essentially a separately excited dynamo as the field winding is connected across the busbars and not to the brushes of the booster. The shunt booster is used in connection with a storage battery to regulate the voltage and to make the load on the main generator more uniform. At hours when the load is light the battery may carry the entire load, the generator not running, while during

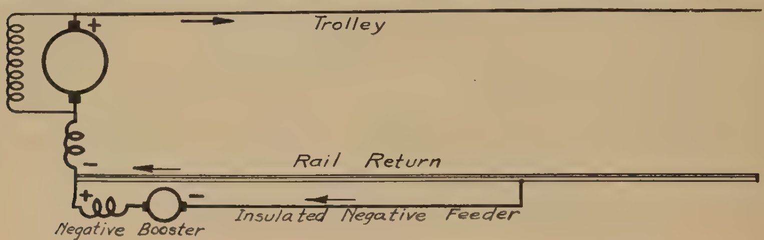


FIG. 28.—Negative return booster.

peak-load periods the battery operates in parallel with the main generator. Circuit connections are shown in Fig. 29.

The rheostat in the shunt-field circuit provides the means for regulating the booster voltage. A reversing switch in the shunt-field circuit will permit the booster voltage to add to the main-generator voltage when the battery is charging while a like increase is given to the battery voltage when it is carrying the load.

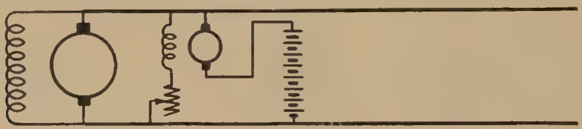


FIG. 29.—Shunt booster and battery connection.

Reversible Booster and Floating Battery.—For systems in which the load fluctuates rapidly above and below the average load value the reversible booster in combination with a storage battery is sometimes used. The reversible booster is differentially wound and while a variety of circuit arrangements have been developed the simple type of circuit connection shown in Fig. 30 serves to illustrate the principle of operation.

The number of cells in the battery is chosen so that on open circuit, with average load on the system, the battery voltage is equal to that of the system. The series field and the shunt field

are so proportioned that, with average load current and normal line voltage, their magnetizing effects are equal in value but opposite in direction and therefore neutralize each other. When the load increases the series field becomes stronger than the shunt field. The e.m.f. generated in the booster adds to the battery voltage and as a consequence the battery discharges, carrying part of the load. When the load drops below the average value the magnetizing effect of the shunt field will prevail and the e.m.f. of the booster will be reversed, thereby causing a charging current to flow into the battery. In this manner the battery in combination with the booster automatically evens up the load fluctuations and as a result the load on the generator becomes much more uniform or more nearly constant than the load on the system. A rheostat in the shunt-field circuit operated by hand is provided to compensate for the

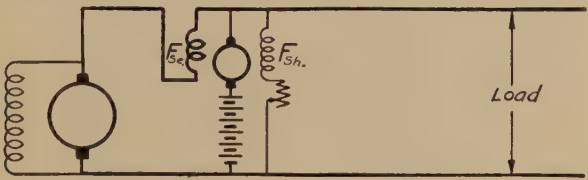


FIG. 30.—Reversible booster and storage battery.

variations in the open-circuit battery voltage due to the state of battery charge.

Battery with Resistance or End-cell Control.—Storage batteries usually have a sufficient number of cells so as to have an e.m.f. somewhat greater than the busbar voltage. The electromotive force of the battery as applied to the load may be regulated either by a variable resistance in series, as shown in Fig. 31, or by end-cell control, as in Fig. 32.

By adjusting the resistance the power delivered by the battery may be controlled. The method is, however, wasteful of electric energy, as the total load current passes through the resistance in series with the battery. Adjustments of the resistance must be made not merely for charging the load but also to compensate for the variations in the battery voltage, depending on its state of charge.

In place of having a variable resistance in series with the battery, the e.m.f. applied to the load may be varied by cutting in or out of circuit one or more cells at one end of the battery.

Necessarily this change in the number of cells in series delivering power must be made without breaking the circuit and also without short circuiting any cell. This is accomplished by means of double contact; a main contact and an auxiliary contact through a resistance R, as indicated by the circuit diagram in Fig. 32. As the main contact is moved from one cell terminal to that of the adjacent cell the battery connection to the load remains intact through the auxiliary contact. When the main and auxiliary contacts are in contact with the + and − terminals of a single cell the resistance in series with the auxiliary contact prevents excessive flow of the current, as would be the case if there were no resistance in series with the auxiliary contact. For large batteries, the end cell contacts are operated by a remote-control motor-driven worm gear and the voltage regulation may be

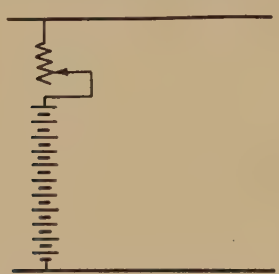


Fig. 31.—Resistance control of storage battery.

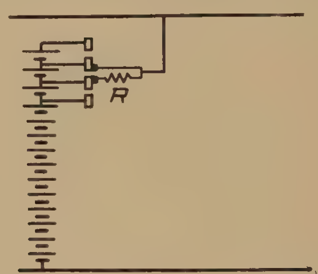


Fig. 32.—End-cell control of storage battery discharge.

made automatic. The use of storage batteries for load control and voltage regulation is an important advantage of direct currents over alternating currents in electric power distribution.

Voltage Regulation. Type TD. Regulator.—The Type TD regulator (Fig. 33) controls the generator voltage by rapidly opening and closing a shunt circuit across the field rheostat, thereby maintaining an average value of field current required by the generator to hold the voltage at normal for all conditions of load and variations in speed for which the machine has been designed. The operation of this type of regulator depends upon a continuous variation of generator voltage, but because of the rapidity of the contact action this variation is very slight and the generator maintains a uniform voltage. The action of the several parts can best be explained by means of the circuit diagram in Fig. 34.

The regulator consists of a main control magnet and a relay magnet. The former has an adjustable stop core at the bottom and a movable core above. The movable core is attached to a lever operating the main contacts. The cores are excited by the potential winding, which is connected across the generator terminals and the pull of these cores is opposed by a helical spring which tends to keep the main contacts closed. The relay magnet is also excited from the bus through an external resistance and is controlled by the main contacts on the main-control magnet

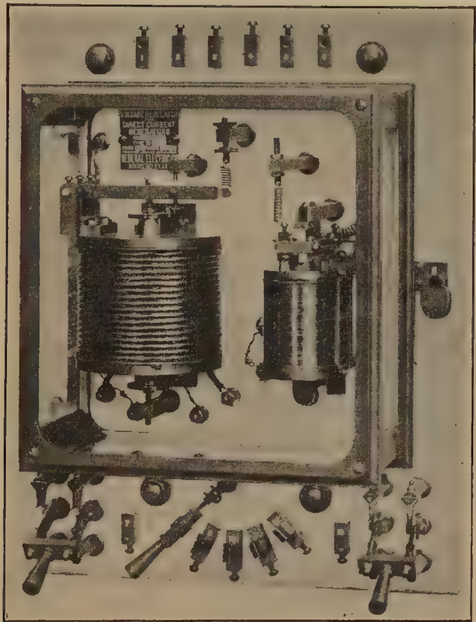


FIG. 33.—Type TD form G regulator. (*General Electric Company.*)

lever which open and close a parallel circuit across the relay winding. Its armature controls the contacts, which open and close the shunt circuit across the field rheostat. When the main contacts are open, the relay contacts are also open, as the relay magnet is then magnetized and its armature overcomes the pull of the opposing spring. A condenser is connected across the relay contacts to absorb the arc which otherwise would destroy the contacts.

The cycle of operation is as follows: The shunt circuit across the generator-field rheostat is first opened by means of a switch

on the base of the regulator and the rheostat turned to a point that will reduce the generator voltage 35 per cent below normal. The main control magnet is at once weakened and allows the spring to pull out the movable core until the main contacts are closed. This closes the parallel circuit across the relay, thus neutralizing its windings. The relay spring then lifts the armature and closes the relay contacts. The switch in the shunt circuit across the generator-field rheostat is now closed, practically short circuiting the rheostat, and the generator voltage at once rises. As soon as it reaches the value for which the regulator has been adjusted, the main-control magnet causes its

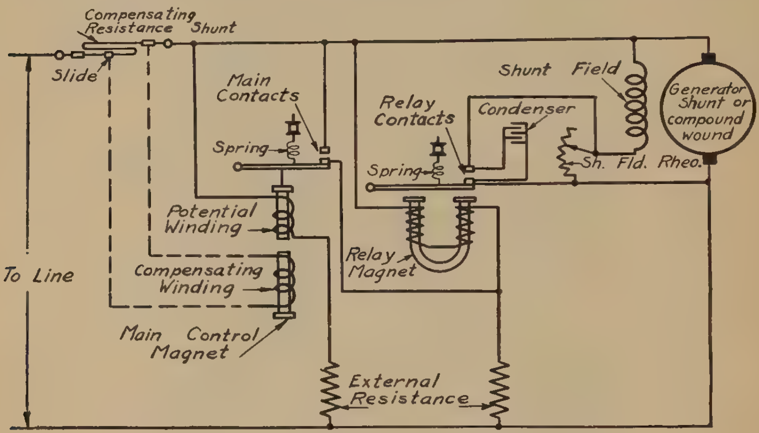


FIG. 34.—Circuit diagram of Type TD regulator.

contacts to open, which in turn open the relay contacts across the rheostat, inserting the resistance in the field circuit. The voltage at once falls off and the main contacts close, the relay armature is released and the shunt circuit across the rheostat is again completed. The voltage then starts to rise and this cycle of operation is continued at a high rate of vibration, maintaining a constant generator voltage.

Train Lighting.—Train lighting offers many special problems in electric power distribution. Satisfactory service requires that an approximately constant voltage be maintained, whatever the number of lamps in use, or the load on the system without regard to the direction of motion or speed of the train. To gain this end in a most economical manner is the problem of train lighting. For electrically propelled trains, interurban lines, and

street cars the solution is comparatively simple as the electric power is directly available. In steam railroads a number of systems have been developed and used which may be grouped on the basis of power supply.

1. The storage-battery system in which each car is equipped with a storage battery for this purpose.

2. The single prime mover-generator or head-end system, in which a generator and prime mover, placed in the baggage car or on the locomotive, provide the electric power for the whole train.

3. The axle-lighting system in which a generator, mounted under each car is belted to an axle of the car.

The storage-battery system is simple in principle but expensive in operation. Also it is difficult to secure satisfactory voltage regulation, particularly on long runs.

The source of electric power in the head-end system is a compound-wound generator, on or near to the locomotive, driven by a steam turbine. This system, however, also requires storage batteries to provide light during switching or when the train is parted.

The axle-lighting system is generally used and a number of systems have been devised on the general basis of letting each car have its own source of electric power and that the axle of the car be used as a prime mover. Generators of special design are necessary and automatic regulation must be included in the system to compensate for the variation in voltage due to change of speed of the train. Accessory storage batteries are required in this system also, to provide current when the train is at a standstill.

Of the mechanical methods for gaining generator voltage regulation at varying speed the slipping of the driving belt or the use of a slipping clutch, as well as the use of a centrifugally operated switch for the automatic making or breaking of the connections between the generator and the large batteries are of special interest.

The electrical or electromagnetic methods for securing voltage regulation are based on two principles:

(*a*) An automatic variation of resistance placed either in series with the leads or in the field circuit of the generator.

(*b*) By using the armature reaction of the generator to secure the desired external characteristics.

The Rosenberg train-lighting generator, for which the circuit diagram is shown in Fig. 35, is a good example of the latter type. A strong battery DD' is connected across the field windings F_1 and F_2 and provides field excitation for the N and S poles as indicated in the diagram. The armature has a single winding but is provided with two pairs of brushes located 90 electrical degrees apart. The main pair of brushes AA' are short circuited and the currents flowing through the armature between the AA' brushes produces a field flux at right angles to the magnetic field set up by the electric current in F_1F_2 and its direction depending on the direction of rotation. The armature conductors cuttting this cross flux cause a current to flow to the second pair of brushes

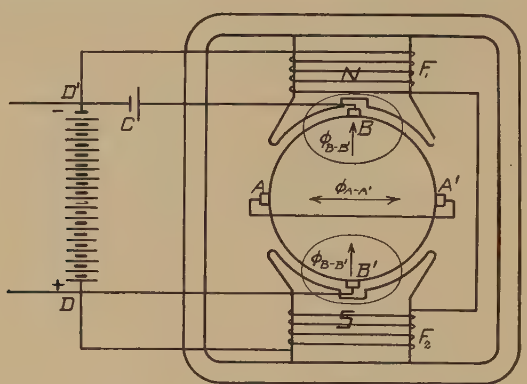


Fig. 35.—Circuit diagram of Rosenberg generator.

BB'. The magnetic flux or armature reaction produced by the BB' currents in the armature is in opposition to the original flux N-S.

The BB' brushes are connected to the battery terminals through the aluminum electrolytic cell C. This cell permits current to flow quite readily from B to D' but because of the valve action of the aluminum oxide film on the aluminum electrode in the cell very little current can flow in the reverse direction, that is, from D' to B. Hence the cell C is a valve, automatic in its action, that permits current to flow from the BB' brushes to the battery or the outside circuit DD' when the speed is sufficiently high, but prevents the discharge of the battery at low speeds or when the train stands still.

The interaction of the armature cross flux produced by currents in brushes AA' with the armature reaction of the flux produced

by the currents in BB' on the original field NS due to the currents in the field windings F_1F_2 result in the generation of essentially constant voltage at BB' for wide variations in load and for all train speeds above a given minimum. For still lower speeds and where the train is at a standstill the batteries supply the load current.

Third-brush Generator.—The circuit diagram of a third-brush generator used as part of the electrical equipment in some automobiles is shown in Fig. 36.

The design of the generator is such that the current which is zero at start increases with the speed until a maximum value is reached and holds fairly constant over a considerable range of speed but falls off if the speed becomes still greater. The effect is

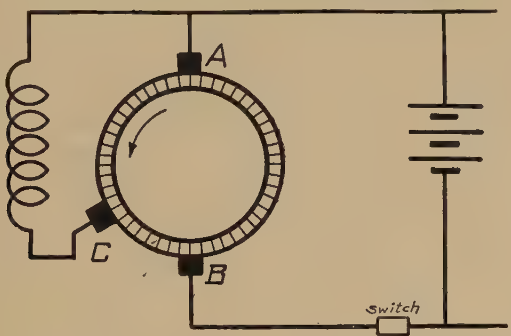


FIG. 36.—Circuit diagram of third-brush generator.

produced by the armature reaction of the current between the main brushes as well as of the current flowing in the third brush. Consider the change that occurs when the speed increases from a low value. The generated voltage increases and as a consequence greater current will flow between the main brushes A and B. The reaction of the increased armature current shifts the field flux away from the leading pole tip, which automatically reduces the voltage between A and C and thereby weakens the field excitation. The relative amount of the reduction depends on the point on the saturation curve of the machine at which it operates for the given speed. The lower the field excitation the greater will be the reduction in flux for any increase in armature reaction. The action of the currents flowing in the coils short circuited by brush C, tends to increase the demagnetizing effect on the field excitation.

An automatic switch opens the circuit when the voltage generated at the brushes AB falls below the battery terminal voltage in order to prevent the battery from discharging through the generator when running at low speed or at a standstill. The speed-current characteristic may be greatly modified by shifting brush C in position. After the brush position has been determined by the requirements of the service to be rendered the auxiliary brush is kept in a fixed position. The third-brush generator is therefore designed to operate over a wide range of speeds and still automatically regulate so as to deliver power at a fairly constant voltage.

PROBLEMS

1. What is the maximum amount of power than can be transmitted over a two-wire transmission line of No. 00 bare copper 2 miles in length, if the generator voltage is 250 volts? Find efficiency of transmission.

2. A load of 120 kw. is transmitted a distance of 2,000 ft. from 235-volt busbars under the condition that the load voltage must not drop below 220 volts. Find the size of copper wire in circular mils. Also find its weight.

3. Repeat Problem 2 for a distance of 4,000 ft. What is the ratio of weights of copper required in the two cases?

4. A load of 150 amp. is located at a distance of 1,050 ft. from the 550-volt busbars and 800 ft. further away is a load of 80 amp. A 300,000 cir. mil. feeder connects the first load to the busbars, while a No. 00 wire extends to the second load. Find voltage at each load, power at each load and efficiency of transmission.

5. Two 125-kw. loads are located 1,000 and 1,500 ft., respectively, from a power station which maintains a constant voltage of 500 volts. The size of the feeder extending from the first load is 400,000 cir. mils. A feeder 500 ft. long and No. 000 in size connects the second load to the first load. Find the current and voltage of each load. Also find the efficiency of transmission.

6. Eight lamps, each taking a current of 10 amp., are connected in parallel as indicated in Fig. 6 (a). Assume that the lamps are equally spaced and that *each* of the connecting conductors has a resistance of 0.1 ohm. If the busbar voltage is 110 volts find the voltage at each lamp and the efficiency of transmission.

7. Arrange the lamps of Problem 6 as shown in Fig. 6 (d). Assuming that the resistance of the connecting wires is the same as in Problem 6 and that the impressed voltage is 110 volts, find the voltage at each lamp and also efficiency of transmission.

8. Repeat Problem 7 except that the circuit is as shown in Fig. 6 (e). Make the connection to the 110-volt power main at two diametrically opposite lamp terminals.

9. A balancer set across 230-volt mains when running idle takes an armature input of 575 watts, this being the rotational losses of the two

machines. The two outer wires of the three-wire system each have a resistance of 0.02 ohms and the neutral wire a resistance of 0.036 ohms. A load of 70 amp. is placed on one side and a load of 10 amp. on the other side. The armature resistance of each machine is 0.030 ohms. The field rheostats are so adjusted that the voltages are equal at the balancer set terminals. Find the voltage at each load and also the induced voltage of each machine.

10. A No. 0000 trolley wire 7 miles long is tapped every $1\frac{1}{2}$ miles from the generator end by individual 300,000 cir. mil. feeders which run directly to the generator from the tapped point. The track resistance is 0.05 ohms per mile. Find the voltage at a car taking 100 amp. located 5 miles from the generator end, if the station voltage is 600 volts.

CHAPTER XXI

THERMIONIC AND GASEOUS CONDUCTION

The Edison Effect.—In 1883, while experimenting with his newly invented incandescent lamp, Edison discovered what was later termed the *Edison effect*. He placed an extra element or plate inside of his lamp and connected it externally to the filament as shown in Fig. 1. By the use of a galvanometer he found that a small current flowed into this plate or anode and that the current was larger when the plate was connected to the positive end of the filament than if connected to the negative end. Edison did not follow up this discovery and it was not until nearly twenty years later that the phenomenon was explained and its great value appreciated through the development of the Electron theory.

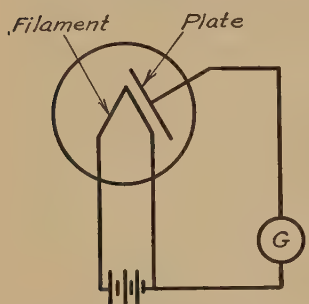


Fig. 1.—Circuit diagram for Edison effect.

According to the Electron theory the electric current is composed of a large number of small electrically charged particles of matter, called *electrons* that flow as a stream in the conductor. In this theory it is assumed that all matter is atomic in structure and that each atom is made up of a positive nucleus surrounded by one or more electrons. In the *neutral* atom the number of electrons which always carry a negative charge of electricity, are just sufficient to neutralize the positive charge on the nucleus. These electrons revolve about their nuclei in well-defined paths and are for the most part rigidly held to their orbits. In certain materials, however, some of the atoms may have one electron which is comparatively free; that is, it is not rigidly held in an orbit but may move around within the boundaries of the material practically independent of the movement of any one atom. Since the free electrons carry electric charges their motion will at all times be governed by electrostatic laws, so that at any instant all portions

of the material will have equal positive and negative charges. If, however, an e.m.f. is applied between two points of the material these free electrons will be carried toward the positive terminal and this movement constitutes a flow of current. Such a material is called a *conductor* and its degree of conductivity is directly proportional to the number of free electrons contained in a unit volume of the material. *A non-conductor*, similarly, is one which contains few, if any, free electrons. It is to be noted that the flow of electrons is from negative to positive, whereas the flow of current is always considered to be from positive to negative. This difference exists because the original assumption of the direction of flow of current was made before the nature of the phenomena was understood, and long before the Electron theory came into existence. It makes little difference in which direction the current is assumed to flow, however, as long as the notation is followed consistently. It is, however, necessary to realize that electrons actually flow in a direction opposite to our customarily assumed direction of flow of electric currents, in order to understand the phenomena discussed in this chapter.

Thermionic Emission.—At ordinary temperatures the electrons are confined in their movements to the space within the surface of the conductor; the kinetic energy due to their motion not being sufficient to break through the surface tension of the material. If, however, their velocity be greatly increased by some means to a value of the order to magnitude of 10^8 cm. per second, the kinetic energy will be large enough to break through the surface tension and let the electron pass into space outside the conductor. This result may be obtained either by heating the metal to incandescence, as in radio vacuum tubes, or by causing other electrons to strike the material at high velocities, as in X-ray tubes. In the latter case, called *secondary emission* each high velocity electron impinging upon the material "knocks out" several other electrons when it strikes.

The driving off of free electrons from a material when heated is known as *thermionic emission.* The phenomenon is very similar to the evaporation of atoms of water from the surface of the liquid when heated, the laws of which were known some time prior to the discovery of thermionic emission. In 1901 Richardson derived an equation for the emission of electrons from a hot metal which is exactly analagous to that previously obtained

for the evaporation of a liquid. Richardson's equation, known as his law of emission, is given in equation (1).

$$i = a\sqrt{T}\,\epsilon^{-\frac{b}{2T}} \tag{1}$$

i = the number of electrons evaporated per second per $\overline{\text{cm.}}^2$ of hot surface.
a = a constant.
T = the absolute temperature or temperature in C° + 273°
b = the latent heat of evaporation of electrons.
ϵ = the Naperian base.

Richardson's equation should be compared with equation (2) for evaporation of a liquid.

$$N = A\sqrt{T}\,\epsilon^{-\frac{a}{2T}} \tag{2}$$

N = the number of atoms evaporated per second per $\overline{\text{cm.}}^2$ of hot surface.
A = a constant.
T = the absolute temperature or C° + 273°.
a = the latent heat of evaporation of the liquid.

In 1923 Dushman developed a more accurate expression for the current, differing somewhat in form from equation (1). However, it should be noted that Richardson's equation conforms remarkably well with experimental results.

Dushman's expression for the current in thermionic emission is given by equation (3):

$$i = AT^2\,\epsilon^{-\frac{b_o}{2T}} \tag{3}$$

i = amperes per $\overline{\text{cm.}}^2$ of hot filament surface.
T = the absolute temperature or C° + 273°
A = a constant; it varies somewhat with method of derivation, but is between 50 and 60.
$b_o = \phi_o \frac{e}{k}$, in which:
e = electron charge.
k = Boltzman's constant.
ϕ_o = Richardson's work function.

ϕ_o has been determined for some metals as follows:

Tungsten	4.53 volts
Thorium	2.94 volts
Molybdenum	4.31 volts
Tantalum	4.40 volts
Calcium	2.24 volts

Equation (3) might appear to give an entirely different value for the current i than given by equation (1) since $\sqrt{T}$ is replaced by T^2. However, the exponential term primarily determines the shape of the curve so that both equations plot to nearly the same form.

As stated above, a metal must be heated to incandescence before thermal emission will occur. If the emitting material is drawn into a filament and placed in a vacuum, the heating of the filament to incandescence is readily accomplished by passing a current through it. The maximum temperature allowable is determined primarily by the material used and to some extent by the dimensions of the filament. A pure tungsten filament must be operated at about 2400° absolute temperature, a tungsten filament coated with thorium at about 1500° and one coated with barium and strontium oxides at about 1150°. The difference in temperature lies in the difference in melting points of the materials and their different surface tensions to electronic emission. Thorium has a lower surface tension for electrons than tungsten while barium and strontium oxides have still lower values. This means that electrons can escape from these latter materials at a lower velocity than is required for tungsten. For this reason all filaments intended for thermal emission are of the thoriated or oxide coated type. Due to the high temperatures required for emission and the requirements of thermionic conduction, the filament must be placed in a fairly high vacuum to prevent oxidation and gas ionization. Thus the name *vacuum tube* has been applied to the devices utilizing this principle.

The Fleming Valve.—The two-electrode vacuum tube is based on the phenomenon discovered by Edison and known as the *Edison effect.* Fleming studied this phenomenon more fully and found that by inserting a battery in the plate or anode lead (Fig. 2) a relatively large current could be passed through this circuit when the *positive* terminal of the battery was connected to the plate and no current would flow if the *negative* terminal

of the battery and the plate were connected together. This tube is, therefore, a perfect rectifier and was used extensively as a detector in the early days of radio service for rectifying the high-frequency radio currents in order to secure an audible sound. Known as the Fleming valve, it was the most sensitive detector available at that time.

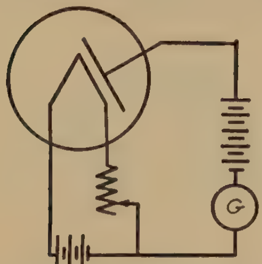


FIG. 2.—Fleming valve with test circuit, as used by Fleming.

Two-electrode Tubes.—The *characteristic curves* of a vacuum tube show the relations between the voltages and currents in the tube. If in Fig. 2, the temperature of the filament be varied by altering the current through it and the current in the galvanometer be noted for each change, a curve such as *a* in Fig. 3 will be obtained. When the temperature is very low no emission occurs and hence no electrons are drawn over to the plate, and, therefore, there is no deflection of the galvanometer. As the temperature is increased a few electrons are "boiled off" and are immediately attracted by the plate giving a slight deflection of the galvanometer. Further increase of temperature will cause a rapid increase of emission current until point *d* is reached. According to Richardson's equation (1) the emission current should increase indefinitely with increasing temperature, but the galvanometer deflections show a definite maximum value. This is because the galvanometer indicates the number of electrons arriving at the plate, whereas Richardson's equation refers to the number of electrons leaving the filament. After an electron leaves the filament a small but definite time interval is required for it to pass over to the plate. As a result of this time interval there must always be a number of electrons in the space between the two electrodes as long as the filament continues to emit electrons. The number of electrons in this space must necessarily increase with increasing filament temperature until the total negative charge in the space between the filament and plate overcomes the effect of the positive plate or anode. Any additional electrons which may be emitted will then be forced back into the filament and no further increase in electron flow can occur. This effect is known as *space charge* and the maximum value of current in curve *a* of Fig. 3 is the *space-charge saturation* value for the given value of plate voltage.

If the plate voltage is increased, a greater number of electrons must accumulate in the space between filament and plate before the effect of the plate voltage is neutralized. The space-charge saturation curve is therefore a higher value of plate current.

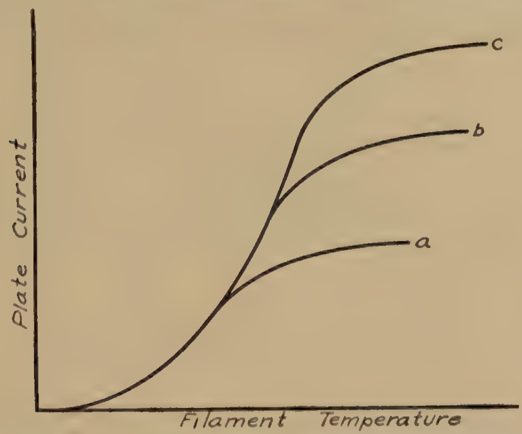


FIG. 3.—Space charge saturation curves. Plate current and filament temperature for three plate voltages in a two-element or diode tube.

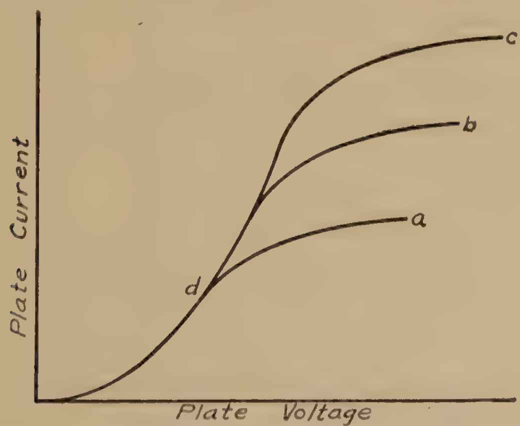


FIG. 4.—Filament saturation curve. Plate current and plate voltage for three filament temperatures in a two-element or diode tube.

If in the circuit of Fig. 2 the filament temperature is kept constant and the plate voltage varied a curve such as *a* in Fig. 4 will be obtained, quite similar in shape to the curves of Fig. 3. When the plate voltage is very low the space charge forces most of the emitted electrons back into the filament again. As the

plate voltage is increased the space charge is gradually overcome, thereby producing an increase in the plate current. At point d nearly every electron emitted has been drawn over and the current soon assumes a constant value, which is the *filament-saturation* value. If higher values of filament temperature are used more electrons are emitted and curves such as b and c are obtained.

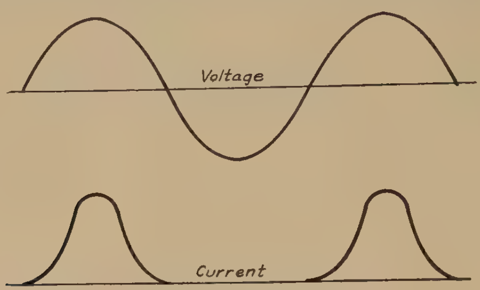


FIG. 5.—Line-wave voltage and resultant rectified current in two-element tube.

From the foregoing it is evident that the two-element tube may be used as a rectifier, transmitting a unidirectional current when an alternating voltage is impressed across its terminals. Figure 5 shows an alternating voltage and the resultant unidirectional current through the tube when using the circuit shown in Fig. 6. The exact shape of the current curve may be determined by means of the corresponding curve of Fig. 4. The

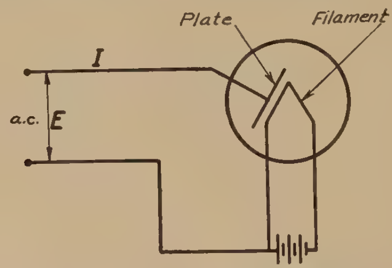


FIG. 6.—Circuit diagram for two element tube.

current and voltage are measured at the points in the circuit indicated in Fig. 6. This type of tube as for example the Kenotron is extensively used as a rectifier of alternating currents at high voltages, the pulsating unidirectional current produced by the tube being smoothed out to a steady direct-current value by suitable filters, as illustrated in Fig. 8. Such a device makes it possible to transmit direct currents at high voltages with

correspondingly low transmission losses and the advantages of having direct currents on the transmission line. The rectified direct-current voltage may be made of any desired value by the selection of the proper ratio in the number of turns in the transformer windings. The use of the vacuum tube for rectification in power systems is limited chiefly by the carrying capacity of the tube. Tubes of up to 1,000 kw. rating have been constructed.

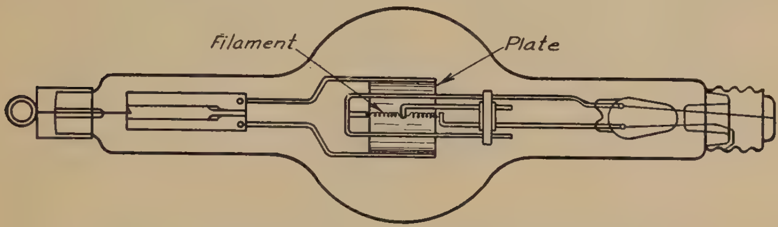


Fig. 7.—Kenetron-two-electrode tube.

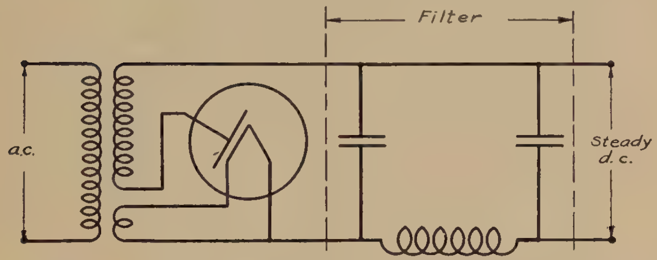


Fig. 8.—Circuit diagram of two-electrode tube with filter.

Three-electrode Tubes.—In 1907 De Forest discovered that the insertion of a *third electrode* in the Fleming valve would greatly increase its field of application. This third electrode, or *grid*, is placed between the filament and plate and consists of a wire mesh through which the electrons may pass as indicated by the circuit diagram in Fig. 9. A negative charge applied to the *grid* electrode will tend to increase the space charge and, therefore, to decrease the current from the filament to the plate. Likewise a positive charge applied to the grid tends to decrease the space charge and, as a consequence, increase the plate current. The three-electrode tube will therefore act as a valve; either opening or closing the passage for the plate current depending on the voltage or charge impressed on the grid.

This action is clearly shown by the *static characteristic curve* of the tube in Fig. 10. For example, a change in grid potential

of 9.5 volts (Fig. 10) has the same effect upon the plate current as a change of approximately 50 volts in plate potential. Hence a comparatively small variation in grid voltage may be used as

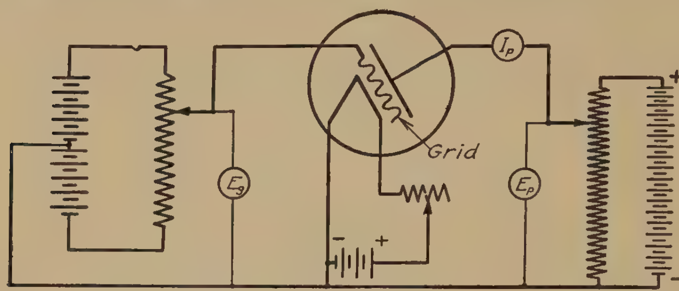


FIG. 9.—Circuit diagram used in taking data for curves in Fig. 10.

equivalent to a large variation in plate potential; that is, the tube serves as an amplifier.

The first commercial application on a large scale of the three-element tube was in the communication field as telephone

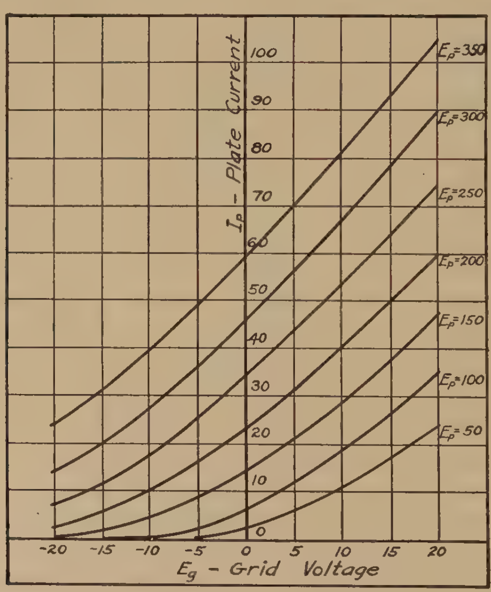


FIG. 10.—Static characteristic curve of three-electrode tube.

repeater. In this work it is used to amplify the very weak alternating potential produced by the human voice in the telephone transmitter. The telephone currents are applied to the grid and

thus produce changes identical in form but of much greater amplitude in the comparatively large plate current. By means of a series of amplifications it is possible to transmit speech from any telephone transmitter in the Bell system to any receiver in any part of the United States or Canada and even to many points in Europe.

In the radio communication field the three-element tube is used for several distinct purposes:

(*a*) As an oscillator to convert direct-current power into high-frequency alternating currents.

(*b*) As a modulator to vary the magnitude of the high-frequency current by the voice-frequency current from a telephone transmitter.

Fig. 11.—Pliotron three-electrode vacuum tube.

(*c*) As a detector to demodulate or rectify the modulated signals when received at some distant point so as to reproduce the voice-frequency currents.

(*d*) As an amplifier of the voice currents before they are applied to a telephone receiver or loud speaker.

By far the greater part of vacuum tube applications are largely or entirely alternating-current phenomena and not primarily concerned with direct currents.

In recent years, applications have been found for both the two- and three-element vacuum tubes in the electric power field. The valve action is used for rectifying alternating currents and in control apparatus for electric power systems.

Vacuum tubes may be classified on the basis of the degree of vacuum used; on the number of elements or electrodes in each tube; on size or on the purpose for which the tube is designed. High vacuum tubes are designated by coined words containing the Greek word "tron" meaning nothing in it. Thus, the *kenotron* is a two-electrode tube, the *pliotron* a three-electrode

tube; and the *magnetron*, a two-electrode tube having an externally wound coil. Electric currents flowing in this coil produce a magnetic field parallel to the path of the filament plate current.

X-rays.—If electrons moving at high velocities strike a metallic target, as T in Fig. 12, the impact sets up a series of very short electromagnetic waves. These waves were first observed by Roentgen in 1896 and as their nature was at that time unknown they were called X-rays or Roentgen rays. The X-rays travel through space with velocity of light and have the properties of reflection and refraction like light waves. In essence X-rays differ from light merely in wave length. The X-ray wave lengths are extremely short even in comparison to light waves.

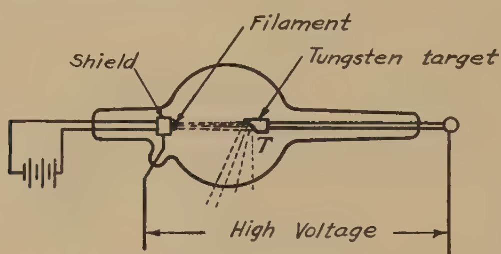


FIG. 12.—Circuit diagram for Coolidge X-ray tube.

Because of the difference in wave length the X-rays can pass through many substances which are opaque to light and possess the power of producing many important physiological and chemical as well as physical effects.

The X-ray is also an important accessory in various engineering tests. As an example, it may be mentioned that, in large castings, blow holes or other imperfections may be discovered by the use of X-rays.

The Coolidge X-ray tube is highly evacuated and has at one end a tungsten filament which is heated to incandescence by an electric current as indicated by the circuit diagram in Fig. 12. A very high direct-current voltage, 100,000 volts or more, is impressed between the hot filament and the target T (Fig. 12). The electrons emitted by the incandescent tungsten filament experience rapid acceleration in passing the very steep voltage gradient between the filament and the plate and quickly acquire high velocities before striking the tungsten target T. As result of these impacts very penetrating X-rays of high intensity are emitted by the target.

Gaseous Conduction.—Gaseous conduction consists in the flow of current through a gas by means of ions as carriers of electric charges. In order to conduct electric currents the gas must be ionized. Examples of such conduction are found in Tungar rectifiers, neon lamps, corona, mercury-arc rectifiers, arcing grounds, etc. Gaseous conduction occurs under more varied conditions than thermionic conduction; the latter requires a fairly high vacuum while gaseous conduction occurs under atmospheric conditions as well as over a wide range of gas pressure.

Ionization of a gas at atmospheric pressure requires very high potentials. If the pressure be reduced the gas will ionize at much lower potentials. The presence of a few electrons travelling at high speeds will also tend to ionize the atoms by the impact of their collisions and thereby lower the voltage otherwise required for ionization. For this reason most commercial applications of gaseous conduction use gas in a tube or bulb at less than atmospheric pressure; many also have a filament or other means for producing thermionic emission as an auxiliary, producing rapid gas ionization.

Tungar Rectifiers.—An example of gaseous conduction is found in gas-filled rectifier bulbs, such as the Tungar tube, used for rectifying compartively large currents at low voltage, as in

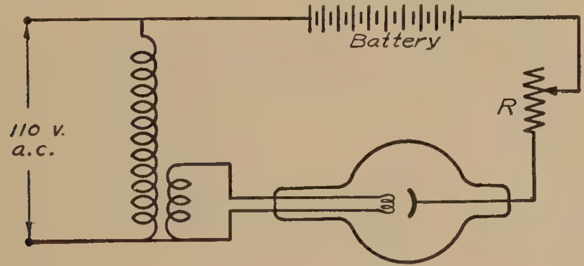


FIG. 13.—Circuit diagram Tungar rectifier.

charging storage batteries from an alternating-current power source. A circuit diagram for the Tungar rectifier is shown in Fig. 13. Tungar tubes (Fig. 14) have a heavy, high-current filament and a small circular plate. The bulb is filled with an inert gas at a low pressure. When the filament is heated electrons are given off by thermionic emission. If the voltage on the plate is in the positive half cycle the electrons will be drawn over to the plate with rapidly increasing velocities. As the

electrons travel through the tube they collide with atoms of the gas, which are broken into ions by the impact; the positive ion being attracted to the filament and the negative ion to the plate. The passage of these ions causes more of the atoms to be broken up by collision so that the gas becomes heavily ionized and transmits a large current. When the alternating-current potential on the plate is in its negative half cycle, the emitted electrons are repelled back into the filament and the ionization of the gas ceases. Because of this difference in action for the positive and negative half cycles the tube is a unidirectional conductor or rectifier.

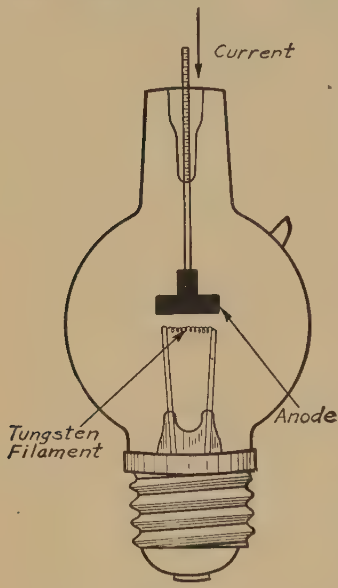


Fig. 14.—Tungar rectifier tube.

This type of tube must not be confused with the Fleming valve described previously which depends for its action upon the flow of electrons only. For this reason the Fleming valve can pass but a small current but will rectify at very high voltages. The gas-filled rectifier can transmit a very large current due to the large number of ions present during its operation but will rectify only at comparatively low voltages since a high voltage would ionize the gas independently of the thermionic emission and in that case the tube would conduct current in both directions.

The *neon lamp* is an example of gaseous conduction without thermionic emission. The lamp consists of a long tube filled with neon gas at a low pressure. Ionization is caused by applying a high potential across the terminals of the tube, sufficient to break down the atoms of the gas. As soon as an atom becomes ionized, the ions travel to their respective terminals under electrostatic attraction. By impact the ion breaks up other atoms until the tube is filled with rapidly moving ions. As each atom breaks up it gives off light of characteristic color producing a glow throughout the gas in the tube. The color of the light may be altered by using different gases in the tube.

CHAPTER XXII

INDUSTRIAL APPLICATIONS

The importance of direct currents in the application and general use of electric energy is seldom appreciated by engineering students. The huge alternating-current central stations, the extensive high-tension power-transmission systems and distribution networks, together with telephones and radio, give the impression that practically all industrial applications require alternating currents, or that from an economic point of view alternating currents are better than direct currents. Even a passing study of the present situation in the use of direct and alternating currents clearly shows such views to be erroneous. A large and very important part of electric machinery and appliances operate on direct currents. The selection of direct-current equipment is made either because the desired service specifically requires direct currents or that under the given conditions direct currents are more economical than alternating currents.

Four outstanding factors that largely affect the choice of direct- or alternating-current machinery for any specific industrial service should be kept in mind: namely, *transformers*, *storage batteries*, *magnetic fields*, and *availability*.

1. By means of the transformer the voltage of alternating-current systems, or any part of an alternating-current system, can be raised or lowered in a very convenient as well as highly economical manner. No apparatus has as yet been developed comparable to the transformer, whether based on the range of voltage ratios, efficiency or operating simplicity, for producing similar changes of voltage in direct-current systems.

2. Storage batteries deliver direct currents and likewise require direct currents for charging. The lack of similar facilities for the storage and automatic return of electric energy in alternating-current systems is in very many cases the determining factor for using direct currents.

3. Not only direct-current generators and motors but likewise alternators, synchronous motors, and rotary converters require

direct currents for field excitation. Direct currents are also necessary for lifting magnets, many types of relays and control magnets and in other appliances in which unidirectional magnetic fields are essential or desirable.

4. In places where only direct currents or only alternating currents can be obtained at reasonable cost it is evident that the apparatus must be selected so as to conform with the available power supply.

To gain a general view of the more important industrial applications of direct currents, consider the following fields or divisions: (*a*) *metropolitan direct-current areas*, (*b*) *transportation*, (*c*) *communication*, (*d*) *power*, (*e*) *illumination*, (*f*) *electrolytic processes*, (*g*) *minor applications*.

Metropolitan Direct-current Areas.—In many large cities the metropolitan area is supplied with electric power for all purposes by Edison three-wire, direct-current distribution networks, either exclusively or sharing the load with alternating-current distribution systems. For theaters, churches, large stores, large office buildings, and even for street illumination in congested districts, *continuity of service* becomes a controlling factor in the choice of systems for light and power. Direct-current distribution, with storage batteries of adequate capacity floating on the line, is generally considered as the most reliable system. The storage batteries provide the best guarantee against even momentary interruptions in the service.

Although metropolitan areas in most cities are limited to comparatively small districts, the load density is very high and as a consequence large quantities of electric energy are delivered over direct-current distribution networks.

Transportation.—In the transportation field large quantities of electric power are used, of which practically all is supplied to the motors in the form of direct currents. The choice of direct currents in the transportation field is mainly due to two factors:

(*a*) The wellnigh perfect adaptation of the direct-current series motor to the requirements of the traction load.

(*b*) The storage-battery service required for starting and for ignition purposes in internal-combustion engines.

Electrically propelled *automobiles* and trucks are few in number and therefore comparatively little electric power is used. However, for starting, ignition, and lighting purposes all automobiles and auto-trucks are provided with electrical equipment; and

the requirements can be met to best advantage by direct-current apparatus. Practically all cars are provided with a low-voltage,

FIG. 1.—New York Central railroad electrification, 600-volt, direct-current, third-rail system, showing Twentieth Century Limited drawn by 130-ton, gearless locomotive. (*General Electric Company.*)

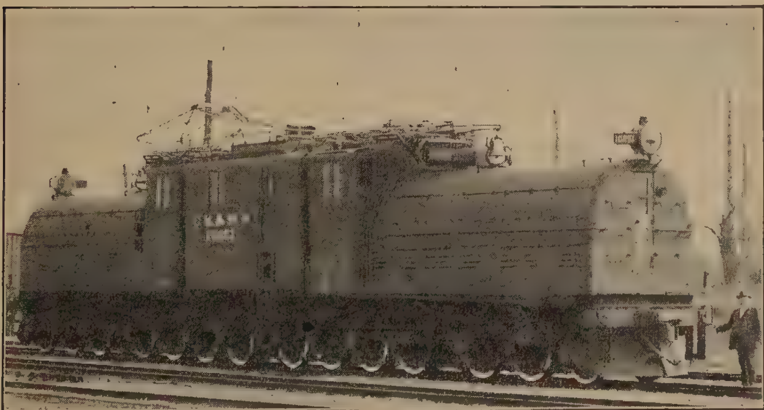


FIG. 2.—A 3,000-volt, direct-current, gearless passenger locomotive. Chicago, Milwaukee, St. Paul and Pacific R. R.

direct-current generator and a storage battery for ignition and lighting and in the majority of cases a motor for starting purposes. The combined rating of generators, battery and motor as installed

in automobiles is, for a single car, rather small, but the total installed kilowatts in the 30,000,000 or more automobiles and trucks in operation is necessarily a large figure, even in comparison to the generator capacity of large central stations.

For *elevators, hoists, cranes,* etc. direct-current series motors are greatly to be preferred. The current coming to the motor is, in many cases, supplied by a near-by direct-current generator driven by an alternating-current motor. For *high speed elevators* the motor-generator method of direct-current supply is used, even if the power is obtained from direct-current mains. More effective torque-speed regulation can be obtained by varying the

Fig. 3.—The Olympian in the Cascades. Chicago, Milwaukee, St. Paul and Pacific R. R.

generator-field excitation and thereby the voltage impressed on the elevator-motor terminals than by any method of regulation if power is supplied to the elevator motor directly from constant potential mains.

By far the more important applications of direct currents in the transportation field are to provide tractive power and illumination for *electric street cars, electric interurban trains* and *electrified sections of trunk railways.* The choice of direct currents for electric traction is based on the highly satisfactory torque-speed characteristics of the direct-current series motor in relation to the requirements of the traction load. Practically all electric street cars and interurban trains operate on direct-current series motors. The more important electrified sections of trunk rail-

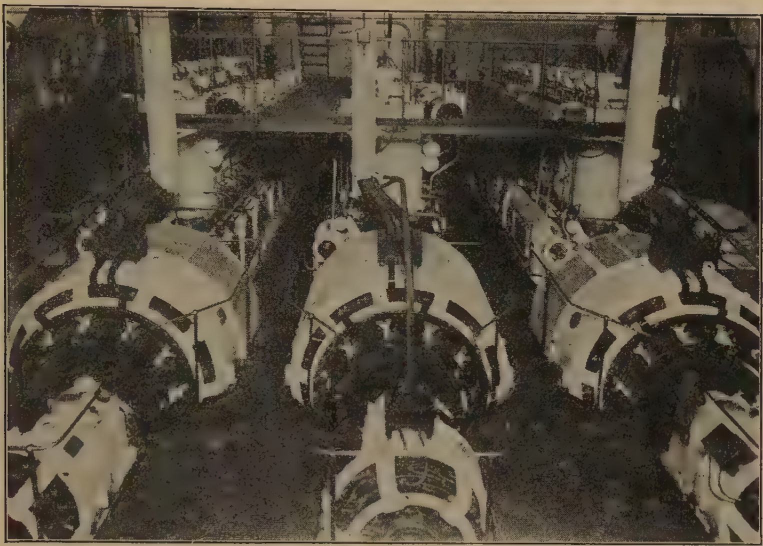


FIG. 4.—Engine room of Diesel-electric motor ship "Point Breeze," showing three 600 hp. Ingersoll-Rand Diesel engines and direct-current generators. (*General Electric Company.*)

FIG. 5.—Diesel electric motor ship Sharon, equipped with direct-current motors for propulsion and for all auxiliary purposes.

ways, as for example the 660 miles of electrified main line of the Chicago, Milwaukee, St. Paul and Pacific Railroad, likewise use

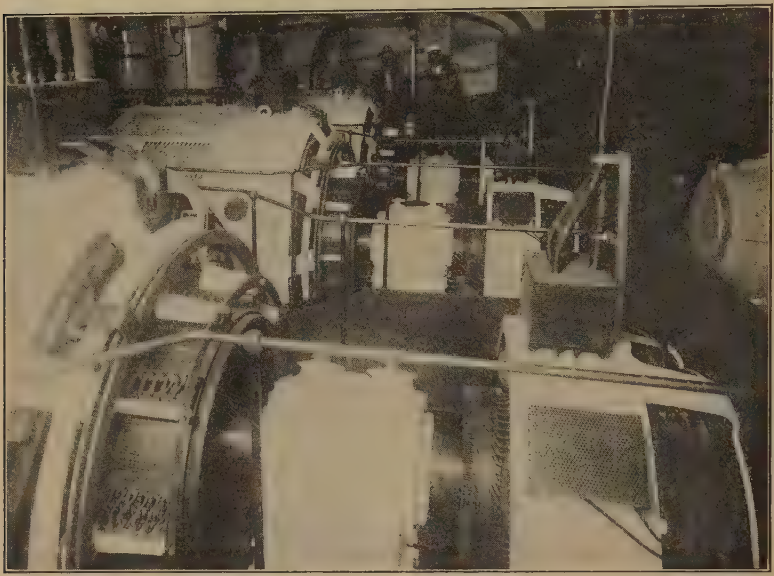


Fig. 6.—Athwartship view of engine room of motor ship Sharon, showing three 525-kw. direct-current generators and three 50-kw. exciters. (*General Electric Company.*)

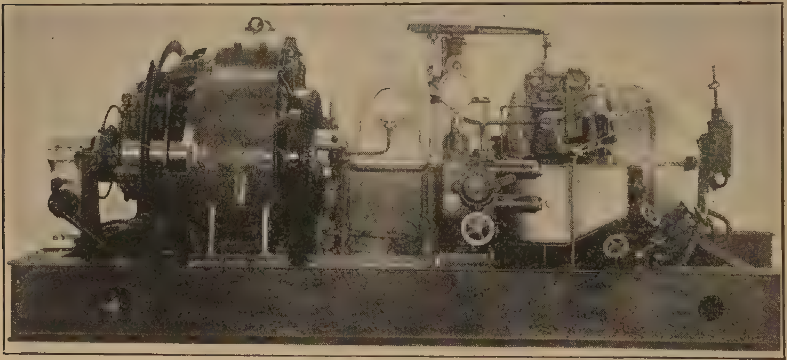


Fig. 7.—A 250-kw. auxiliary turbine-generator set for U. S. scout cruiser. (*General Electric Company.*)

direct-current series motors. The power supply in practically all cases is obtained from alternating-current generating stations over trunk transmission lines. In order to gain the torque-speed

advantages of the direct-current series motor for traction the alternating-current supply is converted into direct currents either on the locomotive itself or in substations located near the railway line. The conversion from alternating currents to direct currents is accomplished by means of motor-generator sets, rotary converters or mercury-arc rectifiers. When the direct currents pass through the series motors the electric power is converted into tractive effort that moves the train.

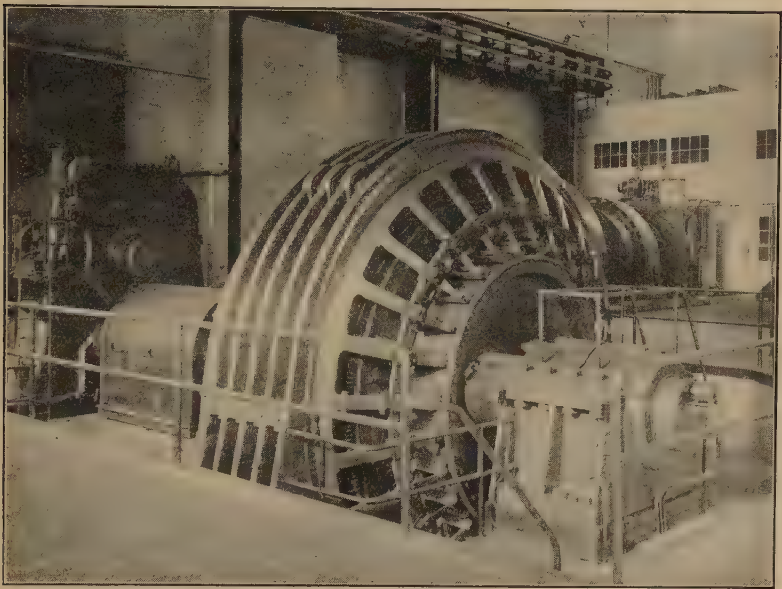


Fig. 8.—A 7,000-hp., direct-current, reversing motor driving a 54-in. blooming mill; Bethlehem Steel Company. (*General Electric Company.*)

Direct-current systems and equipment are largely used in marine installations for lighting and power. Electrically propelled ships are in many cases equipped with direct-current motors, directly connected to the propeller shaft. Storage batteries and direct-current motors and appliances are essential to the equipment of submarines and other naval vessels.

Communication.—In telegraph systems direct currents are used for transmitting the message, the power being supplied by primary cells or storage batteries. Central energy in telephone systems is obtained from storage batteries with power supplied by motor-generator sets. Similarly in radio-broad-

casting stations large storage batteries form the immediate source of electric energy.

In radio receiving sets both primary cells and storage batteries are used. The power required for each set is small but as the number of sets in operation is very large the total direct-current equipment in the radio field measured in kilowatts is a large figure. Direct currents are also used in railway signal systems, in fire-alarm systems and in other signal systems used for a

FIG. 9.—Electric shovel in strip mine of Northern Illinois Coal Corp., Wilmington, Ill. (*General Electric Company.*)

variety of purposes. In many cases direct currents are selected in order to use energy stored in primary cells or storage batteries.

Power.—Although the bulk of electric power used is generated and distributed as alternating currents, the advantages offered by direct-current motors are for many purposes sufficient to more than compensate for the cost of converting the alternating currents into direct currents before the electric power is applied to the motor load. In addition to the almost exclusive use of direct currents in electric traction over 30 per cent of the electric power required by the industries is delivered by means of direct-current motors.

Direct currents are used extensively in the iron and steel industry, both for major and minor operations. In machine

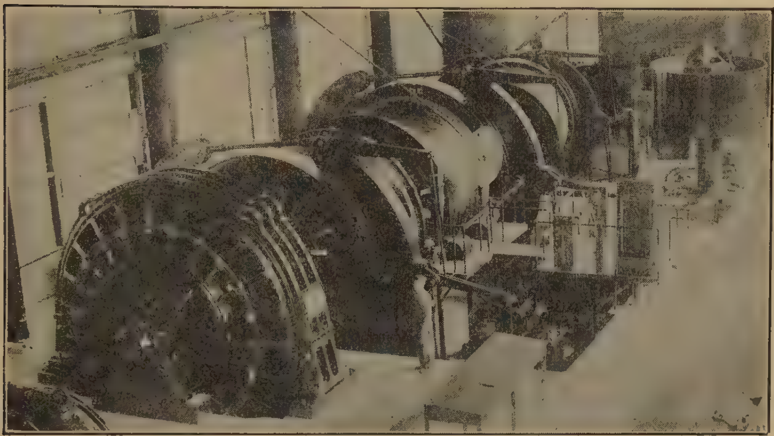


Fig. 10.—Ore hoist, Inspiration Consolidated Copper Co., Inspiration, Arizona. Direct-current motor, 215-hp., 51-r.p.m., 650-volt. (*General Electric Company.*)

shops and factories, variable speed direct-current motors are widely used. Machine tools which are essential in very many

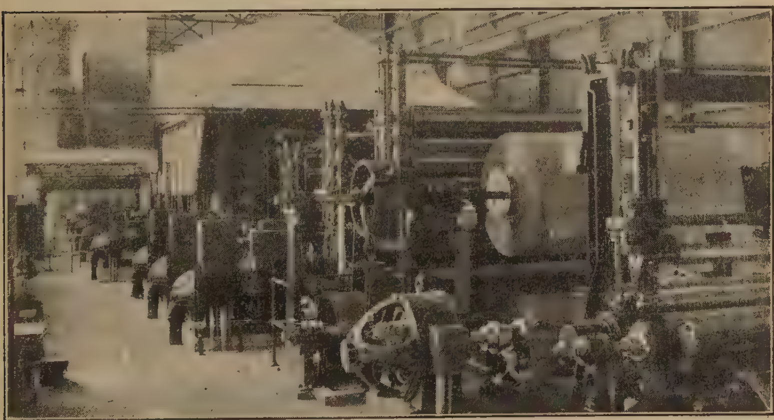


Fig. 11.—Direct-current motors in sectional paper machine drive using variable voltage for speed regulation. National Paper Products Co., Port Townsend, Wash. (*Western Electric and Manufacturing Co.*)

industries are largely equipped with direct-current motors. Elevators, from high-speed passenger service to slow-speed,

heavy-duty, freight equipment, are in most cases operated by direct-current motors. Material handling machinery in all

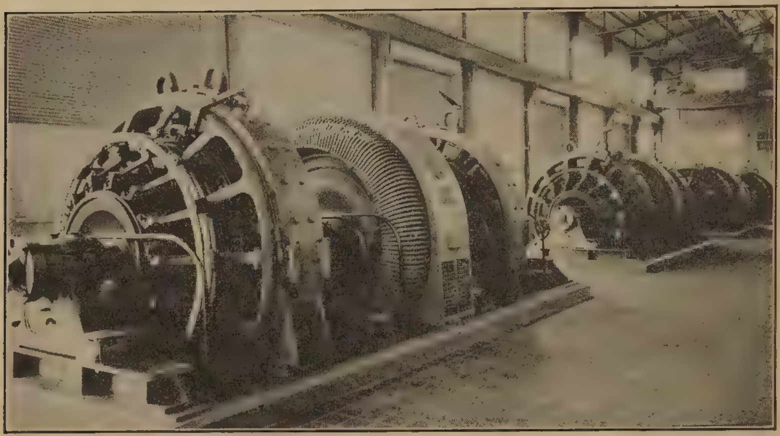


Fig. 12.—Four motor-generator sets for electrolytic service. Each set has two shunt-wound direct-current generators, each rated at 2,750 kw., 360 r.p.m. 500/550 volts. Anaconda Copper Co., Anaconda, Mont. (*General Electric Company.*)

fields requires flexibility in speed and torque which can to best advantage be met by direct-current motors. Likewise, in mining,

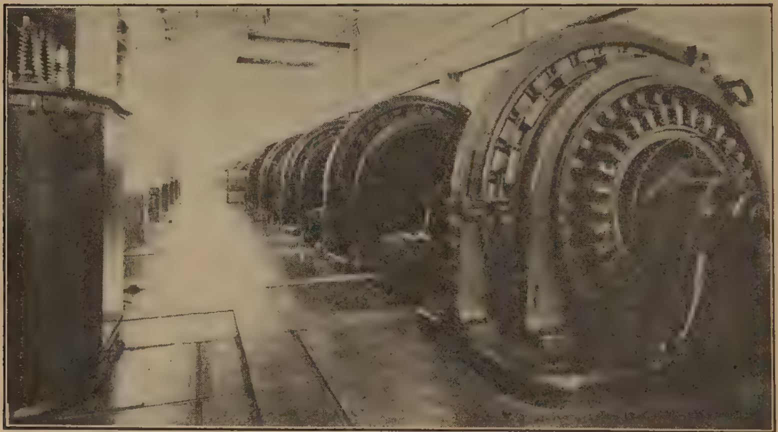


Fig. 13.—Nine synchronous converters for electrolytic service, each converter rated 5,800 kva., at 580 volts. Anaconda Copper Mining Co., Great Falls, Mont. (*General Electric Company.*)

direct-current motors are used for cutting, hauling, hoisting, and other purposes.

In the paper industry direct-current motors are indispensible. The motors that operate the successive sections through which the paper ribbon (in some cases 25 feet wide) passes, must work without fail on most exacting speed schedules. Precise speed regulation is of prime importance for satisfactory operation of a paper mill.

Illumination.—The incandescent lamp operates satisfactorily on either direct or alternating currents and hence other factors than the tungsten lamp determine the choice between alternating- and direct-current systems. In cities outside metropolitan

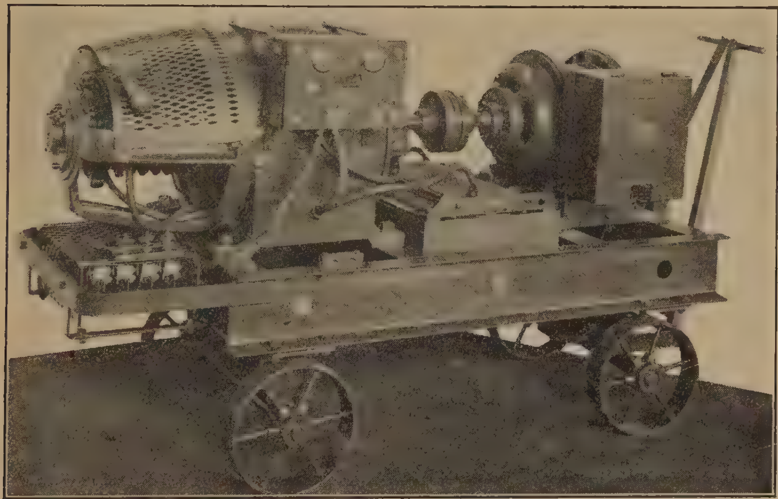


Fig. 14.—Portable, arc-welding set. 300 amp., direct current.

areas direct-current power for illumination is practically limited to street lighting by magnetite-arc lamps; the lighting of drafting rooms, shops, photographic studios, etc., by mercury-arc lamps; lighting of street cars, railway trains, automobiles, ships having direct-current equipment, on farms and other isolated small power units that generally use storage batteries as accessory equipment.

Electrolytic Processes.—All electrolytic processes require direct currents. The voltage used is generally low, but the currents are very large so that the power in kilowatts is large. Electrolytic reduction of zinc, the refining of copper and other metals, electroplating and electrotyping, all require direct currents for operation. The applications are many and for a variety of pur-

poses, but direct currents are necessary for all electrolytic processes.

Applications.—The minor applications of direct currents are many, of which only three will be mentioned.

(*a*) Arc welding, which is rapidly increasing in magnitude and economic importance, can best be done by the use of direct currents.

(*b*) Dust and smoke precipitation by the Cottrell process requires high-voltage direct currents.

(*c*) Control relays, used for a great variety of purposes operate to best advantage on direct currents.

Even the above brief outline of industrial applications gives ample evidence of the great importance of direct currents in the industries; that the study of direct-current phenomena is an end in itself and not merely an introduction to the study of alternating currents; that the basic laws and principles of direct currents form a large and very important division in the application of electric energy for practical purposes.

APPENDIX

TREATMENT FOR ELECTRICAL SHOCK

(Issued by the N.E.L.A.)

An accidental electrical shock usually does not kill at once, but may only stun the victim and for a while stop the breathing.

The shock is not likely to be immediately fatal, because:

1. The conductors may make only a brief and imperfect contact with the body.
2. The skin, unless it is damp with perspiration or wet, offers some resistance to the current.

The life of the victim depends upon the prompt and continued use of artificial respiration. The reasons for this are:

1. The body continuously depends on an exchange of air, as shown by the fact that we must breathe in and out about fifteen times a minute.
2. If the body is not thus repeatedly supplied with air, suffocation occurs.
3. Persons whose breathing has been stopped by electrical shock have been reported restored after artificial respiration has been continued for approximately 4 hr., and the treatment should be continuously applied until rigor mortis (stiffening of the body due to death) sets in.

The Schaefer, or Prone-pressure, method of artificial respiration, *slightly modified, is illustrated and described in the following resuscitation rules.* The advantages of this method are:

1. It is immediately available.
2. Easy performance; no apparatus and little muscular exertion required.
3. Larger ventilation of the lungs than by the supine method.
4. Simplicity: the operator makes no complex motions and readily learns the method.
5. No trouble from the tongue falling back into the air passages. The first impulse is expiration and any foreign substance in the mouth or air passage will probably be expelled.

6. No risk of injury to the liver or ribs if the method is executed with proper care.

Aid can be rendered best by one who has studied the rules and has learned them by practice on a volunteer subject.

INTRUCTIONS FOR RESUSCITATION

Follow These Instructions Even if Vicim Appears Dead

I. Free the Victim from the Current Immediately.—1. Quickly release the victim from the current, being very careful to avoid receiving a shock. Use any dry non-conductor (rubber gloves, clothing, wood, rope, etc.) to move either the victim or the conductor. Beware of using metal or any moist material. If both of the victim's hands are grasping live conductors, endeavor to free them one at a time. If necessary shut off current.

Begin at once to get the subject to breathe (resuscitation), for a moment of delay is serious. Use "Prone-pressure method" for four (4) hr. if necessary, or until a doctor has advised that *rigor mortis* has set in.

Observe the Following Precautions.—(*a*) The victim's loose clothing, if dry, may be used to pull him away; do not touch the soles or heels of his shoes while he remains in contact—the nails are dangerous. If this is impossible, use rubber gloves, a dry coat, a dry rope, a dry stick or board, or any other *dry non-conductor* to move either the victim or the conductor, so as to break the electrical contact.

(*b*) If the bare skin of the victim must be touched by your hands, be sure to cover them with rubber gloves, mackintosh, rubber sheeting, or dry cloth; or stand on a dry board or on some other dry insulating surface. If possible, use only *one* hand.

If the man receives a shock while on a pole, first see that his belt is secure around the pole, if possible above crossarm so victim will not fall, then break the current. Pass a hand-line under his arms, preferably through his body belt, securely knot it, and pass the end of the line over the first crossarm above the victim. If you are alone, pass the line once around this crossarm. If you are not alone, drop the line to those at the base of the pole. As soon as the rope is taut, free the victim's safety belt and spurs and descend the pole, guiding the victim.

2. Open the nearest switch, if that is the quickest way to break the circuit.

3. If necessary to cut a live wire, use an ax or a hatchet with a dry wooden handle, turning your face away to protect it from electrical flash.

II. **Attend Instantly to Victim's Breathing.**—1. As soon as the victim is clear of the live conductor, quickly feel with your finger in his mouth and throat and remove any foreign body (tobacco, false teeth, etc.). If the mouth is tightly shut, pay no attention to the above-mentioned instructions until later, but immediately begin resuscitation. The patient will breathe through his nose and after resuscitation has been carried on a short time, the jaws will probably relax, and any foreign substance in the mouth can then be removed. Do not stop to loosen the patient's clothing; *every moment of delay is serious.*

2. Lay the patient on his belly, one arm extended directly overhead, the other arm bent at the elbow and with the face resting on hand or forearm so that the nose and mouth are free for breathing. (See Fig. 1.)

3. Kneel, straddling the patient's hips, with the knees just below the patient's hip bones or opening of pants pockets. Place the palms of the hands on the small of the back with fingers resting on the ribs, the little finger just touching the lowest rib, the thumb alongside of the fingers, the tips of the fingers just out of sight. (See Fig. 1.)

4. With arms held straight, swing forward slowly so that the weight of your body is gradually brought to bear upon the subject (see Fig. 2). This operation, which should take from two to three seconds, *must not be violent*—internal organs may be injured. The lower part of the chest and also the abdomen are thus compressed, and air is forced out of the lungs, the diaphragm is kept in natural motion, other organs are massaged, and the circulation of the blood accelerated.

5. Now *immediately* swing backward so as to completely remove the pressure, thus returning to the position shown in Fig. 3. Through their elasticity, the chest walls expand, and the pressure being removed the diaphragm descends, and the lungs are thus supplied with fresh air.

6. After two seconds swing forward again. Thus repeat deliberately twelve to fifteen times a minute the double movement of compression and release—a complete respiration in four or five seconds. If a watch or a clock is not visible, follow the natural rate of your own deep breathing; the proper rate may be

FIG. 1.

FIG. 2.

FIG. 3.

determined by counting—swinging forward with each expiration and backward with each inspiration.

7. As soon as this artificial respiration has been started and while it is being continued, an assistant should loosen any tight clothing about the patient's neck, chest, or waist. *Keep the patient warm.* Place ammonia near the nose, determining safe distance by first trying how near it may be held to your own. Do not give any liquids whatever by mouth until the patient is fully conscious.

8. Continue artificial respiration without interruption (if necessary, for four hours) until natural breathing is restored. Cases are on record of success after three and one-half hours of effort. The ordinary tests for death are not conclusive in cases of electric shock and doctors must be so advised by *you*, if necessary.

9. When the patient revives, he should be kept lying down and not allowed to get up or be raised under any consideration unless on the advice of a doctor. If the doctor has not arrived by the time the patient has revived, he should be given some stimulant, such as one teaspoonful of aromatic spirits of ammonia in a small glass of water, or a drink of hot ginger tea or coffee.

The patient should then have any other injuries attended to and be kept warm, being placed in the most comfortable position.

10. Resuscitation should be carried on at the nearest possible point to where the patient received his injuries. He should not be moved from this point until he is breathing normally of his own volition, and then moved only in a lying position. Should it be necessary, due to extreme weather conditions, etc., to move the patient before he is breathing normally, he should be kept in a prone position and placed upon a hard surface (door or shutter) or on the floor of a conveyance, resuscitation being carried on during the time that he is being moved.

11. A brief return of spontaneous respiration is not a certain indication for terminating the treatment. Not infrequently, the patient, after a temporary recovery of respiration, stops breathing again. The patient must be watched, and if normal breathing stops, artificial respiration should be resumed at once.

III. **Send for a Doctor.**—If other persons are present when an accident occurs, send one of them for a doctor without a moment's delay. If alone with the patient, do not neglect the immediate and continued resuscitation of the patient for at least one hour before calling a doctor to assist in further resuscitation efforts.

IV. **First Care of Burns.**—When natural respiration has been restored, burns, if serious should be immediately attended to while waiting for the doctor to arrive.

A raw or blistered surface should be protected from the air. If clothing sticks, do not peel it off—cut around it. The adherent cloth, or a dressing of cotton or other soft material applied to the burned surface, should be saturated with picric acid (0.5 per cent). If this is not at hand, use a solution of baking soda (one teaspoonful to a pint of water), or the wound may be coated with a paste of flour and water, or it may be protected with vaseline, carron oil, olive oil, castor oil, or machine oil if clean. Cover the dressing with cotton, gauze, lint, clean waste, clean handkerchief, or any other soft cloth, held tightly in place by a bandage.

The same coverings should be lightly bandaged over a dry, charred burn, but without wetting the burned region or applying oil to it.

Do not open blisters.

INDEX

53
72
89
83

297 | 79.25 × 0.7 = 51.975
28
17
16
10

51.975
9
60.975
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$\frac{9 \times 100}{20}$ = 45% } 60